

Descomposición de Lu-Chipman para Matrices de Mueller Utilizando Redes Neuronales Informadas con Física

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Universidad de la República**

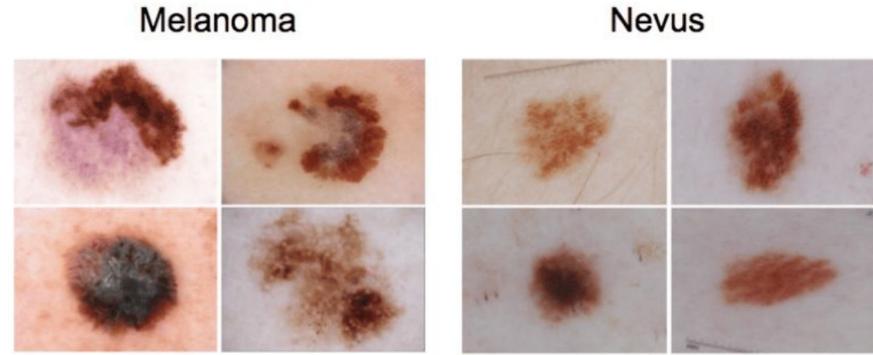
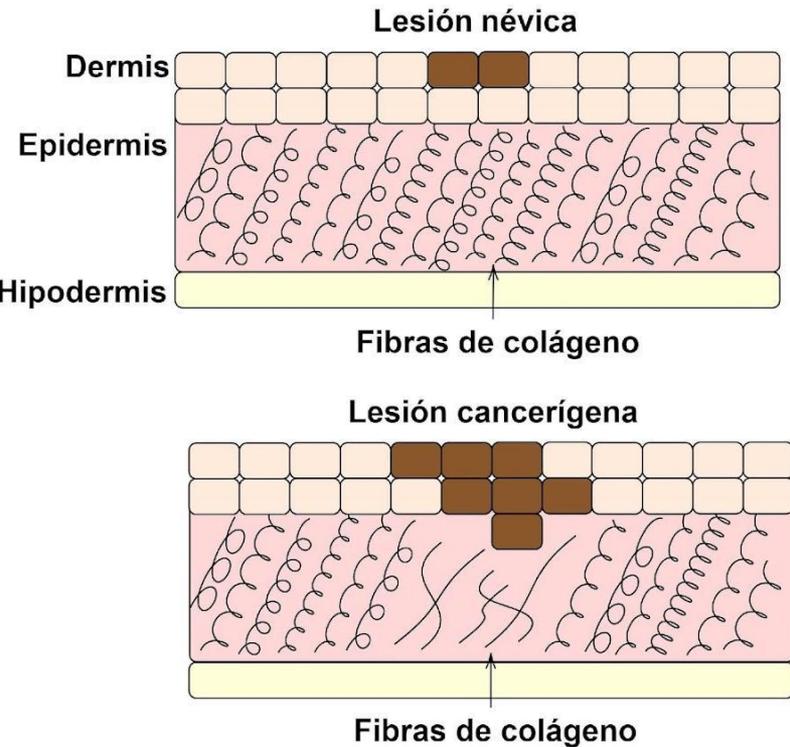


Noviembre de 2024



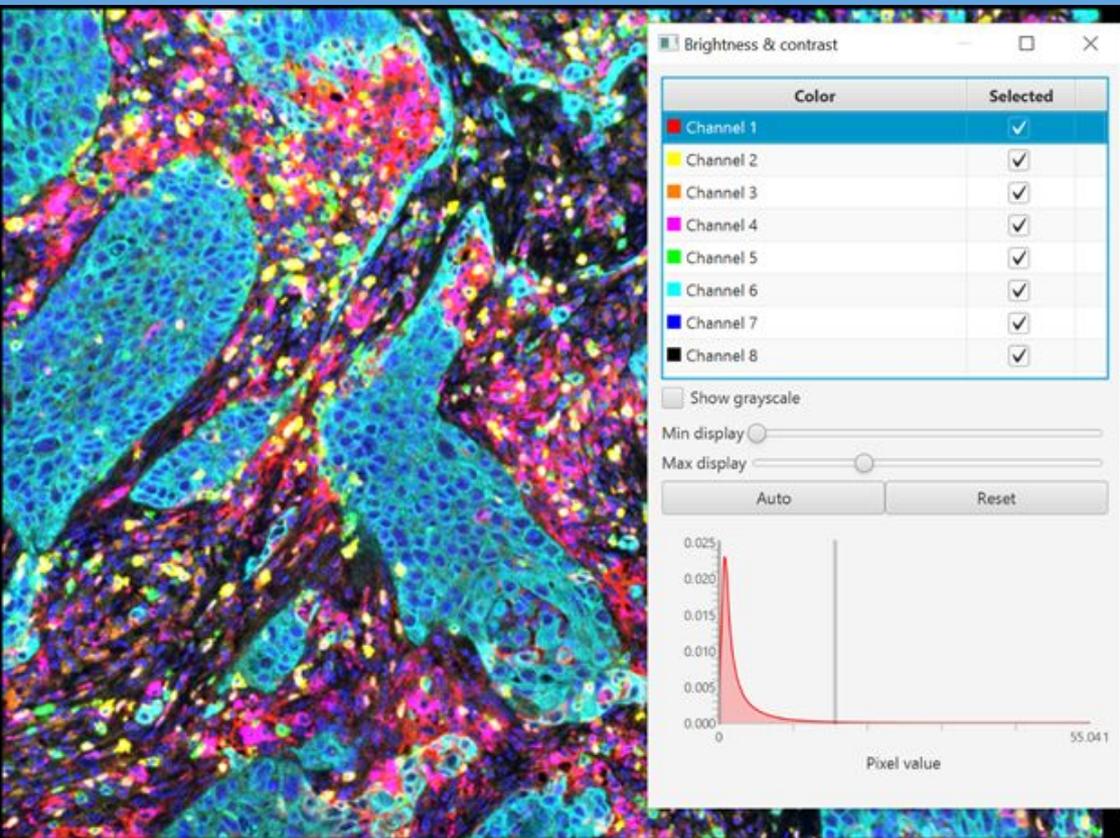
AGENCIA NACIONAL
DE INVESTIGACIÓN
E INNOVACIÓN

Motivación



- **Proliferación melanocítica, Rayos UV**
- **Cáncer más peligroso de la piel**
- **Reglas ABCDE**
- **Destruyen la Matriz de Colágeno**

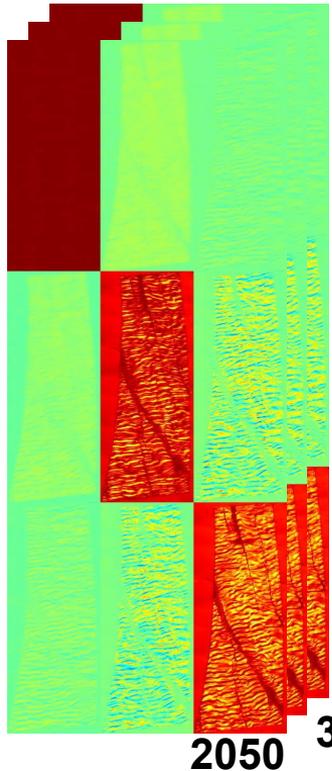
Motivación



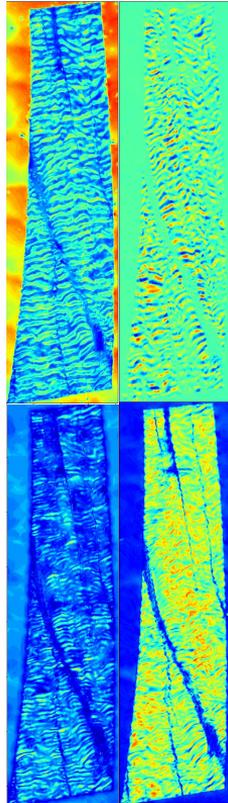
- **Imágenes microscópicas**
- **Tinción con H&E**
- **Sistema multimodal**
- **WSI, Stitching**
- **QuPath**

Motivación

Matriz de Mueller



Depolarización
↓
Diatenuación



Actividad Óptica
Birrefringencia

- Implementar parámetros polarimétricos

- Uso de memoria RAM:

$7390 \times 2050 \times 3 \times 3 \times 3 \times N$ Bytes = 0,38 Giga Bytes x N

- N = 1 (uint8): 0.38 GB.

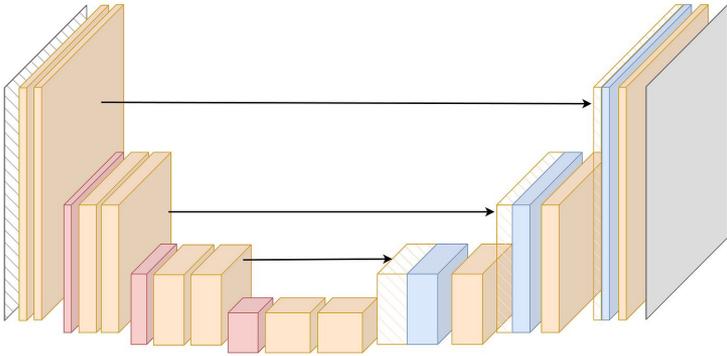
- N = 2 (uint16): 0.76 GB.

- N = 8 (float64): 3.04 GB.

