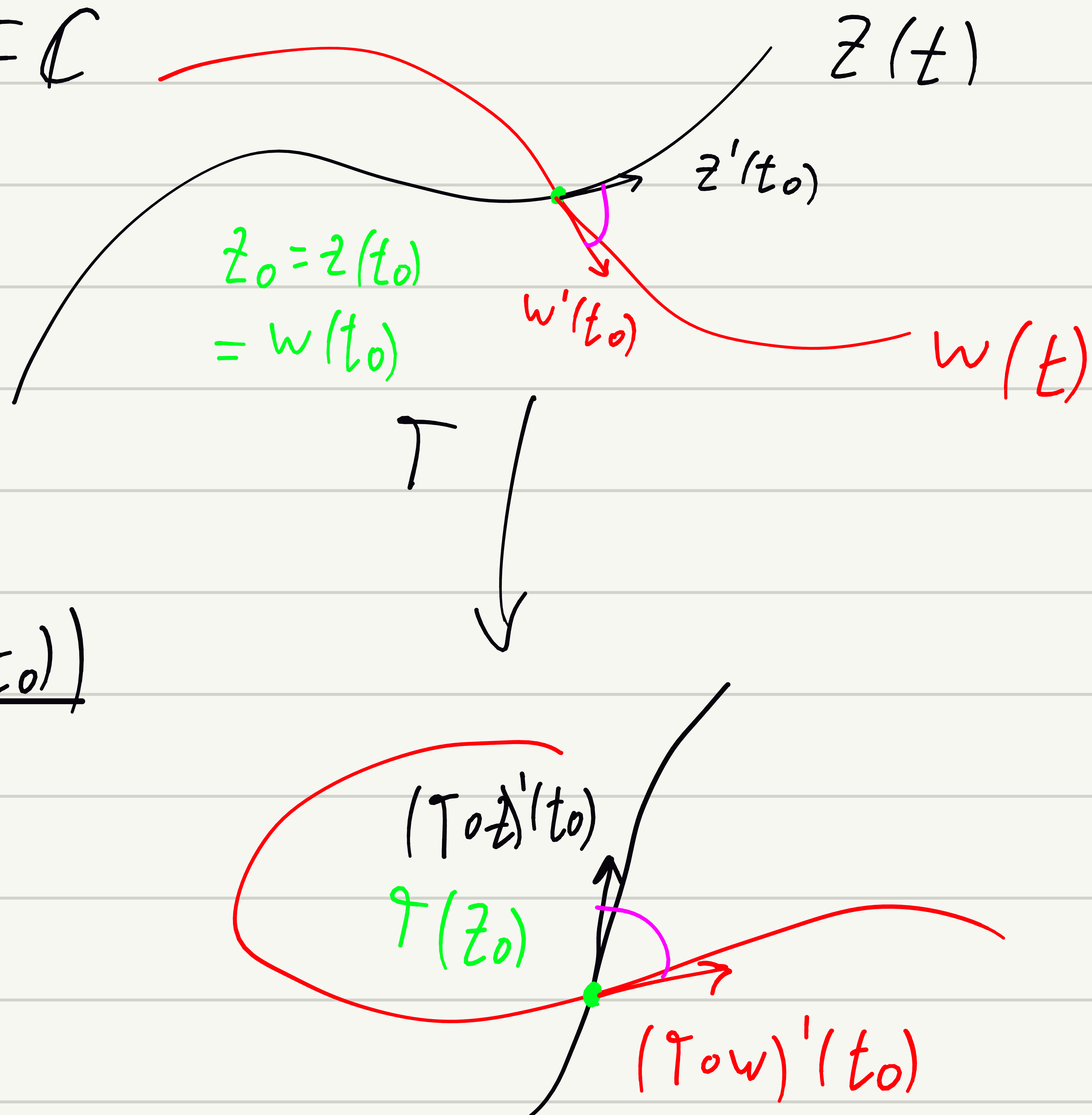
$z, w : I \subseteq \mathbb{R} \to \Omega \subseteq \mathbb{R}^2 = \mathbb{C}$

$T : \Omega \subseteq \mathbb{C} \to \mathbb{C}$

$T \circ z : I \to \mathbb{C}$



$$(T\circ z)'(t_0) = \lim_{t\to t_0} \frac{T(z(t)) - T(z(t_0))}{t - t_0}$$

Producto por escalar y resta de $\mathbb{R}^2$

$$= \lim_{t\to t_0} \frac{T(z(t)) - T(z(t_0))}{t - t_0} \cdot \frac{z(t) - z(t_0)}{z(t) - z(t_0)}$$

Producto complejo

$$= \lim_{t\to t_0} \frac{T(z(t)) - T(z(t_0))}{z(t) - z(t_0)} \cdot \frac{z(t) - z(t_0)}{t - t_0}$$

$$= \lim_{t\to t_0} \frac{T(z(t)) - T(z(t_0))}{z(t) - z(t_0)} \cdot \lim_{t\to t_0} \frac{z(t) - z(t_0)}{t - t_0}$$

$$= \lim_{u\to z_0} \frac{T(u) - T(z_0)}{u - z_0} \cdot \lim_{t\to t_0} \frac{z(t) - z(t_0)}{t - t_0} = T'(z(t_0)) \cdot z'(t_0)$$

$$(T\circ w)'(t_0) = T'(w(t_0)) \cdot w'(t_0) = T'(z(t_0)) \cdot w'(t_0)$$

Ambos rotan por $\arg(T'(z(t_0)))$

3. Hallar una transformación de Möbius g tal que: 1 e i son puntos fijos y $g(0) = -1$

$$g(1) = \frac{a+b}{c+d} = 1 \Rightarrow a+b = c+d \Rightarrow d = a+b-c$$

$$g(z) = \frac{az+b}{cz+a+b-c} \Rightarrow g(i) = \frac{ai+b}{ci+a+b-c} = i \Rightarrow \cancel{a}i+b = (\cancel{a}+b-c)i - c$$

$$\frac{z}{\bar{z}}, \quad z = \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$\Rightarrow b(1-i) = -c(1+i) \Rightarrow b = -c\,\frac{1+i}{1-i} = -ci$$

$$\Rightarrow g(z) = \frac{az - ci}{cz + a - ci - c} \Rightarrow g(0) = \frac{-ci}{a-ci-c} = -1$$

$$\Rightarrow ci = a - ci - c \Rightarrow c(2i+1) = a$$

$$\Rightarrow g(z) = \frac{c(2i+1)z - ci}{cz + c(2i+1) - c(i+1)} = \frac{(2i+1)z - i}{z+i}$$