FENOMENOS DE TRANSPORTE EN INGENIERÍA DE PROCESOS

Material Para La Realización De Controles Y Exámenes

PRIMER PARCIAL

Operadores en coordenadas rectangulares

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z}$$

$$\nabla \rho = \vec{\delta}_x \frac{\partial \rho}{\partial x} + \vec{\delta}_y \frac{\partial \rho}{\partial y} + \vec{\delta}_z \frac{\partial \rho}{\partial z}$$

$$\nabla \bullet \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Ecuaciones de variación

TABLA 3.4—1

LA ECUACIÓN DE CONTINUIDAD EN DISTINTOS SISTEMAS COORDENADOS

Coordenadas rectangulares (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (A)$$

Coordenadas cilíndricas (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (B)$$

Coordenadas esféricas (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \operatorname{sen}\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \operatorname{sen}\theta) + \frac{1}{r \operatorname{sen}\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad (C)$$

TABLA 3.4—2

LA ECUACIÓN DE MOVIMIENTO EN COORDENADAS RECTANGULARES (x, y, z)

En función de $\boldsymbol{\tau}$:

componente x
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x \qquad (A)$$

componente y
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho g_y \qquad (B)$$

componente z
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \qquad (C)$$

En función de los gradientes de velocidad para un fluido newtoniano de ρ y μ constantes:

componente x
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x \qquad (D)$$

componente y
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y \qquad (E)$$

componente z
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z \qquad (F)$$

TABLA 3.4—3

LA ECUACIÓN DE MOVIMIENTO EN COORDENADAS CILÍNDRICAS (r, θ, z)

En función de $\boldsymbol{\tau}$:

componente r^a

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r \qquad (A)$$

componente θ^b

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta \qquad (B)$$

componente z

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \qquad (C)$$

En función de los gradientes de velocidad para un fluido newtoniano de ρ y μ constantes:

componente r^a

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \qquad (D)$$

componente θ^b

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \qquad (E)$$

componente z

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (F)$$

LA ECUACIÓN DE MOVIMIENTO EN COORDENADAS ESFÉRICAS (r, θ, ϕ)

En función de τ:

Spherical coordinates (r, θ, ϕ):[c]

Componente r

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \tau_{rr}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r} \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r \quad \text{(B.5-7)}$$

Componente θ

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3 \tau_{r\theta}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta} \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right] + \rho g_\theta \quad \text{(B.5-8)}$$

Componente ϕ

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}\right) = -\frac{1}{r \sin\theta}\frac{\partial p}{\partial \phi} - \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3 \tau_{r\phi}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi} \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi \quad \text{(B.5-9)}$$

[c] These equations have been written without making the assumption that $\boldsymbol{\tau}$ is symmetric. This means, for example, that when the usual assumption is made that the stress tensor is symmetric, $\tau_{r\theta} - \tau_{\theta r} = 0$.

En función de los gradientes de velocidad para un fluido newtoniano de ρ y μ constantes:

Spherical coordinates (r, θ, ϕ):

Componente r

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r \quad \text{(B.6-7)}^a$$

Componente θ

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta)\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2 \sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta \quad \text{(B.6-8)}$$

Componente ϕ

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}\right) = -\frac{1}{r \sin\theta}\frac{\partial p}{\partial \phi} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi \sin\theta)\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2 \sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2 \sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi \quad \text{(B.6-9)}$$

[a] The quantity in the brackets in Eq. B.6-7 is *not* what one would expect from Eq. (M) for $[\nabla \cdot \nabla \mathbf{v}]$ in Table A.7-3, because we have added to Eq. (M) the expression for $(2/r)(\nabla \cdot \mathbf{v})$, which is zero for fluids with constant ρ. This gives a much simpler equation.

