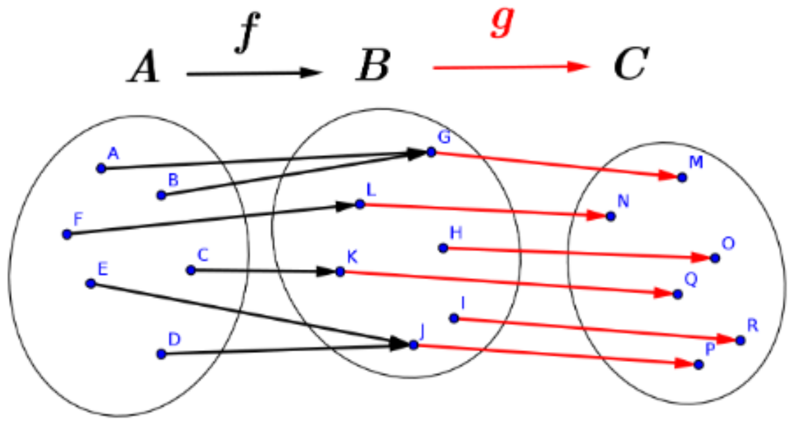
1. (*) Sean $f : A \to B$ y $g : B \to C$ funciones dadas por el siguiente diagrama:



$f(A) = G$

a) $g \circ f : A \to C \qquad g \circ f(x) = g(f(x))$

$g \circ f(A) = M \qquad g \circ f(D) = P$

$g \circ f(B) = M \qquad g \circ f(E) = P$

$g \circ f(C) = Q \qquad g \circ f(F) = N$

b) $f : A \to B$

Definición inyectiva: $f(x) = f(y) \Rightarrow x = y$

sobreyectiva: $\forall z \in B, \exists x \in A$ tal que $f(x) = z$

f no es inyectiva ni sobreyectiva

$f(A) = f(B) = G \qquad \to \nexists x \in A, f(x) = H$

g es inyectiva y sobre-yectiva

$g \circ f$ no es inyectiva ni sobreyectiva

Si $g \circ f$ es inyectiva, entonces f es inyectiva.

Si $g \circ f$ es sobreyectiva, entonces g es sobreyectiva.

3 a) $f, g : \mathbb{R} \to \mathbb{R}$ $\qquad$ x es raíz si $f(x) = 0$

1) Probar: $\underbrace{f \text{ no tiene raíces}}_{P} \Rightarrow \underbrace{f \circ g \text{ no tiene raíces}}_{Q}$

$P : \forall x \in \mathbb{R},\ f(x) \neq 0$ $\qquad$ tomando $x = g(y) \in \mathbb{R}$

$Q : \forall y \in \mathbb{R},\ f \circ g(y) \neq 0$

$\Leftrightarrow \forall y \in \mathbb{R},\ f(g(y)) \neq 0$

2) Contraejemplo de $Q \Rightarrow P$

$f(x) = x^2$ $\Rightarrow$ tiene raíz $\qquad f \circ g(x) = f(1) = 1 \to$ no tiene raíces

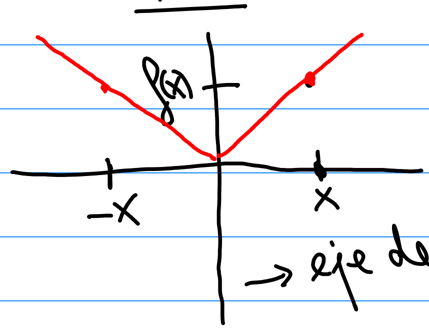
$g(x) = 1$

5. $f : \mathbb{R} \to \mathbb{R}$

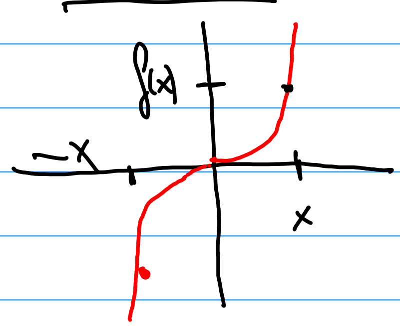
f es par si $f(x) = f(-x),\ \forall x \in \mathbb{R}$