- Tiempo de ejecución: varios minutos, infinito

Motivación

UNet



Encoder Name:

Resnet50

Encoder Weights:

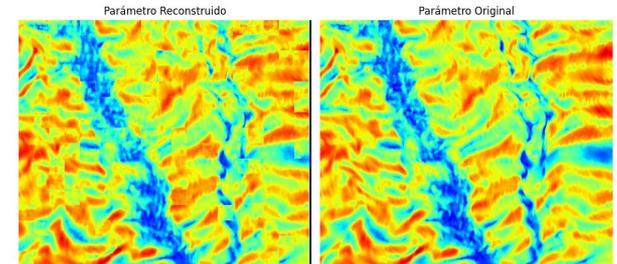
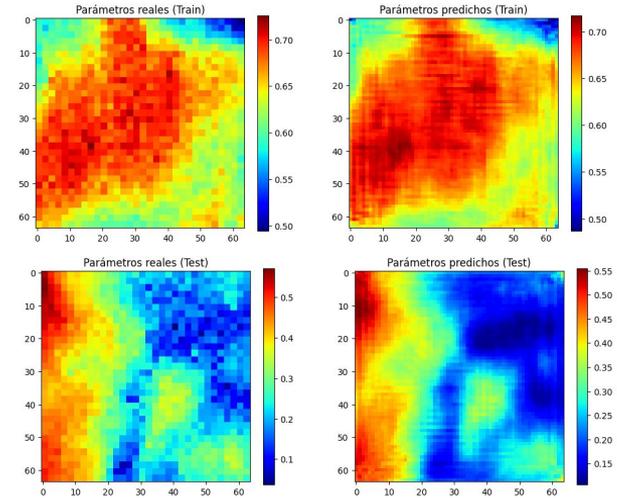
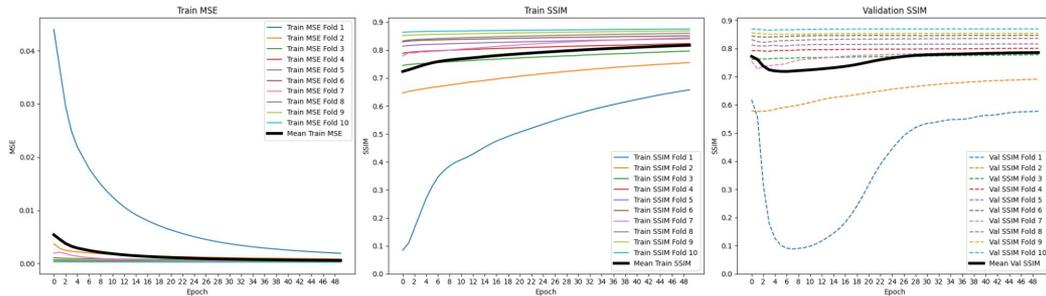
ImageNet

Canales:

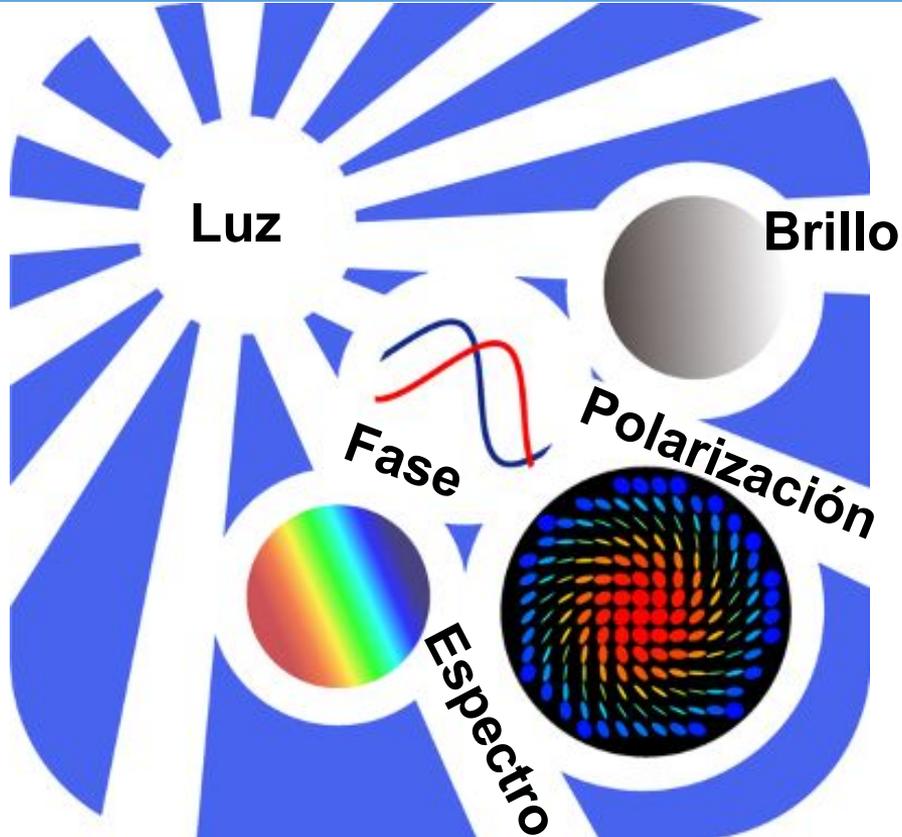
(12 input, 1 output)

Activación:

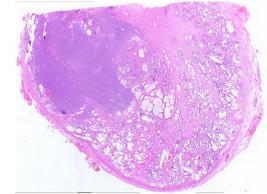
Sigmoid



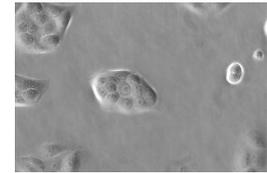
Luz Natural



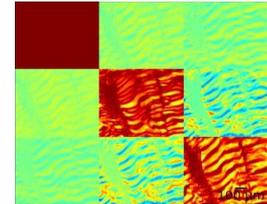
**Microscopía
de campo claro**



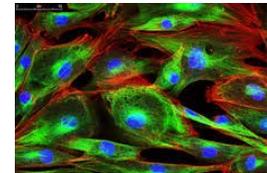
**Microscopía de
reconstrucción
de fase**



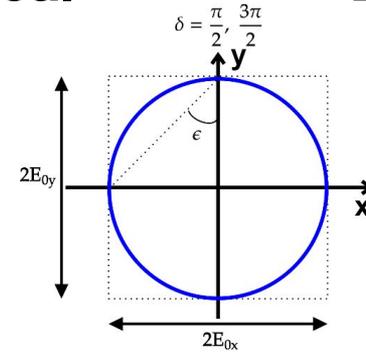
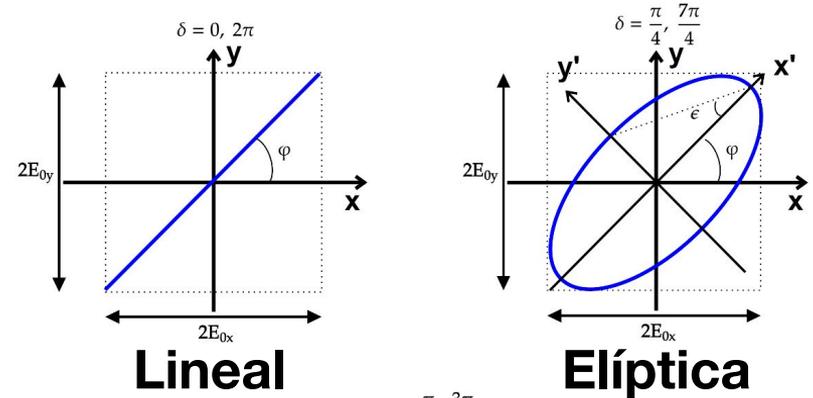
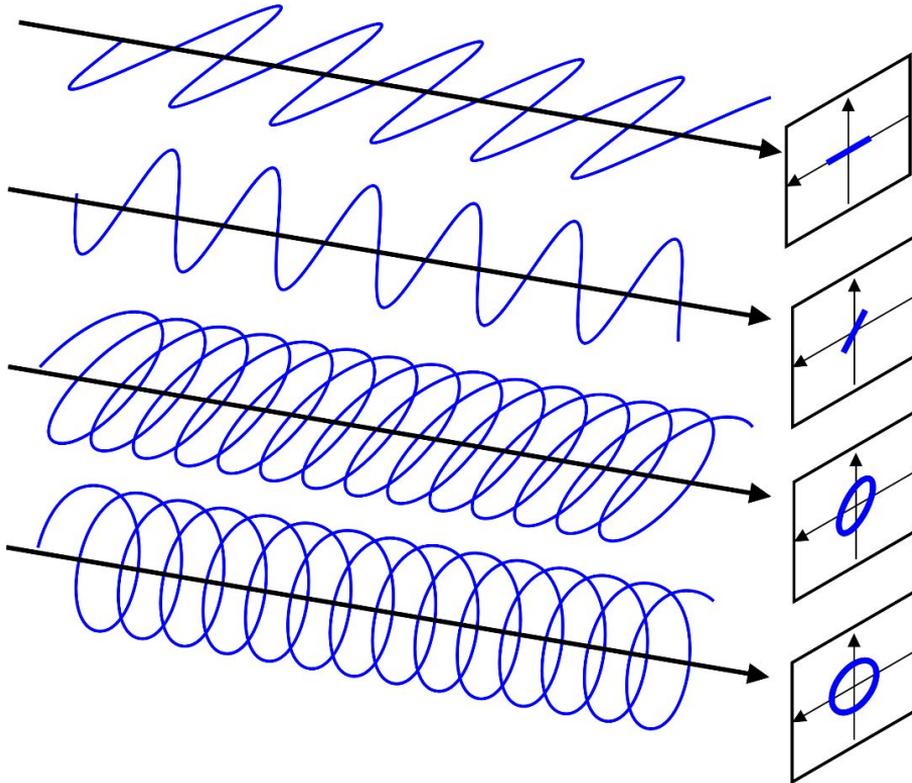
**Microscopía de
luz polarizada**