TABLA 3.4–5

COMPONENTES DEL TENSOR ESFUERZO EN COORDENADAS RECTANGULARES (x, y, z)

$$\tau_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{yy} = -\mu\left[2\frac{\partial v_y}{\partial y} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{xy} = \tau_{yx} = -\mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right] \qquad (D)$$

$$\tau_{yz} = \tau_{zy} = -\mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right] \qquad (E)$$

$$\tau_{zx} = \tau_{xz} = -\mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \qquad (G)$$

TABLA 3.4—6

COMPONENTES DEL TENSOR ESFUERZO EN COORDENADAS CILÍNDRICAS

(r, θ, z)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \tfrac{2}{3}(\nabla \cdot v)\right] \tag{A}$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \tag{B}$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \tfrac{2}{3}(\nabla \cdot v)\right] \tag{C}$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \tag{D}$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right] \tag{E}$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right] \tag{F}$$

$$(\nabla \cdot v) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \tag{G}$$

TABLA 3.4–7

COMPONENTES DEL TENSOR ESFUERZO EN COORDENADAS ESFÉRICAS

(r, θ, ϕ)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{\phi\phi} = -\mu\left[2\left(\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right) - \frac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad (D)$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\mathrm{sen}\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\mathrm{sen}\,\theta}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi}\right] \qquad (E)$$

$$\tau_{\phi r} = \tau_{r\phi} = -\mu\left[\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}(v_\theta\,\mathrm{sen}\,\theta) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} \qquad (G)$$

Balances macroscópicos en sistemas isotérmicos

Balance de masa: $\frac{\partial m}{\partial t} = -\Delta\omega \quad$ con $\omega = \rho < v > A_{flujo}$

Balance de cantidad de movimiento: $\frac{\partial \vec{p}}{\partial t} = -\Delta\left(\frac{< v^2 >}{< v >}\vec{\omega} + P\vec{S}\right) + m_{TOTAL}\vec{g} - \vec{F}$

Balance de energía mecánica general en estado estacionario: $\Delta\left[\left(\frac{1}{2}\frac{< v^3 >}{< v >} + gh + \frac{P}{\rho}\right)\omega\right] + E_v + W = 0$

Balance de energía mecánica con una sola entrada y una sola salida en estado estacionario:

$$\frac{1}{2}\Delta\left(\frac{< v^3 >}{< v >}\right) + g\Delta h + \frac{\Delta P}{\rho} + \hat{E}_v + \hat{W} = 0$$

Capa límite hidrodinámica sobre una placa plana

RÉGIMEN LAMINAR	RÉGIMEN TURBULENTO
$\delta_H = \frac{5\cdot x}{\sqrt{Re_x}}$	$\delta_H = \frac{0.376\cdot x}{{Re_x}^{1/5}}$
$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$	$C_{fx} = \frac{0.0575}{{Re_x}^{1/5}}$
$C_{fm} = \frac{1.328}{\sqrt{Re_L}}$	$C_{fm} = \frac{1}{L}\left[\frac{1.328 x_c}{\sqrt{Re_{x_c}}} + 0.072\left(\frac{\mu}{\rho v_\infty}\right)^{1/5}\left(L^{4/5} - {x_c}^{4/5}\right)\right]$ Si la capa límite laminar es despreciable: $C_{fm} = \frac{0.072}{{Re_L}^{1/5}}$

Factor de fricción en tubos

RÉGIMEN LAMINAR	RÉGIMEN TURBULENTO
$C_f = \frac{16}{Re}$	$C_f = \frac{0.0791}{Re^{1/4}}$

Segundo Parcial

Ecuaciones de variación de energía térmica

TABLA 10.2–2

LA ECUACIÓN DE ENERGÍA EN FUNCIÓN DE LAS DENSIDADES DE FLUJO DE ENERGÍA Y DE CANTIDAD DE MOVIMIENTO

Coordenadas rectangulares:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right]$$
$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{xx} \frac{\partial v_x}{\partial x} + \tau_{yy} \frac{\partial v_y}{\partial y} + \tau_{zz} \frac{\partial v_z}{\partial z} \right\}$$
$$- \left\{ \tau_{xy} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) + \tau_{xz} \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) + \tau_{yz} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \right\} \quad (A)$$