f es impar si $f(x) = -f(-x),\ \forall x \in \mathbb{R}$ $\to f(-x) = -f(x)$

Par

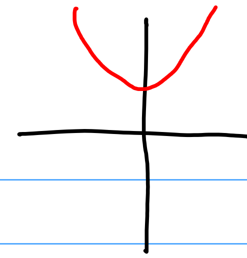


Imapar



x^n es par si n es par

impar si n es impar



$f(x) = x + 1$ no es par ni impar $\quad f(-x) = -x + 1$

$\neq f(x)$

$\neq -f(x)$

$|x|$ es par

$\frac{1}{x}$ es impar $: \mathbb{R} \setminus \{0\} \to \mathbb{R}$

6) $f(x) = e^x$ ni par ni impar

f es impar $\Rightarrow f(0) = 0$

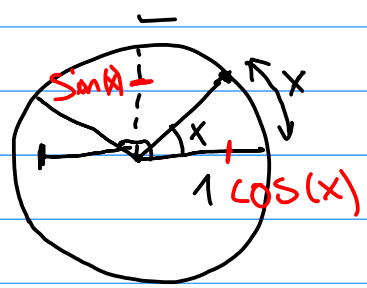
$f(0) = -f(-0) \Rightarrow f(0) = -f(0)$

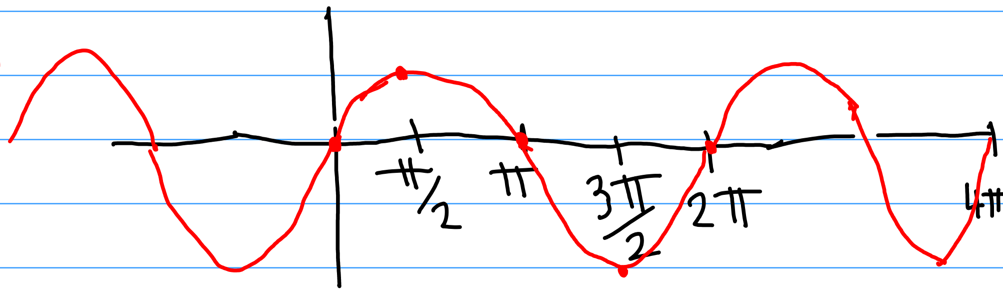
$f(-x) = e^{-x} = \frac{1}{e^x} \neq e^x$

$-e^x$

$\sin : \mathbb{R} \to \mathbb{R}$ $\quad x$ en radianes $\quad \frac{\pi}{2} = 90°$

$2\pi = 360°$

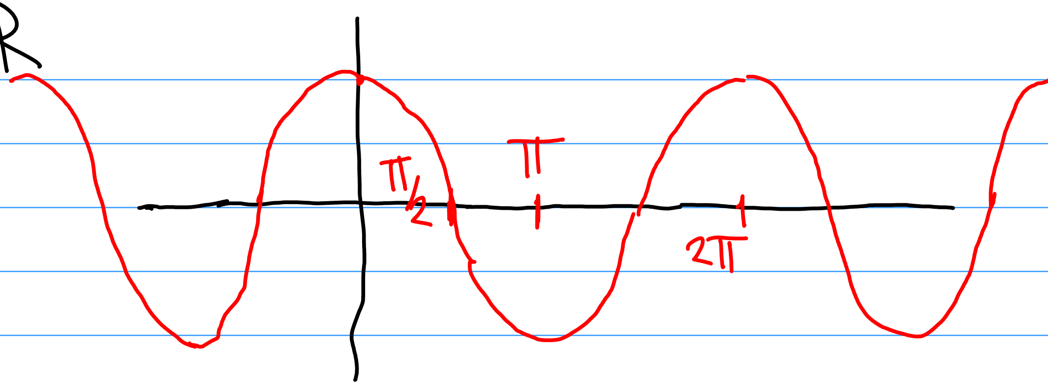




Impar

$\sin(x) = -\sin(-x)$

$\cos : \mathbb{R} \to \mathbb{R}$



Par

$(f+g)(x) = f(x) + g(x)$

$(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x)$

$= (f+g)(x)$

7. a) $f(x) = 2x+1, \quad g(x) = 3x-1$

$f \circ g(x) = f(g(x)) = f(3x-1) = 2(3x-1)+1$

$= 6x - 1$

$g \circ f(x) = g(f(x)) = g(2x+1) = 3(2x+1) - 1$

$= 6x + 2$

$(f+g)(x) = f(x) + g(x) = 5x$

g) $f(x) = x+1, \quad g(x) = \max\{1, x-1\} = \begin{cases} 1 & 1 \geq x-1 \\ x-1 & 1 < x-1 \end{cases}$

