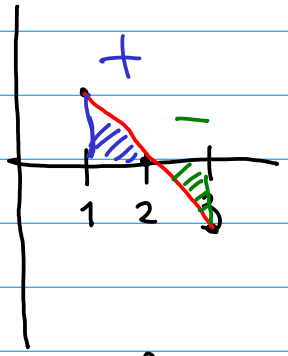
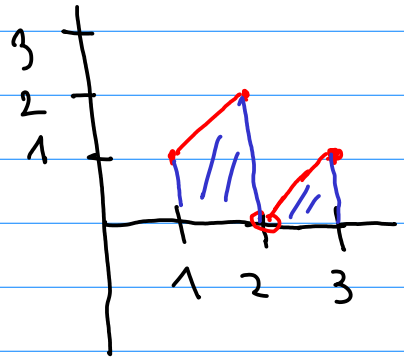
3.1.1. a) $f(x) = 2 - x$



$$\int_1^3 f(x)\,dx = \text{área azul} - \text{área verde} = \frac{1\cdot 1}{2} - \frac{1\cdot 1}{2} = 0$$

d) $$f(x) = \begin{cases} x & 1 \le x \le 2 \\ x-2 & 2 < x \le 3 \end{cases}$$



$$\int_1^3 f(x)\,dx = 1 + \frac{1}{2} + \frac{1}{2} = 2$$

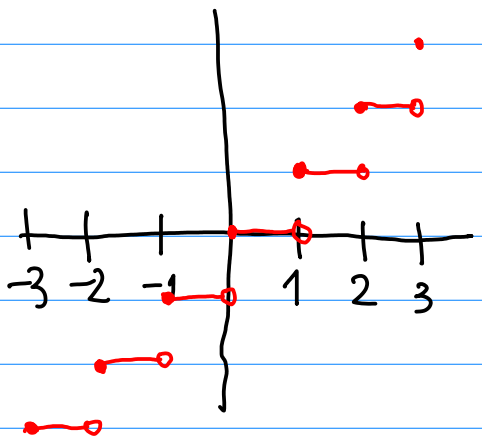
$\lfloor x \rfloor$ = parte entera de x

$= \max\{n \in \mathbb{Z} : n \le x\}$

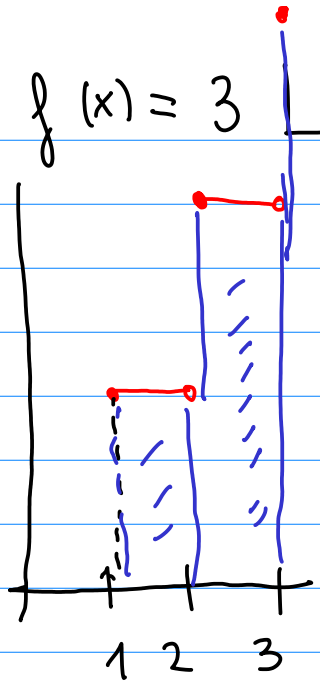
Ejemplos $\lfloor \frac{1}{2} \rfloor = 0$ $\lfloor \pi \rfloor = 3$

$\lfloor -\frac{1}{2} \rfloor = -1$ $\lfloor 45 \rfloor = 45$

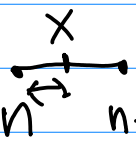
Gráfica



h) $f(x) = 3\lfloor x \rfloor$ $\int_1^3 f(x)\,dx = 3+6 = 9$



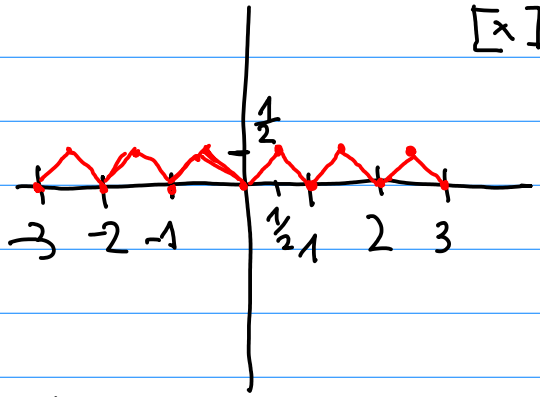
$[x]$ = distancia al entero más cercano
$= |x-n|$ donde n es el entero más cercano