Coordenadas cilíndricas:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{1}{r} \frac{\partial}{\partial r} (r q_r) + \frac{1}{r} \frac{\partial q}{\partial \theta} + \frac{\partial q_z}{\partial z} \right]$$
$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \frac{1}{r} \left(\frac{\partial v_\theta}{\partial \theta} + v_r \right) \right.$$
$$\left. + \tau_{zz} \frac{\partial v_z}{\partial z} \right\} - \left\{ \tau_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] + \tau_{rz} \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \right.$$
$$\left. + \tau_{\theta z} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right) \right\} \quad (B)$$

Coordenadas esféricas:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \operatorname{sen} \theta} \frac{\partial T}{\partial \phi} \right) = -\left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 q_r) \right.$$
$$\left. + \frac{1}{r \operatorname{sen} \theta} \frac{\partial}{\partial \theta} (q_\theta \operatorname{sen} \theta) + \frac{1}{r \operatorname{sen} \theta} \frac{\partial q_\phi}{\partial \phi} \right] - T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) \right.$$
$$\left. + \frac{1}{r \operatorname{sen} \theta} \frac{\partial}{\partial \theta} (v_\theta \operatorname{sen} \theta) + \frac{1}{r \operatorname{sen} \theta} \frac{\partial v_\phi}{\partial \phi} \right) - \left\{ \tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) \right.$$
$$\left. + \tau_{\phi\phi} \left(\frac{1}{r \operatorname{sen} \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right) \right\} - \left\{ \tau_{r\theta} \left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} \right) \right.$$
$$\left. + \tau_{r\phi} \left(\frac{\partial v_\phi}{\partial r} + \frac{1}{r \operatorname{sen} \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r} \right) + \tau_{\theta\phi} \left(\frac{1}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{1}{r \operatorname{sen} \theta} \frac{\partial v_\theta}{\partial \phi} - \frac{\cot \theta}{r} v_\phi \right) \right\} \quad (C)$$

TABLA 10.2—3

LAS ECUACIONES DE ENERGÍA EN FUNCIÓN DE LAS PROPIEDADES DE TRANSPORTE

(para fluidos newtonianos de ρ, μ y k constantes; obsérvese que la constancia de ρ implica que $\hat{C}_v = \hat{C}_p$)

Coordenadas rectangulares:

$$\rho\hat{C}_v\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right]$$
$$+ 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2\right.$$
$$\left. + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \quad (A)$$

Coordenadas cilíndricas:

$$\rho\hat{C}_v\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right]$$
$$+ 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left[\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)\right]^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)^2\right.$$
$$\left. + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)^2 + \left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]^2\right\} \quad (B)$$

Coordenadas esféricas:

$$\rho\hat{C}_v\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\,\mathrm{sen}\,\theta}\frac{\partial T}{\partial \phi}\right) = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right.$$
$$\left. + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\mathrm{sen}^2\,\theta}\frac{\partial^2 T}{\partial \phi^2}\right] + 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2\right.$$
$$\left. + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)^2\right\}$$
$$+ \mu\left\{\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2\right.$$
$$\left. + \left[\frac{\mathrm{sen}\,\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\mathrm{sen}\,\theta}\right) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2\right\} \quad (C)$$

