Sumatorios y productorios

$$\sum_{i=1}^{4}\left(\sum_{k=1}^{i} 2(k+1)\right) = \sum_{k=1}^{1} 2(k+1) + \sum_{k=1}^{2} 2(k+1)$$

$$+ \sum_{k=1}^{3} 2(k+1) + \sum_{k=1}^{4} 2(k+1) = 2(1+1) + 2(1+1) + 2(2+1)$$

$$+ 2(1+1) + 2(2+1) + 2(3+1) + 2(1+1) + 2(2+1) + 2(3+1)$$

$$+ 2(4+1)$$

$x^6 - 4 \nearrow$ 6 raíces complejas

2 reales

$x^6 = 4 \quad x^3 = \pm 2 \quad \underbrace{x = \sqrt[3]{2}, \; x = -\sqrt[3]{2}}$

Capítulo 1

1.1.1 a) $A = \{1, 2, a, b, c\}$

Pregunta: subconjuntos de 2 elementos

P. ej. $\{1, 2\}$ es un subconjunto de 2 elementos

Número total: $4 + 3 + 2 + 1 = 10 = \boxed{\binom{5}{2} = \frac{5!}{2!\,(5-2)!}}$

3 elementos: 10 subconjuntos

b) $A = \{1, a, u\}$

Conjunto potencia

$$P(A) = \{\phi, \{1\}, \{a\}, \{u\}, \{1, a\}, \{1, u\},$$
$$\{a, u\}, A\}$$

↳ 8 subconjuntos $= 2^3$

$|A|$ = número de elementos

En general $|P(A)| = 2^{|A|}$

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2. a) $A \cap B = \{3, 4, 5, 6\}$

e) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

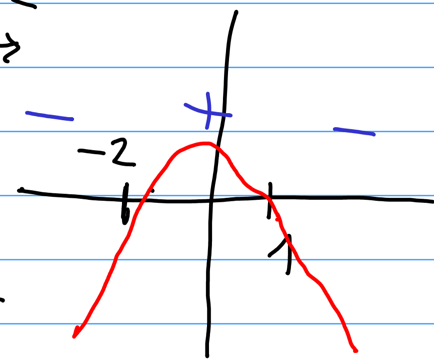
i) $A \cup (B \cap C) = A \cup \{7, 8\} = \{1, \ldots, 8\}$
$$\neq (A \cup B) \cap C$$

l) $(A \setminus B) \cup (B \setminus A) = \{1, 2\} \cup \{7, 8\} = \{1, 2, 7, 8\}$

3 a) $\{n \in \mathbb{N} : n \leq 5\}$ $\quad \mathbb{N} = \{0, 1, 2, 3, \ldots\}$
$$\shortparallel$$
$$\{0, 1, 2, 3, 4, 5\}$$

d) $\{x\in\mathbb{R} : -x^2-x+2<0\} = (-\infty,-2)\cup(1,+\infty)$

Estudiamos el signo de $-x^2-x+2$



$-x^2-x+2=0$

$x^2+x-2=0$

$$x=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2}=\left\langle\begin{matrix}-2\\ 1\end{matrix}\right.$$

6. $A\times B=\{(x,y) : x\in A$ y $y\in B\}$

a) $A=\{1,2,3\}$, $B=\{a,b,c\}$

$$A\times B=\{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$$

$$|A\times B|=|A|\cdot|B|$$

b) A, B conjuntos

pertenece

Probar que si $(x,y)\in A\times B$ y $(x,y)\in B\times A$
entonces $\{x,y\}\subset A\cap B$

subconjunto

Prueba:

Sabemos que $x\in A$ y $y\in B$. También $x\in B$ y $y\in A$.
Entonces $x\in A\cap B$ y $y\in A\cap B$.
Es decir $\{x,y\}\subset A\cap B$.