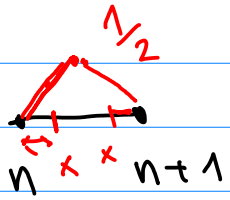
$n \in \mathbb{Z}$

Ejemplo $[2] = 0$ $\left[\frac{1}{2}\right] = \frac{1}{2}$, $\left[\frac{3}{4}\right] = \frac{1}{4}$

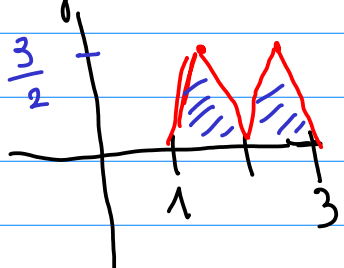
Gráfico



$$[x] = \begin{cases} x & x \in [0, \frac{1}{2}) \\ 1-x & x \in [\frac{1}{2}, 1) \\ x-1 & x \in [1, \frac{3}{2}) \\ 2-x & x \in [\frac{3}{2}, 2) \end{cases}$$

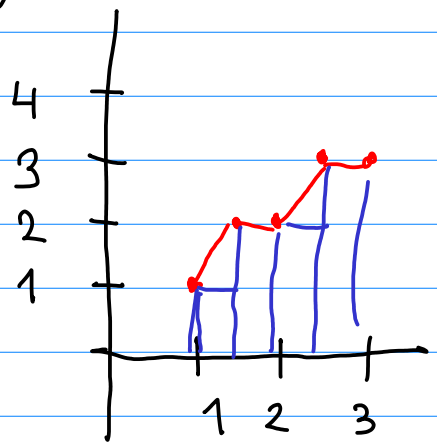


i) $f(x) = 3[x]$ $\int_1^3 f(x)\,dx = \frac{3}{2}$



$= 2 \cdot \frac{1 \cdot \frac{3}{2}}{2}$

j) $f(x) = x + [x]$ $\quad f(1) = 1$

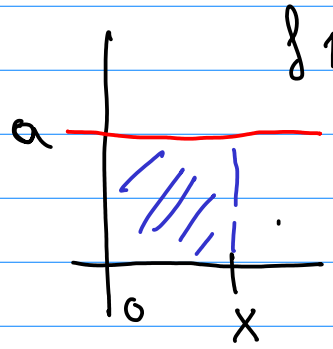


$f\left(\frac{3}{2}\right) = \frac{3}{2} + \frac{1}{2} = 2$

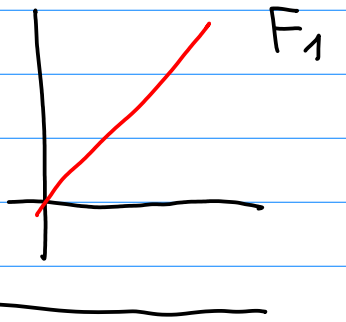
$$\int_1^3 f(x)\,dx = \frac{1}{2} + 1 + 1 + \frac{3}{2}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{2}$$

5.



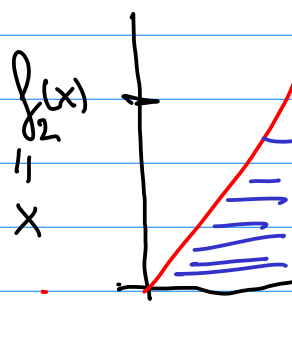
$$F_1(x) = \int_0^x f_1(t)\,dt = a \cdot x$$