$= \begin{cases} 1 & \text{si } x \leq 2 \\ x-1 & \text{si } x > 2 \end{cases}$

$$f \circ g(x) = f(g(x)) = f(\max\{1, x-1\}) =$$
$$= \max\{1, x-1\} + 1 = \max\{2, x\}$$
$$g \circ f(x) = g(f(x)) = g(x+1) = \max\{1, x\}$$
$$(f+g)(x) = f(x) + g(x) = x + 1 + \max\{1, x-1\} =$$
$$= \max\{x+2, 2x\}$$

i) $f(x) = \begin{cases} 2x+1 & x \leq 0 \\ x-1 & x > 0 \end{cases}$, $g(x) = \begin{cases} x & x \leq 0 \\ 2x & x > 0 \end{cases}$

$$f \circ g(x) = f(g(x))$$

<u>Caso $x \leq 0$</u> $\quad g(x) = x$

$$f \circ g(x) = f(x) = 2x+1$$

<u>Caso $x > 0$</u> $\quad g(x) = 2x > 0$

$$f \circ g(x) = f(2x) = 2x - 1$$

$$f \circ g(x) = \begin{cases} 2x+1 & x \leq 0 \\ 2x-1 & x > 0 \end{cases}$$

$g \circ f(x) = g(f(x))$ <u>Caso $x \leq 0$</u> $\quad f(x) = 2x+1$

$$\longrightarrow g(2x+1) = \begin{cases} 2x+1 & 2x+1 \leq 0 \\ 4x+2 & 2x+1 > 0 \end{cases}$$

$$= \begin{cases} 2x+1 & x \leq -\frac{1}{2} \\ 4x+2 & x > -\frac{1}{2} \end{cases} = \begin{cases} 2x+1 & x \leq -\frac{1}{2} \\ 4x+2 & -\frac{1}{2} < x \leq 0 \end{cases}$$

Caso $x > 0$

$$g \circ f(x) = g(x-1) = \begin{cases} x-1 & x-1 \leq 0 \\ 2x-2 & x-1 > 0 \end{cases}$$

$$= \begin{cases} x-1 & x \leq 1 \\ 2x-2 & x > 1 \end{cases} = \begin{cases} x-1 & 0 < x \leq 1 \\ 2x-2 & x > 1 \end{cases}$$

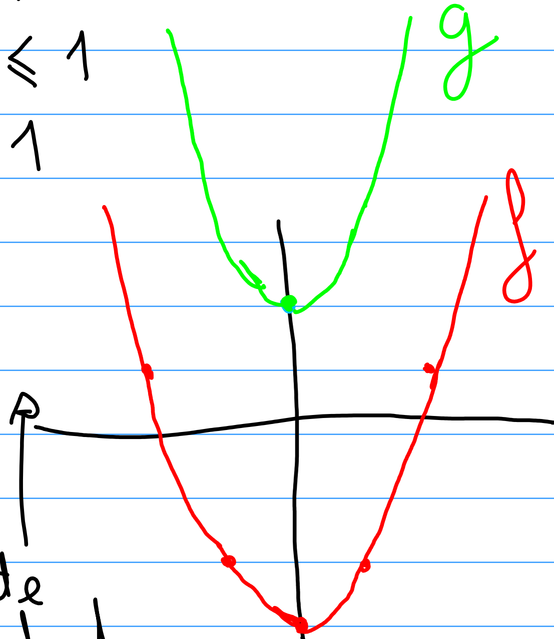
$$g \circ f(x) = \begin{cases} 2x+1 & x \leq -\frac{1}{2} \\ 4x+2 & -\frac{1}{2} < x \leq 0 \\ x-1 & 0 < x \leq 1 \\ 2x-2 & x > 1 \end{cases}$$

12.