**Microscopía
hiperespectral
y fluorescencia**



Polarización de la Luz Monocromática



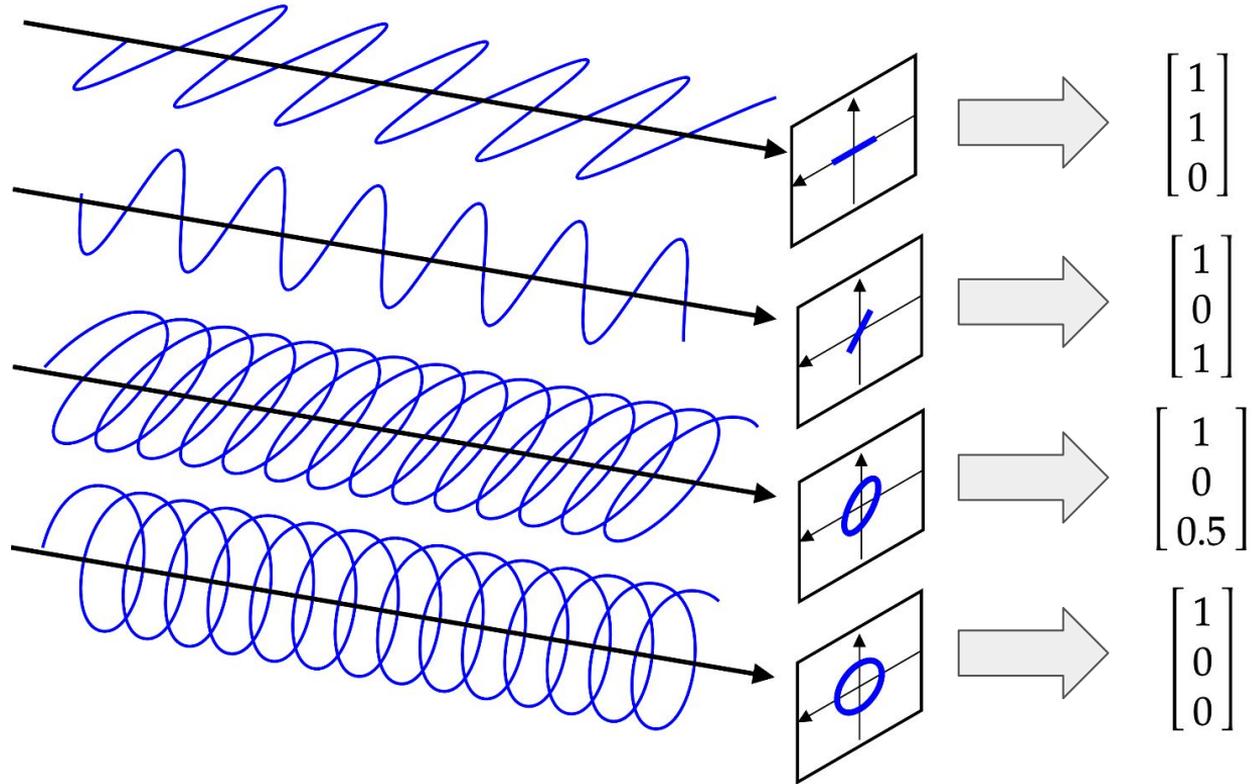
Circular

Vector de Stokes para Luz Monocromática

**Polarizador
lineal**



$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \end{bmatrix} \quad \text{Vector de Stokes}$$



Vector de Stokes para Luz Estructurada

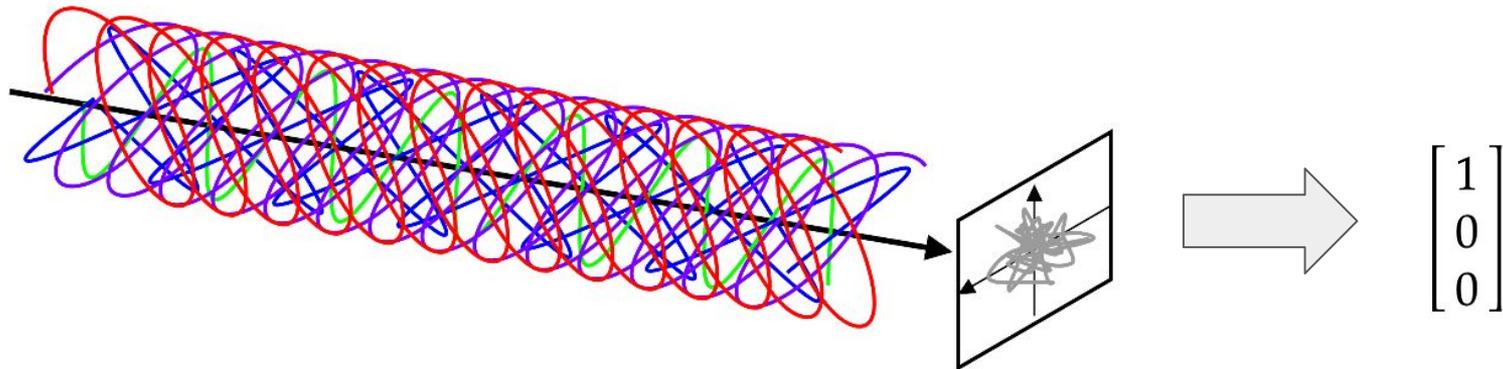
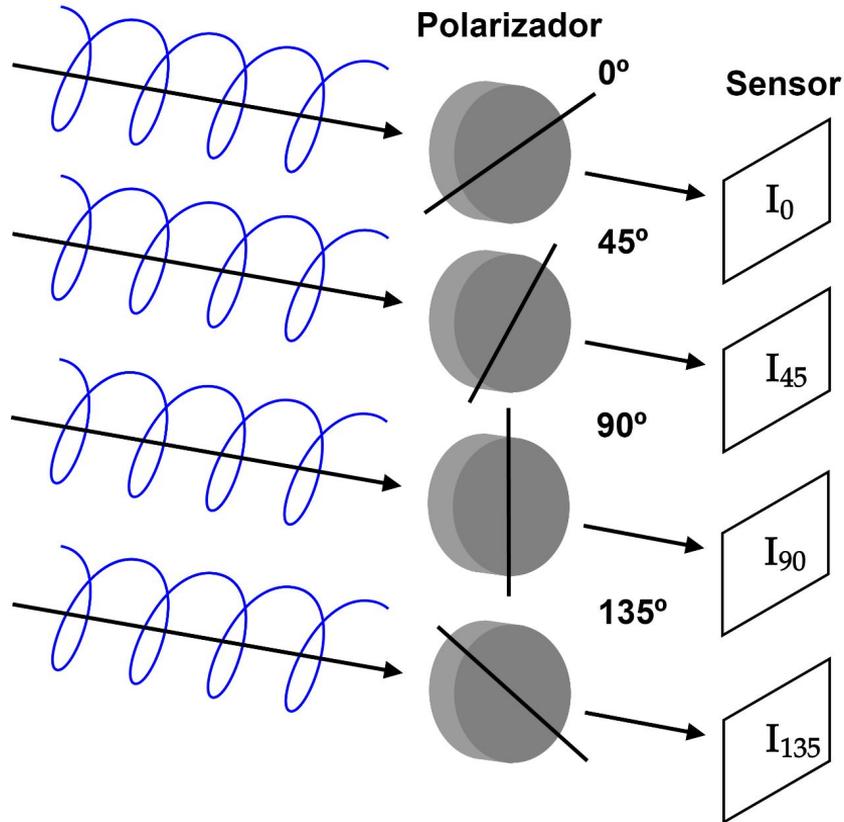


Lámina de
retardo



- Tiene origen en la interferencia
- Ambigüedad: requiere un 4.º parámetro
- Polarizar todas las longitudes de onda

Polarimetría de Stokes



$$S_0 = \frac{I_0 + I_{45} + I_{90} + I_{135}}{2}$$

$$S_1 = I_0 - I_{90}$$

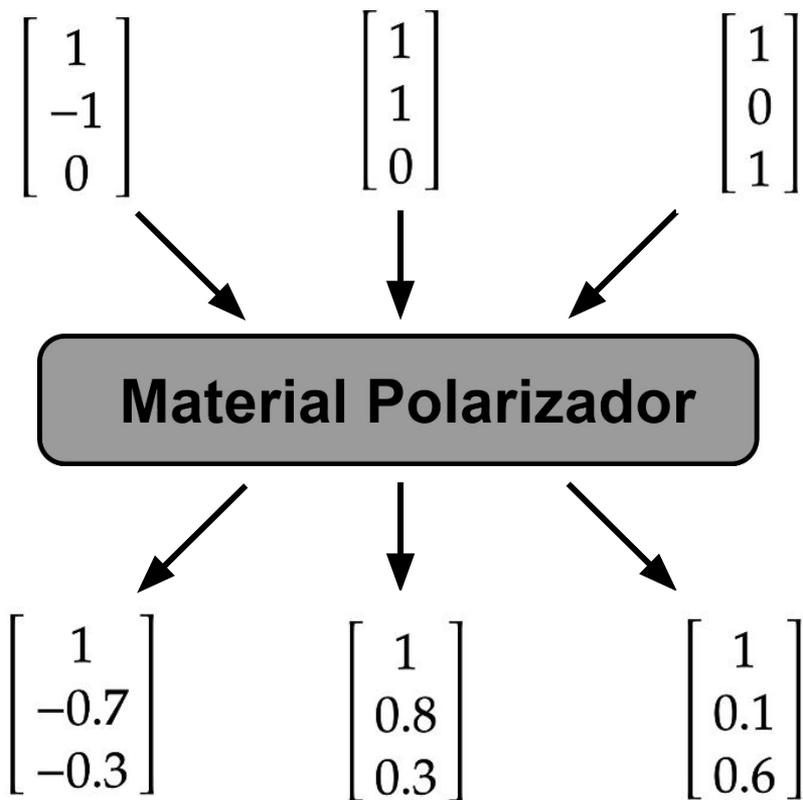
$$S_2 = I_{45} - I_{135}$$

$$|S_3| = \sqrt{S_0^2 - S_1^2 - S_2^2}$$

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

**Vector de
Stokes**

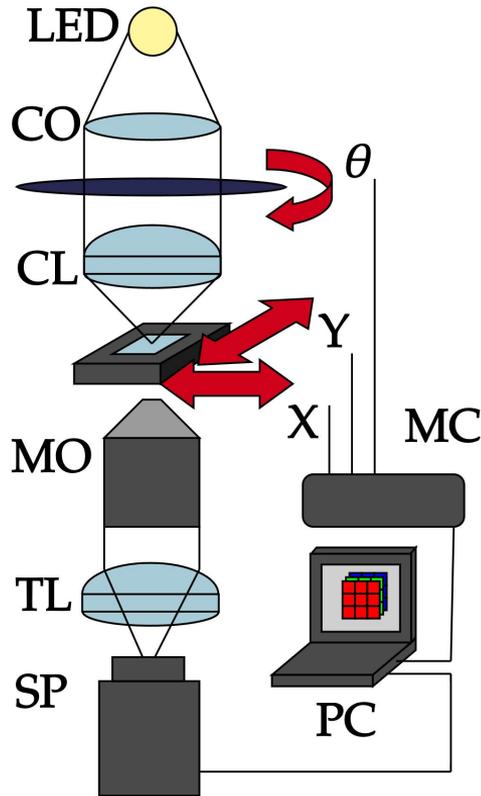
Matrices de Mueller



$$\begin{bmatrix} S'_0 \\ S'_1 \\ S'_2 \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} \\ m_{10} & m_{11} & m_{12} \\ m_{20} & m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \end{bmatrix}$$

