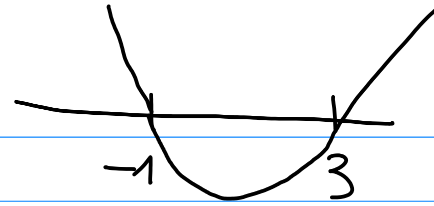
2.1.2.



a) $x^2 - 2x > 3$

$$x^2 - 2x - 3 > 0$$

$$(x+1)(x-3) > 0 \iff x < -1 \text{ o } x > 3$$

$$\iff x \in (-\infty, -1) \cup (3, +\infty)$$

$x+1$	$-$	-1	$+$
$x-3$	$-$	3	$+$

d) $$\frac{2-x}{1+x} \le \frac{1+x}{2-x}$$

$a \le b$, $c < 0$
$ac \ge bc$

$1 \le 2$, -1
$-1 \ge -2$

$1+x$	$-$		$+$		$+$
$2-x$	$+$	-1	$+$	2	$-$

Caso $-1 < x < 2$

$$(2-x)^2 \le (1+x)^2$$

$$4 - 4x + \cancel{x^2} \le 1 + 2x + \cancel{x^2}$$

$$3 \le 6x$$

$$\frac{1}{2} \le x$$

Caso $x < -1$ o $x > 2$

$$(2-x)^2 \ge (1+x)^2$$

$$x \le \frac{1}{2}$$

$$\Rightarrow x \in \left[\frac{1}{2}, 2\right) \cup (-\infty, -1)$$

$\overset{\circ}{\wedge}$ $\overset{\circ}{\vee}$

g) $\sqrt{x+4} < x-1$ hay que estudiar $x \geq -4$

Caso $-4 \leq x < 1$

$x-1 < 0 \leq \sqrt{x+4}$ $\quad 0 \leq a < b$

no hay solución $\quad a^2 < b^2$

Caso $x > 1$, $x-1 > 0$

$$x+4 < (x-1)^2$$

$$x+4 < x^2-2x+1$$

$$x^2-3x-3 > 0$$

↪ raíces $\dfrac{3 \pm \sqrt{9-4(-3)}}{2} = \dfrac{3 \pm \sqrt{21}}{2}$

$$x < \frac{3-\sqrt{21}}{2} \quad o \quad x > \frac{3+\sqrt{21}}{2} > 1$$

Conclusión: $x \in \left(\frac{3+\sqrt{21}}{2}, +\infty\right)$

m) $|nx| > x^2$, $n \in \mathbb{N}$

$x = 0$ no es solución

Caso $x \geq 0$: $nx > x^2 \to x > 0$, $n > x$

Caso $x < 0$: $\underset{|nx|}{\underset{\shortparallel}{-nx}} > x^2 \to -n < x$

$n=0$ no hay solución, $n \geq 1$ $\quad (-n,0) \cup (0,n)$

6. $A \overset{\subset \mathbb{R}}{,} \ \alpha \in \mathbb{R}$

$$\alpha A = \{\alpha \cdot a : a \in A\}$$

a) $\alpha = 2, \ A = \{0, 2, 4\}$

$$\alpha A = \{0, 4, 8\}$$

$$A + B = \{a + b : a \in A, \ b \in B\}$$

b) $A = \{1, 2, 3\}, \ B = \{1, \pi\}$

$$A + B = \{2, 1+\pi, 3, 2+\pi, 4, 3+\pi\}$$

c) $\mathbb{Z} = \{\dots, -1, 0, 1, 2, \dots\}$

$$\mathbb{Z} + \{-1, 2, 5\} = \mathbb{Z}$$

$n.5$

$$\mathbb{Z} + \{1, 2, \tfrac{5}{2}\} = \mathbb{Z} \cup (\mathbb{Z} + \tfrac{1}{2})$$

$$= \tfrac{1}{2}\mathbb{Z}$$

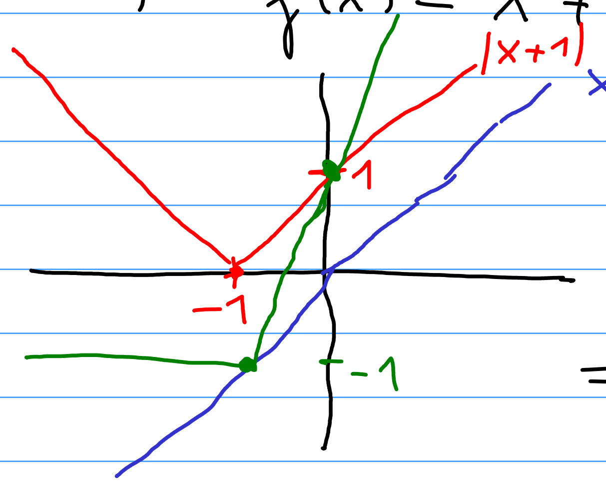
$$\cup$$

$$\mathbb{Z}$$

d) $\{p\} + [a, b] = [a + p, b + p]$

2.2.1.

a) $f(x) = x + |x+1|$ → monótona creciente no estrictamente



$$f(x) = x + |x+1| = \begin{cases} x + x + 1 & x \geq -1 \\ x - (x+1) & x < -1 \end{cases} = \begin{cases} 2x + 1 & x \geq -1 \\ -1 & x < -1 \end{cases}$$