a) $f(x) = x^2 - 3$

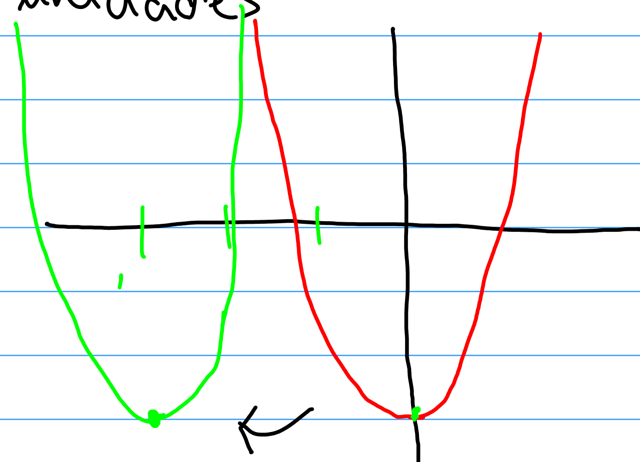
$g(x) = f(x) + 5$

trasladar verticalmente 5 unidades

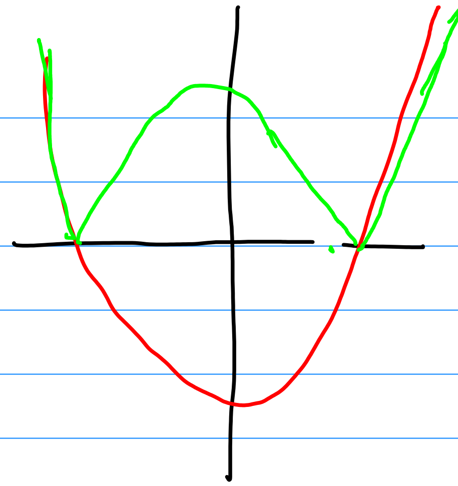


b) $g(x) = f(x+3)$

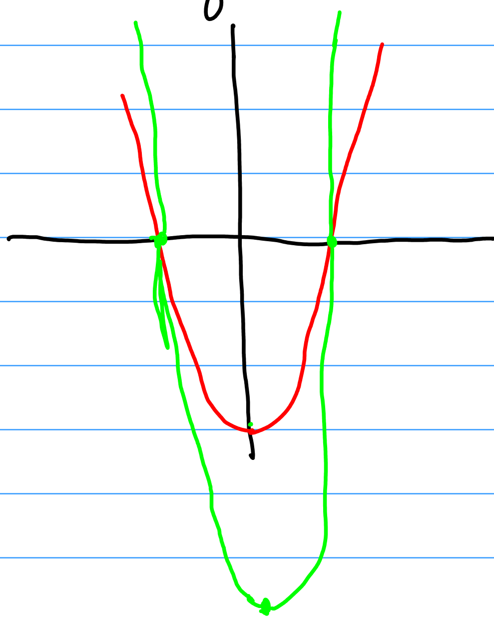
traslada 3 a la izquierda



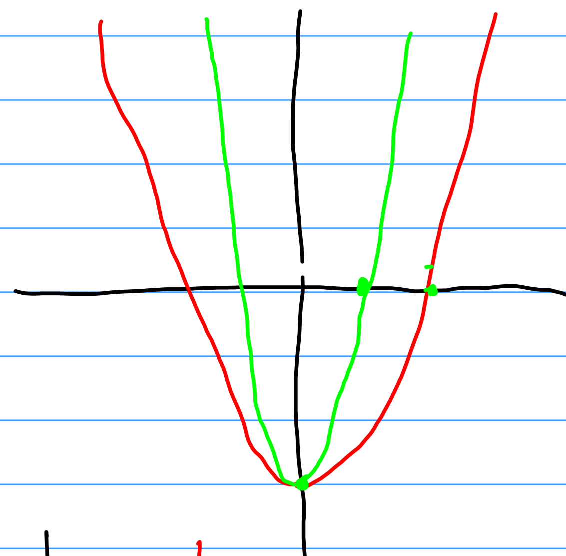
c) $g(x) = |f(x)|$



d) $g(x) = 2f(x)$



e) $g(x) = f(2x)$



f) $g(x) = \dfrac{f(2x)}{2}$