- **Transmitancia/Reflectancia**
- **Quiralidad/Actividad Óptica**
- **Propiedades mecánicas**

Polarimetría de Matrices de Mueller

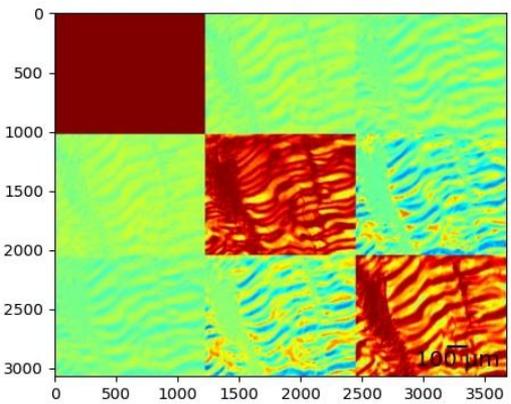
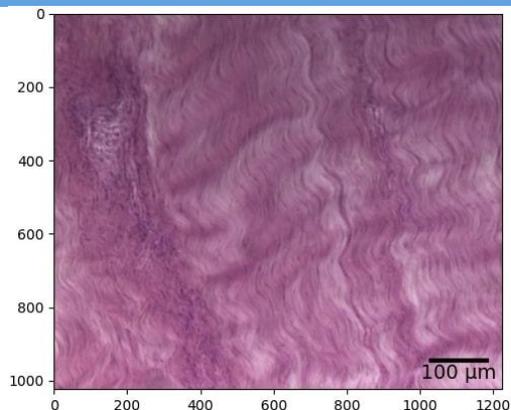


- Stokes input: $\theta = \{0^\circ, 30^\circ, 60^\circ\}$
- Para cada θ , medir $\{I_0, I_{45}, I_{90}, I_{135}\}$
- Calcular cada vector de Stokes e inferir:

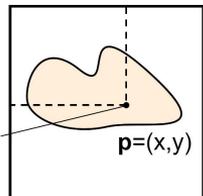
$$\begin{bmatrix} m_{00} & m_{01} & m_{02} \\ m_{10} & m_{11} & m_{12} \\ m_{20} & m_{21} & m_{22} \end{bmatrix} =$$

$$\begin{bmatrix} S_0^{(1)} & S_0^{(2)} & S_0^{(3)} & \dots & S_0^{(n)} \\ S_1^{(1)} & S_1^{(2)} & S_1^{(3)} & \dots & S_1^{(n)} \\ S_2^{(1)} & S_2^{(2)} & S_2^{(3)} & \dots & S_2^{(n)} \end{bmatrix} \left\| \begin{bmatrix} S_0^{(1)} & S_0^{(2)} & S_0^{(3)} & \dots & S_0^{(n)} \\ S_1^{(1)} & S_1^{(2)} & S_1^{(3)} & \dots & S_1^{(n)} \\ S_2^{(1)} & S_2^{(2)} & S_2^{(3)} & \dots & S_2^{(n)} \end{bmatrix}^+$$

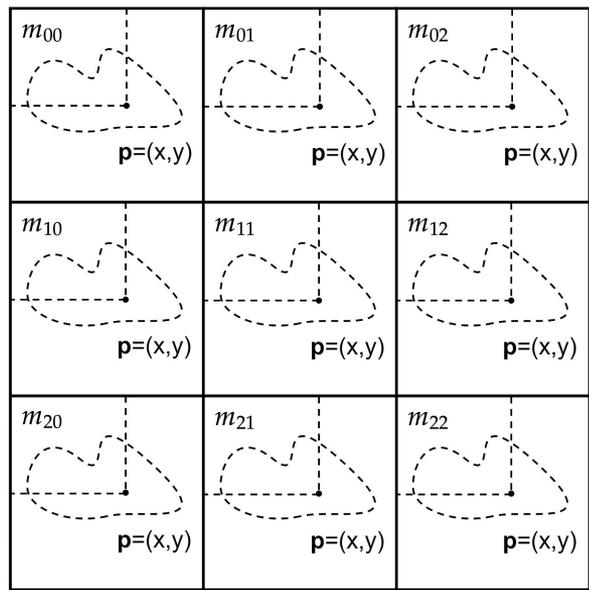
Imaginería de Matrices de Mueller



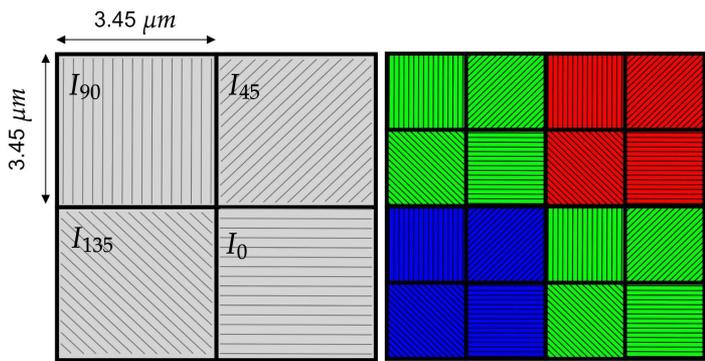
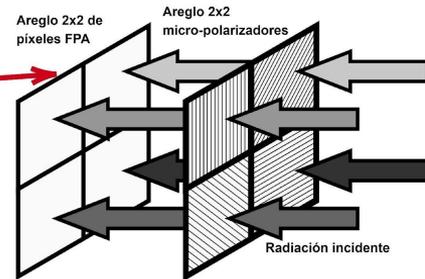
$$\begin{bmatrix} m_{00}(x, y) & m_{01}(x, y) & m_{02}(x, y) \\ m_{10}(x, y) & m_{11}(x, y) & m_{12}(x, y) \\ m_{20}(x, y) & m_{21}(x, y) & m_{22}(x, y) \end{bmatrix}$$



FoV del Sensor

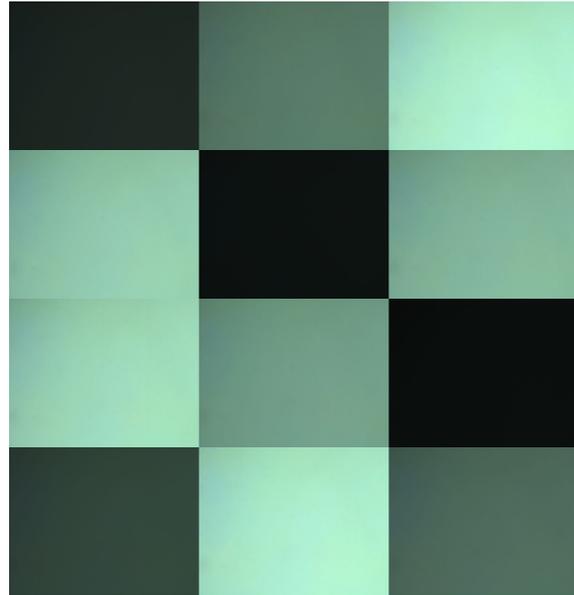


Cámara polarizada



Imaginería de Matrices de Mueller

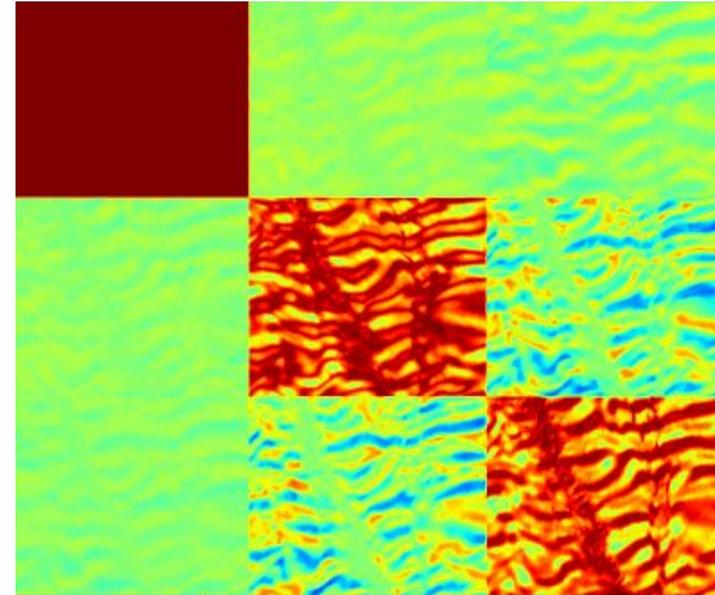
I_{in}



I_{out}



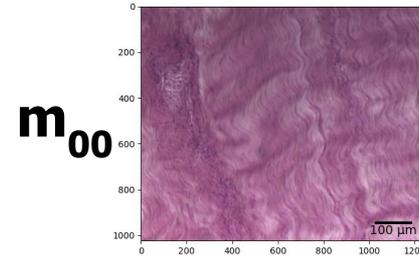
M



Descomposición de Lu-Chipman

Si M no es singular:

$$M = m_{00} \underbrace{\begin{bmatrix} 1 & D^T \\ P & m \end{bmatrix}}_{\widehat{M}} = m_{00} \underbrace{\begin{bmatrix} 1 & 0^T \\ P_\Delta & m_\Delta \end{bmatrix}}_{M_\Delta} \underbrace{\begin{bmatrix} 1 & 0^T \\ 0 & m_R \end{bmatrix}}_{M_R} \underbrace{\begin{bmatrix} 1 & D^T \\ D & m_D \end{bmatrix}}_{M_D}$$

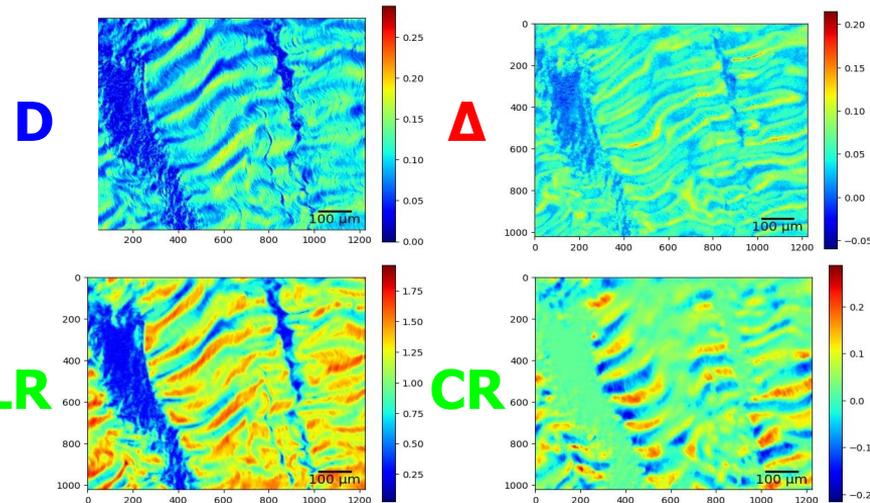


m_{00} - Transmitancia

Δ - **Despolarización** → Irregularidad

R - **Retardancia** → Elasticidad y Quiralidad

D - **Diatenuación** → Dicroísmo



Algoritmo de Descomposición de Lu-Chipman

Si M es una matriz de Mueller no singular, existe una única descomposición que tiene la siguiente forma:

$$M = m_{00}M_{\Delta}M_R M_D, \quad (\text{A.121})$$

dónde m_{00} es la transmitancia bajo luz despolarizada, M_{Δ} es la matriz de despolarización, M_R la matriz de retardancia y M_D la matriz de diatenuación.