TABLA 16.2—1

FORMAS EQUIVALENTES DE LA PRIMERA LEY DE FICK DE LA DIFUSIÓN

Densidad de flujo	Gradiente	Forma de la primera ley de Fick	
$\mathbf{n}_A$	$\nabla \omega_A$	$\mathbf{n}_A - \omega_A(\mathbf{n}_A + \mathbf{n}_B) = -\rho \mathscr{D}_{AB} \nabla \omega_A$	(A)
$\mathbf{N}_A$	∇x_A	$\mathbf{N}_A - x_A(\mathbf{N}_A + \mathbf{N}_B) = -c \mathscr{D}_{AB} \nabla x_A$	(B)
$\mathbf{j}_A$	$\nabla \omega_A$	$\mathbf{j}_A = -\rho \mathscr{D}_{AB} \nabla \omega_A$	(C)
$\mathbf{J}_A^\star$	∇x_A	$\mathbf{J}_A^\star = -c \mathscr{D}_{AB} \nabla x_A$	(D)
$\mathbf{j}_A$	∇x_A	$\mathbf{j}_A = -\left(\frac{c^2}{\rho}\right) M_A M_B \mathscr{D}_{AB} \nabla x_A$	(E)
$\mathbf{J}_A^\star$	$\nabla \omega_A$	$\mathbf{J}_A^\star = -\left(\frac{\rho^2}{c M_A M_B}\right) \mathscr{D}_{AB} \nabla \omega_A$	(F)
$c(\mathbf{v}_A - \mathbf{v}_B)$	∇x_A	$c(\mathbf{v}_A - \mathbf{v}_B) = -\frac{c \mathscr{D}_{AB}}{x_A x_B} \nabla x_A$	(G)

TABLA 18.2−1

LA ECUACIÓN DE CONTINUIDAD DE *A* EN DIVERSOS SISTEMAS COORDENADOS

(Ec. 18.1−10)

Coordenadas rectangulares:

$$\frac{\partial c_A}{\partial t} + \left(\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z}\right) = R_A \tag{A}$$

Coordenadas cilíndricas:

$$\frac{\partial c_A}{\partial t} + \left(\frac{1}{r}\frac{\partial}{\partial r}(rN_{Ar}) + \frac{1}{r}\frac{\partial N_{A\theta}}{\partial \theta} + \frac{\partial N_{Az}}{\partial z}\right) = R_A \tag{B}$$

Coordenadas esféricas:

$$\frac{\partial c_A}{\partial t} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2N_{Ar}) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}(N_{A\theta}\,\mathrm{sen}\,\theta) + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial N_{A\phi}}{\partial \phi}\right) = R_A \tag{C}$$

TABLA 18.2−2

LA ECUACIÓN DE CONTINUIDAD DE *A* PARA ρ Y $\mathscr{D}_{AB}$ CONSTANTES

(Ec. 18.1−17)

Coordenadas rectangulares:

$$\frac{\partial c_A}{\partial t} + \left(v_x\frac{\partial c_A}{\partial x} + v_y\frac{\partial c_A}{\partial y} + v_z\frac{\partial c_A}{\partial z}\right) = \mathscr{D}_{AB}\left(\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} + \frac{\partial^2 c_A}{\partial z^2}\right) + R_A \tag{A}$$

Coordenadas cilíndricas:

$$\frac{\partial c_A}{\partial t} + \left(v_r\frac{\partial c_A}{\partial r} + v_\theta\frac{1}{r}\frac{\partial c_A}{\partial \theta} + v_z\frac{\partial c_A}{\partial z}\right)$$
$$= \mathscr{D}_{AB}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_A}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 c_A}{\partial \theta^2} + \frac{\partial^2 c_A}{\partial z^2}\right) + R_A \tag{B}$$

Coordenadas esféricas:

$$\frac{\partial c_A}{\partial t} + \left(v_r\frac{\partial c_A}{\partial r} + v_\theta\frac{1}{r}\frac{\partial c_A}{\partial \theta} + v_\phi\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial c_A}{\partial \phi}\right)$$
$$= \mathscr{D}_{AB}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c_A}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial \theta}\left(\mathrm{sen}\,\theta\frac{\partial c_A}{\partial \theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2 c_A}{\partial \phi^2}\right) + R_A \tag{C}$$

ANALOGÍAS ENTRE TRANSFERENCIA DE CANTIDAD DE MOVIMIENTO, TRANSFERENCIA DE CALOR Y TRANSFERENCIA DE MATERIA.

Analogía entre transferencia de cantidad de movimiento y calor	Analogía entre transferencia de cantidad de movimiento y materia
Analogía de Reynolds	**Analogía de Reynolds**
$St_H = \frac{C_f}{2}$	$St_D = \frac{C_f}{2}$
Analogía de Colburn	**Analogía de Colburn**
$St_H Pr^{2/3} = \frac{C_f}{2}$	$St_D Sc^{2/3} = \frac{C_f}{2}$