7.6. $|A| = 20$, $|B| = 30$, $|A \cup B| = 37$

$|A \cap B| = 13$

En general, $|A| + |B| = |A \cup B| + |A \cap B|$

8.

a) $(A \cup B) \cup C = A \cup (B \cup C)$ $\quad A \cup B = B \cup A$

$$A \cup B \cup C = \{x : x \in A \text{ o } x \in B \text{ o } x \in C\}$$

b) $(A \cap B) \cap C = A \cap (B \cap C) = A \cap B \cap C$

c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\neq$

$(A \cup B) \cap C$

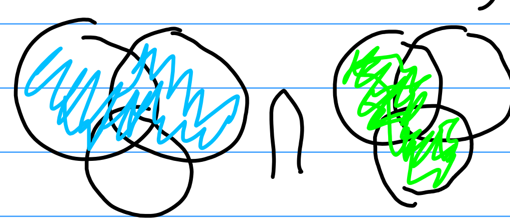
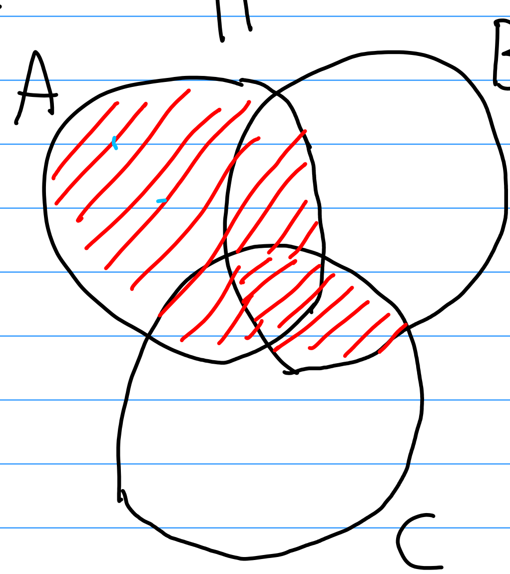
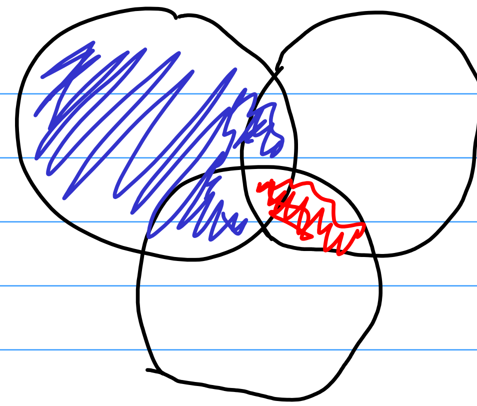


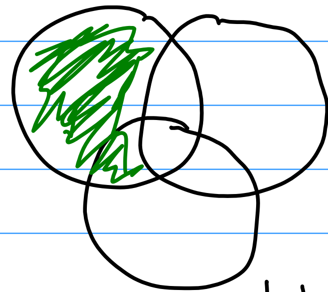
Diagrama de Venn



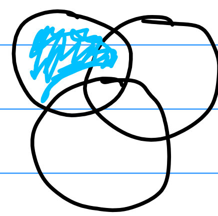
e) $A\setminus(B\cap C) \rightarrow$



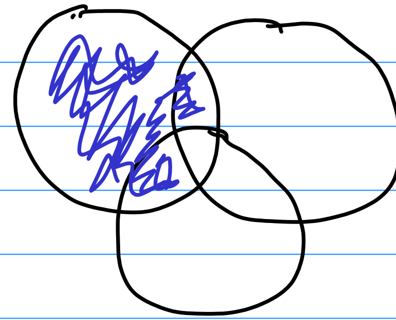
$A\setminus B$



$A\setminus C$



$\cup$ =



$x\in A\setminus(B\cap C) = (A\setminus B)\cup(A\setminus C) \leftarrow$

$x\in A,\ x\notin B\cap C \Leftrightarrow x\in A,\ x\notin B$ o $x\notin C$

6.c. $A\times B = B\times A$

$$\boxed{\emptyset\times\mathbb{N} = \mathbb{N}\times\emptyset = \emptyset}$$

$\to A\neq\emptyset \Rightarrow \exists\, x\in A$

$y\in B \Leftrightarrow (x,y)\in A\times B = B\times A$

Si $y\in B$, entonces $(x,y)\in B\times A$, por lo tanto $y\in A$. Esto prueba $B\subset A$.

$\to B\neq\emptyset \Rightarrow \exists\, z\in B$. Si $y\in A$, $(y,z)\in A\times B = B\times A$ entonces $y\in B$. Entonces $A\subset B \Rightarrow A=B$