Para probarlo, definamos una matriz auxiliar M' como:

$$M' = MM_D^{-1}. \quad (\text{A.122})$$

Como M es no singular, podemos invertir la matriz de diatenuación:

$$M_D^{-1} = \frac{1}{1-D^2} \begin{pmatrix} 1 & -D^T \\ -D & \sqrt{1-D^2}\mathbf{I} + \frac{1}{(1+\sqrt{1-D^2})}(D \otimes D) \end{pmatrix}, \quad (\text{A.123})$$

siendo D el vector de diatenuación:

$$D = \frac{1}{m_{00}} \begin{pmatrix} m_{11} \\ m_{22} \\ m_{33} \end{pmatrix}. \quad (\text{A.124})$$

Multiplicando M_{Δ} y M_R por un lado:

$$M_{\Delta}M_R = \begin{pmatrix} 1 & \mathbf{0}^T \\ P_{\Delta} & m_{\Delta} \end{pmatrix} \begin{pmatrix} 1 & \mathbf{0}^T \\ \mathbf{0} & m_R \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{0}^T \\ P_{\Delta} & m_{\Delta}m_R \end{pmatrix}. \quad (\text{A.125})$$

Multiplicando M y M_D^{-1} por otra parte:

$$\begin{aligned} MM_D^{-1} &= \frac{m_{00}}{1-D^2} \begin{pmatrix} 1 & D^T \\ P & m \end{pmatrix} \begin{pmatrix} 1 & -D^T \\ -D & \sqrt{1-D^2}\mathbf{I} + \frac{1}{(1+\sqrt{1-D^2})}(D \otimes D) \end{pmatrix} \\ &= m_{00} \begin{pmatrix} 1 & \mathbf{0}^T \\ \frac{P-mD}{1-D^2} & m' \end{pmatrix}. \end{aligned} \quad (\text{A.126})$$

Imponiendo Lu-Chipman, se deben cumplir las igualdades:

$$P_{\Delta} = \frac{P-mD}{1-D^2}, \quad m' = m_{\Delta}m_R. \quad (\text{A.127})$$

Definamos una matriz auxiliar n de la siguiente forma:

$$n = (m')^T m' = m_{\Delta}m_R m_R^T m_{\Delta}^T = m_{\Delta}^2. \quad (\text{A.128})$$

Como m_{Δ} es simétrica, n es diagonalizable y tiene valores propios reales positivos:

$$n = U\Lambda U^{-1}. \quad (\text{A.129})$$

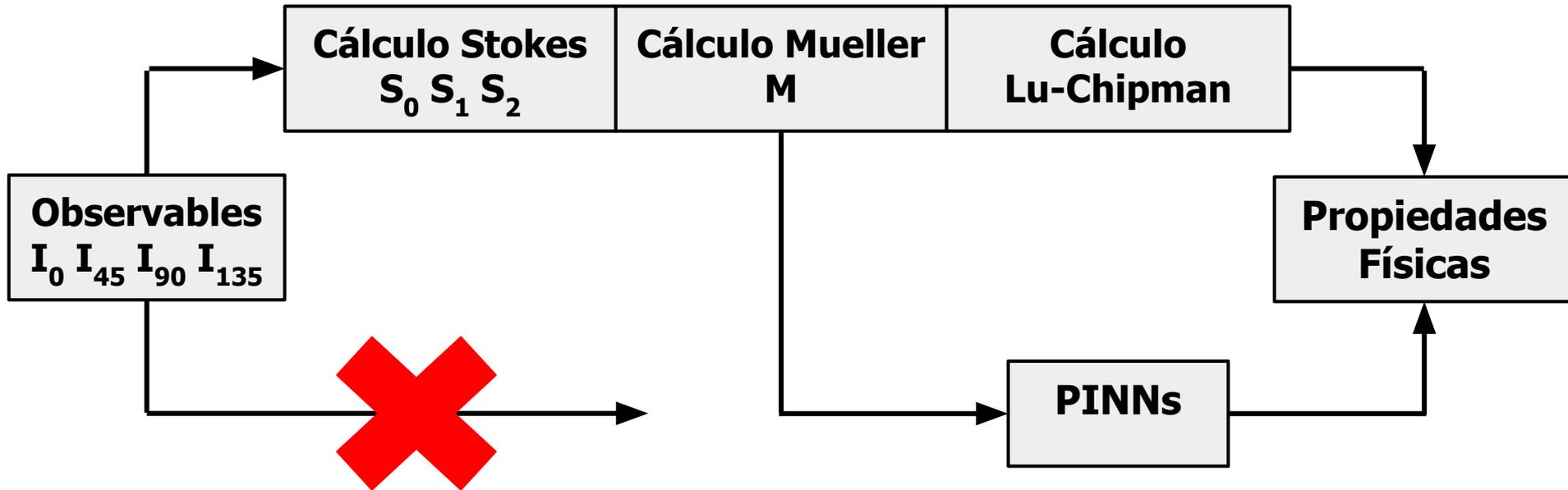
Si λ_1, λ_2 y λ_3 son los valores propios de n , entonces:

$$m_{\Delta} = n^{\frac{1}{2}} = U\Lambda^{\frac{1}{2}}U^{-1}. \quad (\text{A.130})$$

Solo queda determinar m_R :

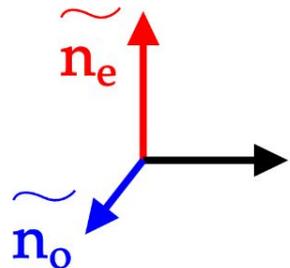
$$m_R = m_{\Delta}^{-1}m'. \quad (\text{A.131})$$