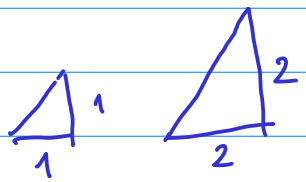


$$F_2(x) = \int_0^x f_2(t)\,dt =$$

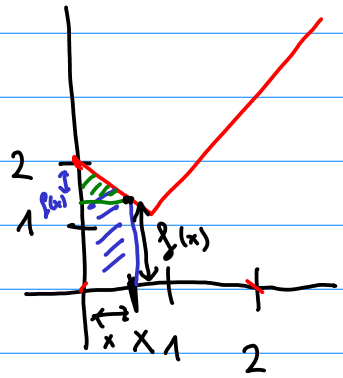
área del triangulo de base x y altura $f_2(x)$

$$= \frac{1}{2} x^2$$





6. $f_1(t) = \max\{t, 2-t\}$

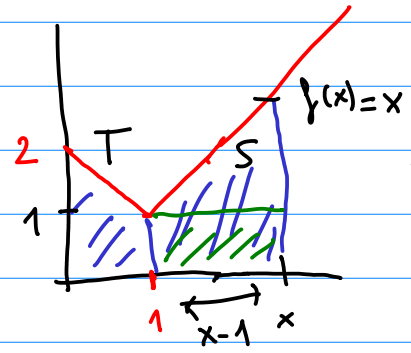


$$F(x) = \int_0^x f_1(t)\,dt$$

Caso $x \in [0,1]$, $f_1(x) = 2-x$

$F(x) = \int_0^x f_1(t)\,dt =$ área del trapezoide azul $= x \cdot \underbrace{(2-x)}_{f_1(x)} + \frac{x\cdot(2-f_1(x))}{2}$

$= x(2-x) + \frac{x\cdot x}{2} = 2x - x^2 + \frac{x^2}{2} = 2x - \frac{x^2}{2}$



Caso $x > 1$

$F(1)$

$F(x) = \text{área}(T) + (x-1)\cdot 1 + \frac{(x-1)\cdot(x-1)}{2}$

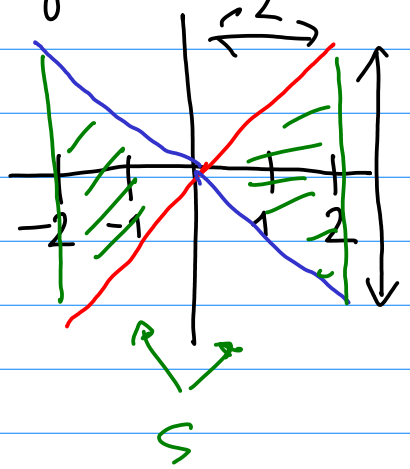
$= \frac{3}{2} + x - 1 + \frac{(x-1)^2}{2}$

$= \frac{1}{2} + x + \frac{x^2 - 2x + 1}{2} =$

$= 1 + \frac{x^2}{2}$

$$F(x) = \begin{cases} 2x - \frac{x^2}{2} & x \in [0,1] \\ 1 + \frac{x^2}{2} & x \in (1, +\infty) \end{cases}$$

7.a) $f(x) = x$, $g(x) = -x$ en $[-2, 2]$



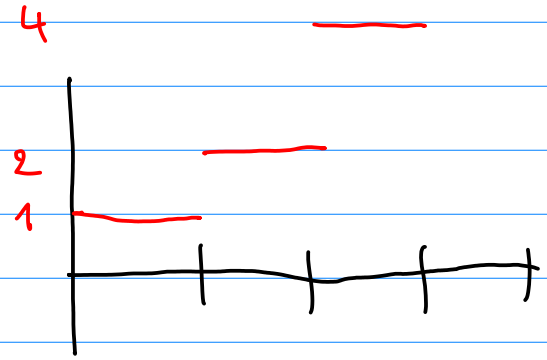
$$\text{área}(S) = 2 \cdot \frac{4 \cdot 2}{2} = 8$$

$\lfloor x \rfloor \in \mathbb{Z}$ $\quad$ $k \in \mathbb{Z}$

$\sin(\pi \cdot k) = 0$

$$\int_0^4 \overline{\sin(\pi \lfloor x \rfloor)}\, dx = \int_0^4 0\, dx = 0$$

c)



$2^{\lfloor x \rfloor}$