Definición $f : \mathbb{R} \to \mathbb{R}$

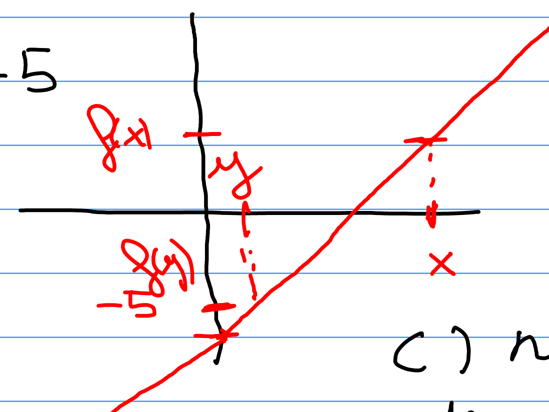
f es monótona creciente si $x \geq y$, entonces $f(x) \geq f(y)$

monótona decreciente si $x \geq y$, entonces $f(x) \leq f(y)$

monótona estrictamente creciente si $x > y \Rightarrow f(x) > f(y)$

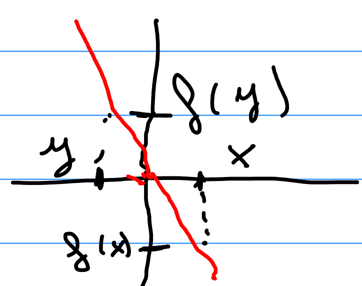
——— decreciente: $x > y \Rightarrow f(x) < f(y)$

a) $f(x) = x - 5$



est. creciente

b)



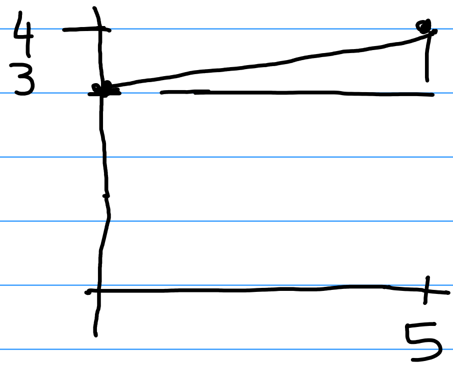
c) ninguna

d) ninguna

e) estrictamente creciente (sin derivada est. positiva) si $x > y \Rightarrow x^3 > y^3$

estrictamente monotona $\Rightarrow$ monotona
$\Rightarrow$ inyectiva

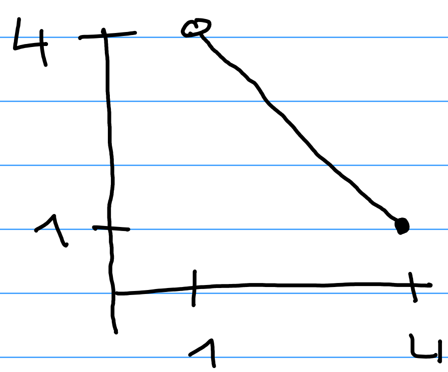
7. $I = [0,5]$, $J = [3,4]$



$f : I \to J$

$f(x) = \frac{1}{5}x + 3$

c) $I = (1,4]$, $J = [1,4)$,



$f(x) = -x + 5$

$f : I \to J$

$f(4) = 1$
$\uparrow \quad \uparrow$
$I \quad J$

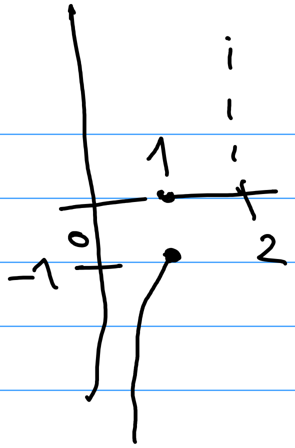
d) $I = (0, +\infty)$, $J = \mathbb{R}$

$\exp : J \to I$ $\qquad \exp(x) = e^x > 0$

$x \in \mathbb{R}$

$\log : I \to J$ ✓

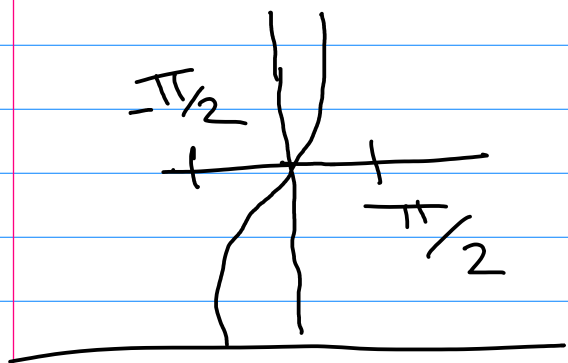
e) $I = (0,2)$, $J = \mathbb{R}$



$$-\frac{1}{x}\ ,\ -\frac{1}{x-2}$$

$$x \in (0,1],\ f(x) = -\frac{1}{x} \leq -1$$

$$x \in (1,2),\ f(x) = \frac{1}{x-2}$$



$$\sin(x) \leq 1$$

$$[-1,1]$$

$$f(x) = \begin{cases} -\frac{1}{x} & x \in (0,1] \\ -\frac{1}{x-2} & x \in (1,2) \end{cases}$$

$$\frac{1}{x} \in [1,+\infty)$$

$$-\frac{1}{x} \in (-\infty,-1]$$