Descomposición de Lu-Chipman vs. PINNs



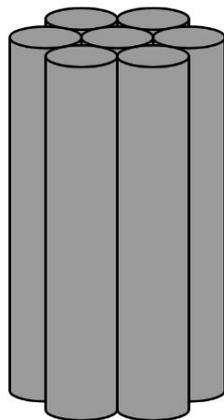
Propiedades Físicas de los Tejidos

$$\tilde{\mathbf{n}} = \mathbf{n} + i\mathbf{k}$$

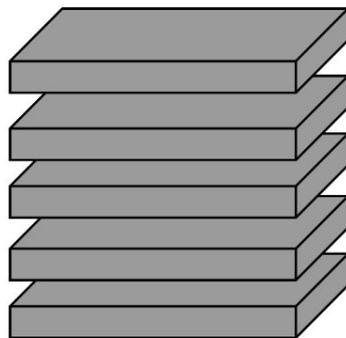


$$(x, y, \lambda)$$

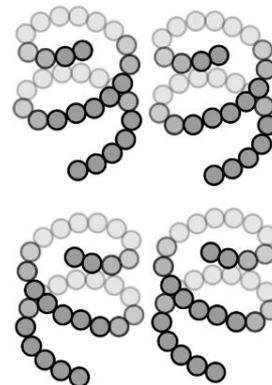
Cilindros



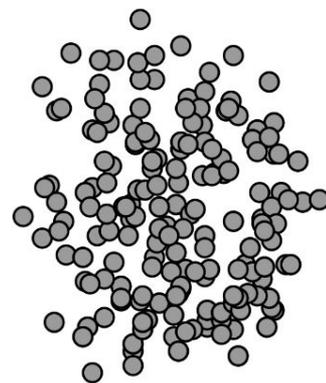
Placas
Dieléctricas



Moléculas
Quirales



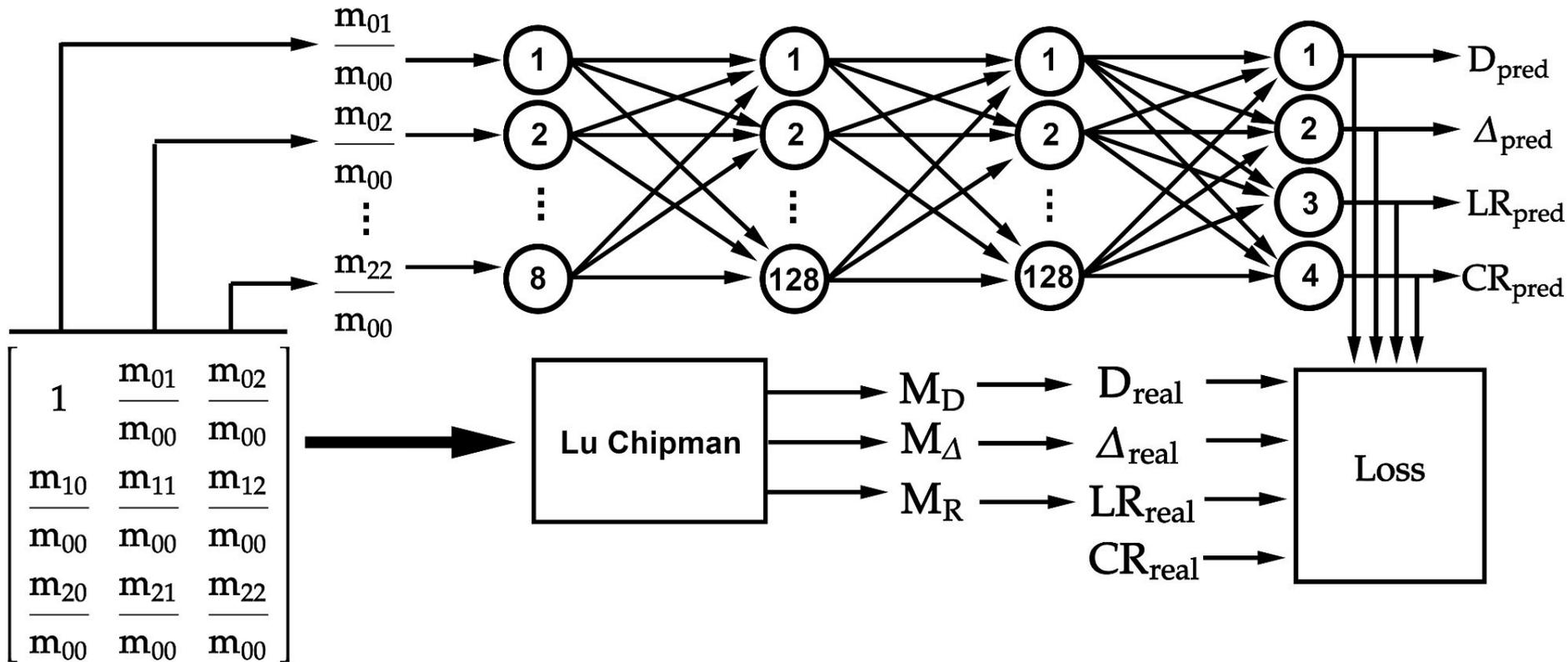
Partículas
Disperas



Orden

Desorden

Modelo PINN



Función de Costo

$$L = L_D + L_{LR} + L_{CR} + L_{\Delta}$$

$$D_{\text{real}} = \frac{\sqrt{m_{01}^2 + m_{02}^2}}{m_{00}}$$

$$L_D = \text{mse}(D_{\text{real}} - D_{\text{pred}})$$

$$\Delta_{\text{real}} = 1 - \frac{\text{Tr}(M_{\Delta,11}) - 1}{3}$$

$$L_{LR} = \text{mse}(LR_{\text{real}} - LR_{\text{pred}})$$

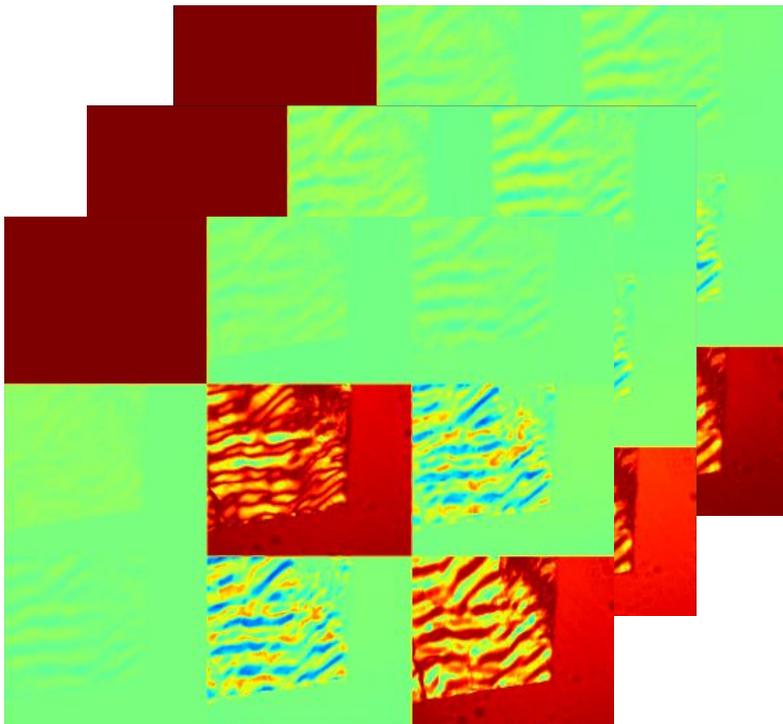
$$L_{CR} = \text{mse}(CR_{\text{real}} - CR_{\text{pred}})$$

$$CR_{\text{real}} = \arctan\left(\frac{M_{R,21} - M_{R,12}}{M_{R,11} + M_{R,22}}\right)$$

$$L_{\Delta} = \text{mse}(\Delta_{\text{real}} - \Delta_{\text{pred}})$$

$$LR_{\text{real}} = \arccos\left(\sqrt{(M_{R,11} + M_{R,22})^2 - (M_{R,21} - M_{R,12})^2} - 1\right)$$

Datos de Entrenamiento



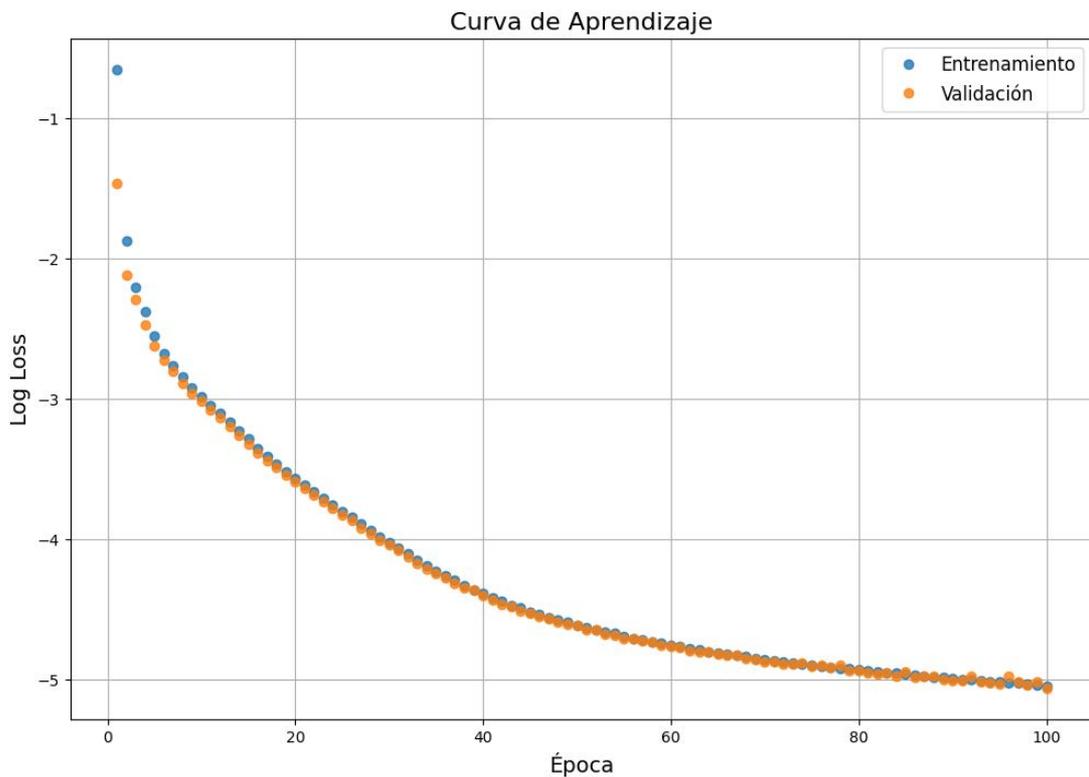
$$X = \begin{bmatrix} \frac{\mathbf{m}_{01}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{02}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{10}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{11}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{12}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{20}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{21}}{\mathbf{m}_{00}} & \frac{\mathbf{m}_{22}}{\mathbf{m}_{00}} \\ \vdots & \vdots \end{bmatrix}$$

$$y = \begin{bmatrix} D_{\text{real}} & LR_{\text{real}} & CR_{\text{real}} & \Delta_{\text{real}} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Número de píxeles:

$$1024 \times 1224 \times 3 = 3.760.128$$

Entrenamiento



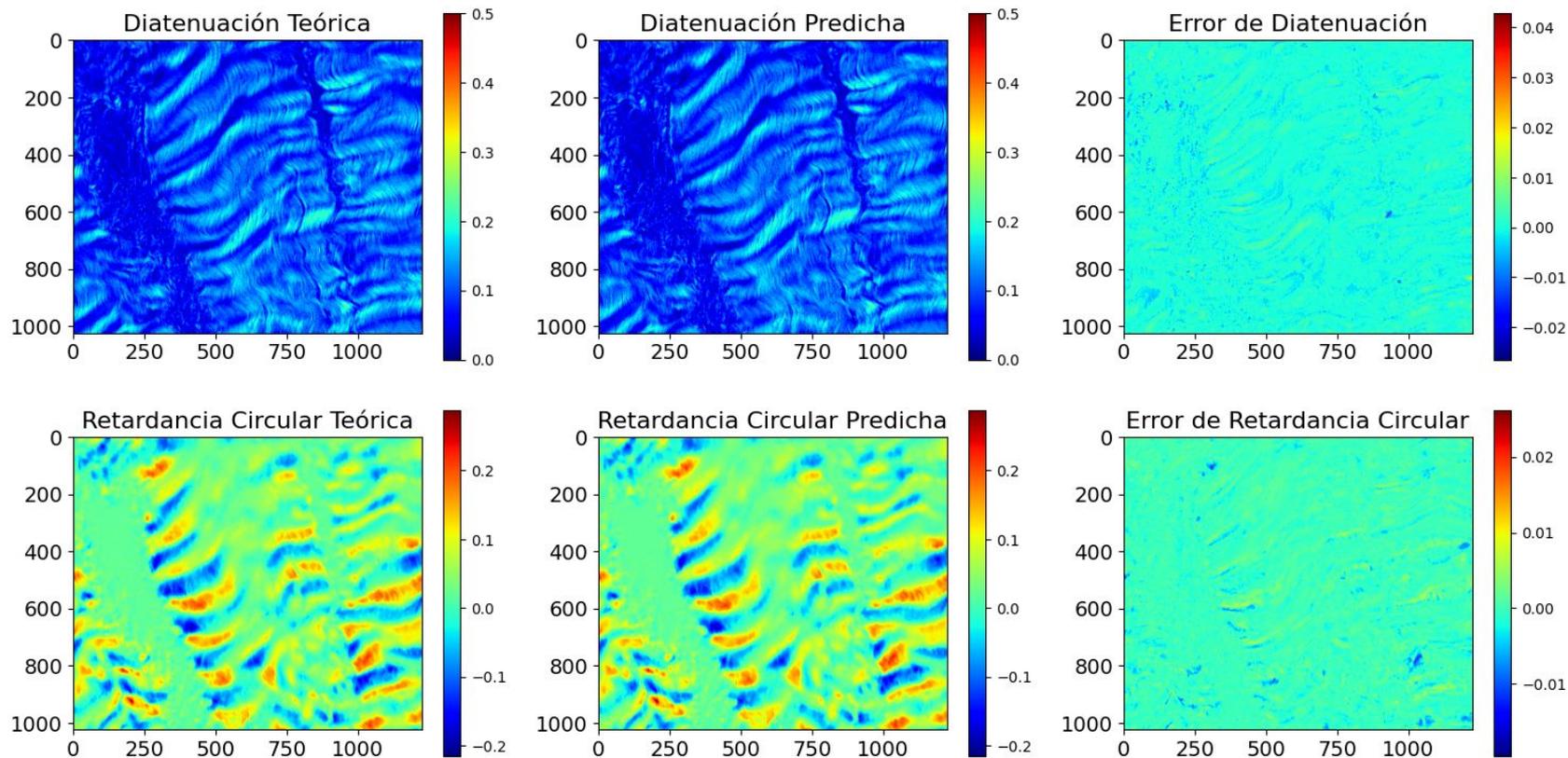
GPU:
Google Colab

Optimizador:
Adam (lr = 0.0001)

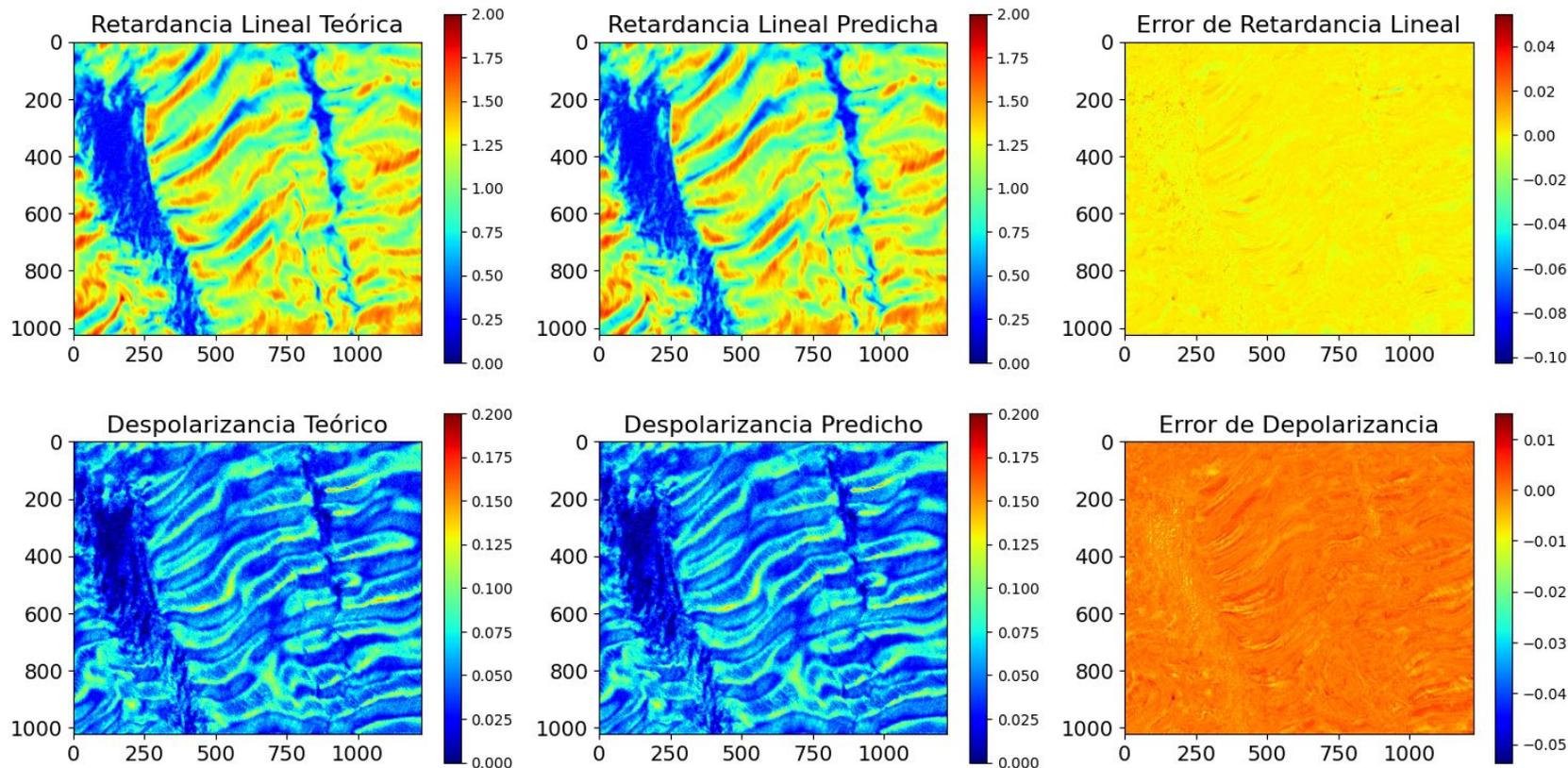
Épocas:
100

Batch Size:
8192

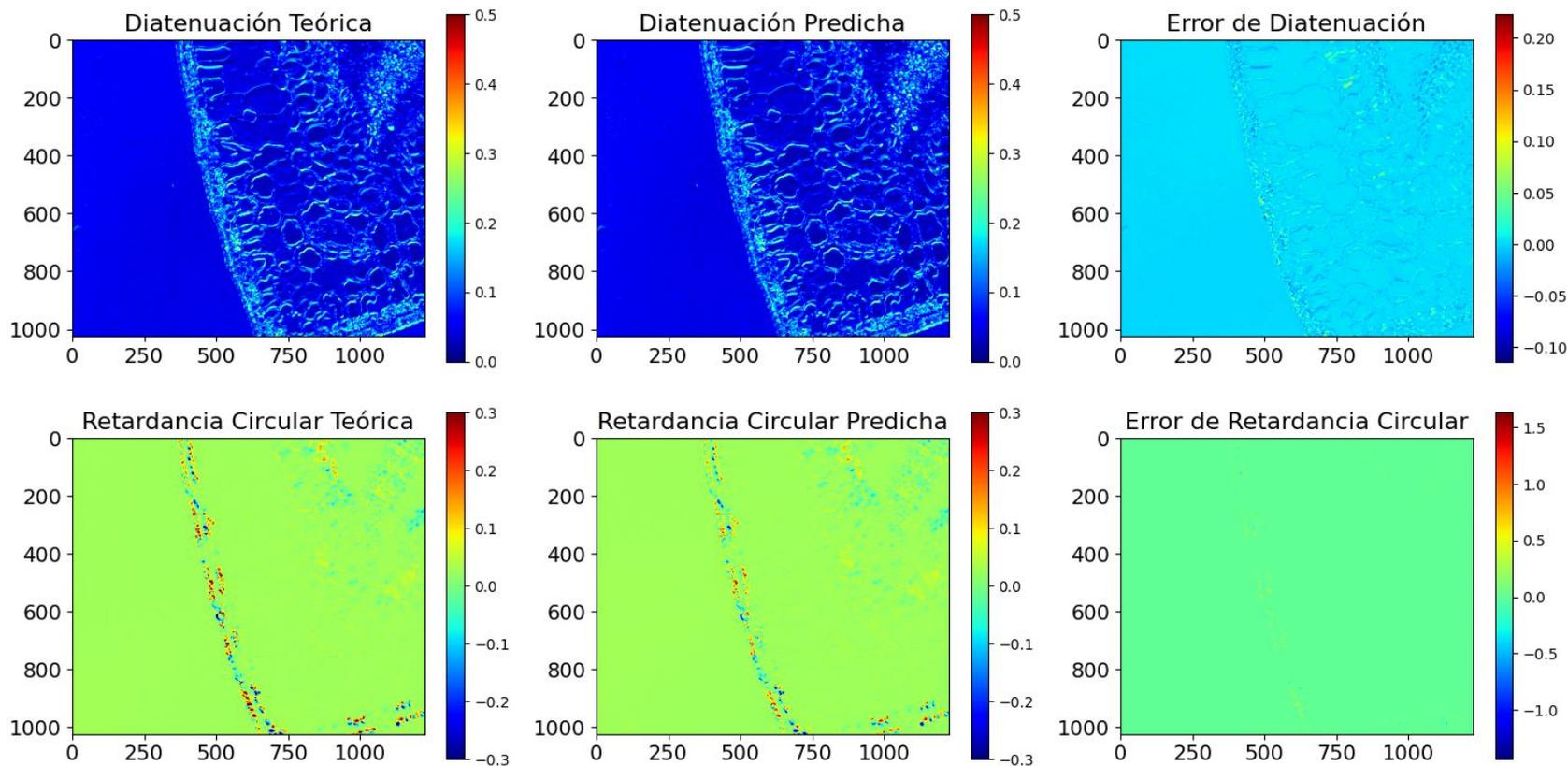
Tejido Denso Conectivo (otra zona)



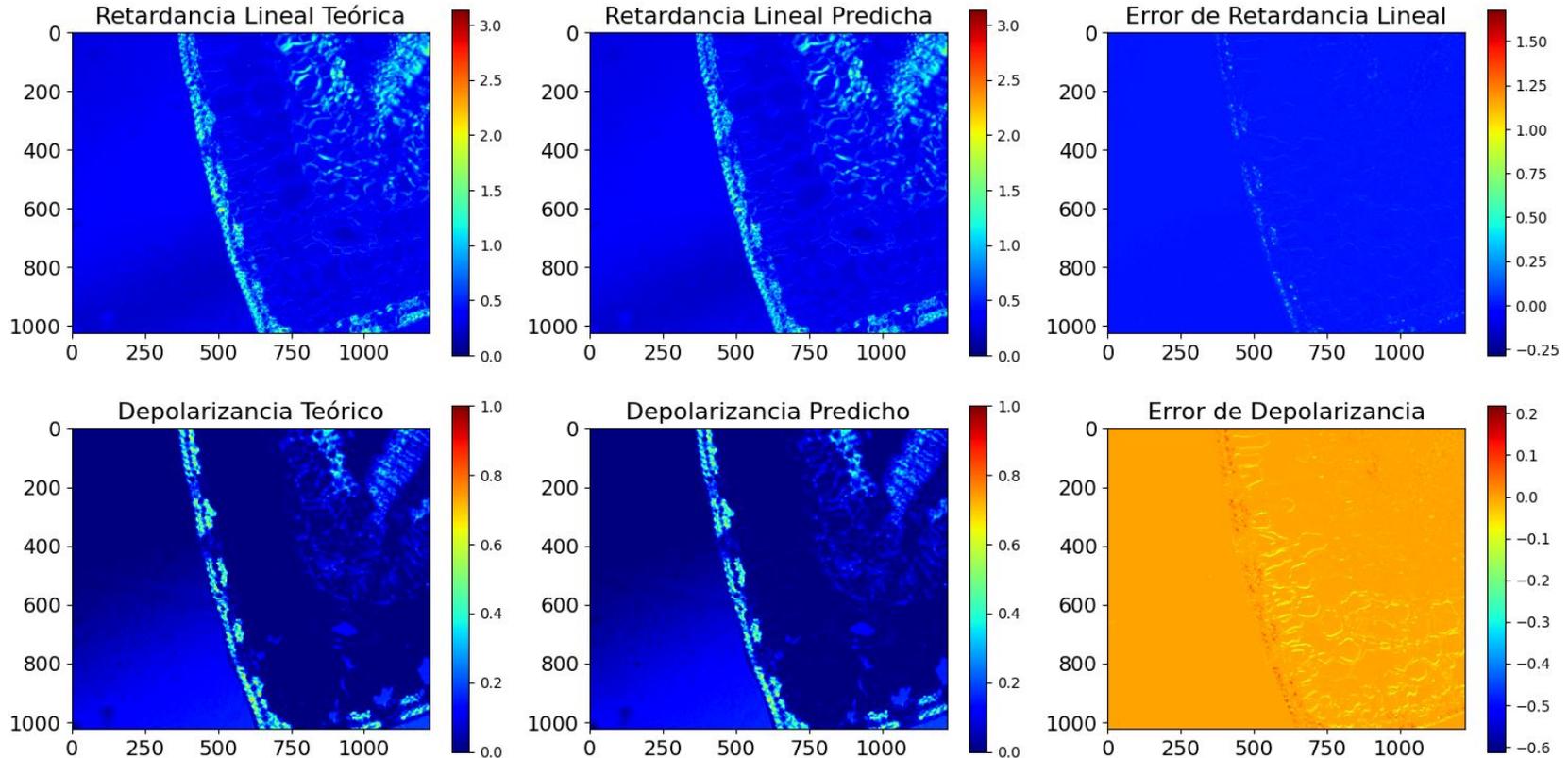
Tejido Denso Conectivo (otra zona)



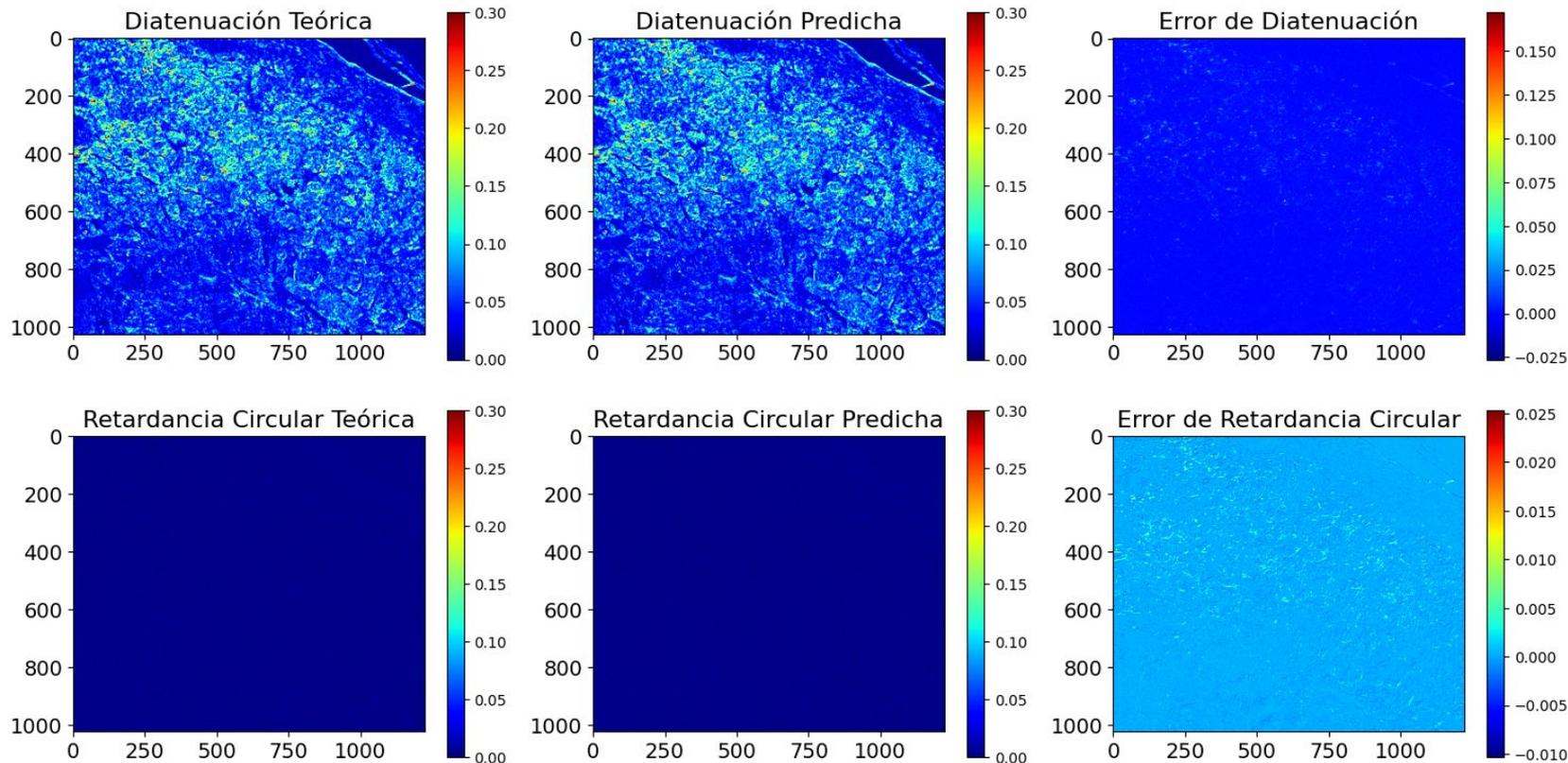
Tallo de Pino



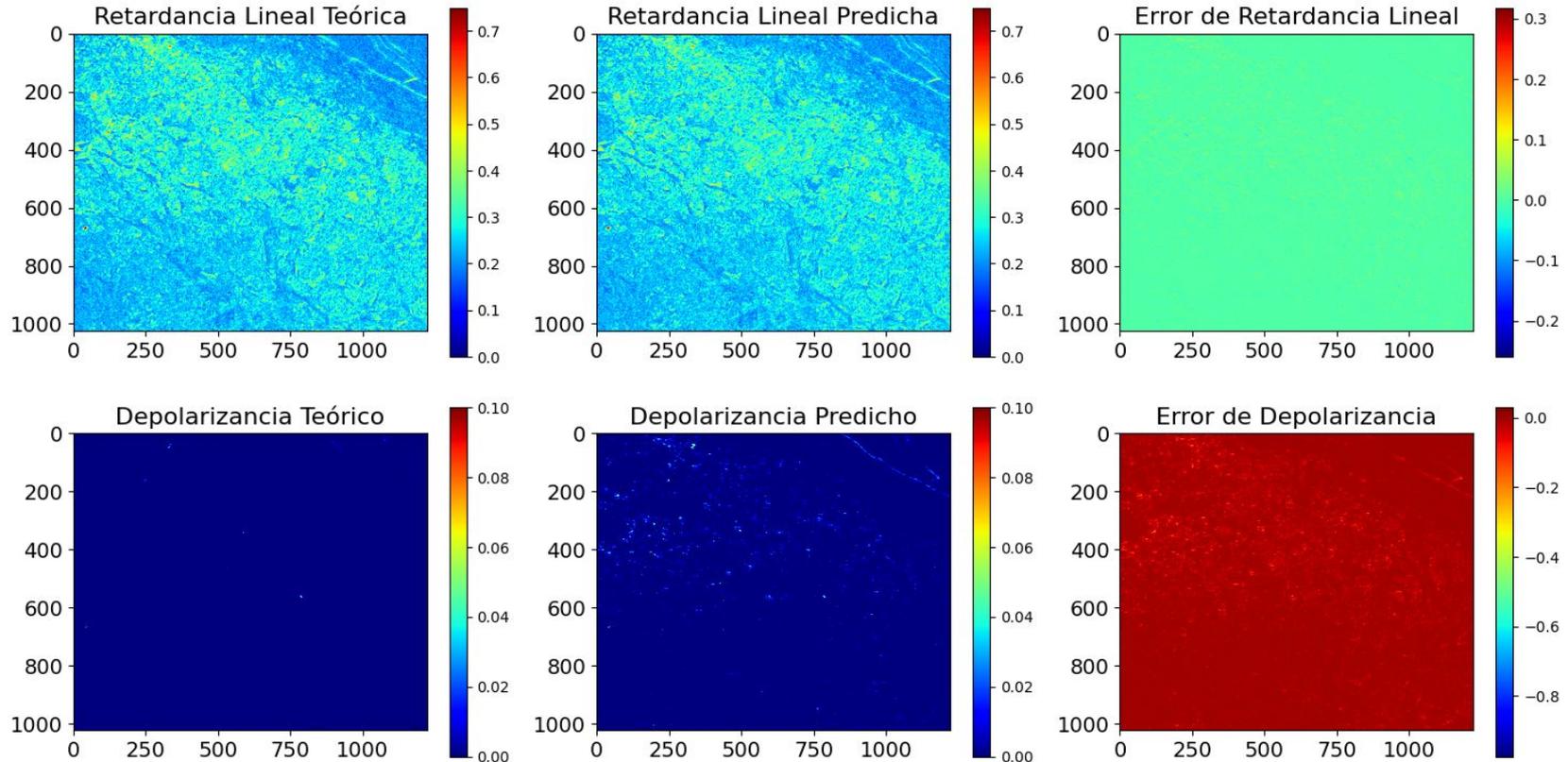
Tallo de Pino



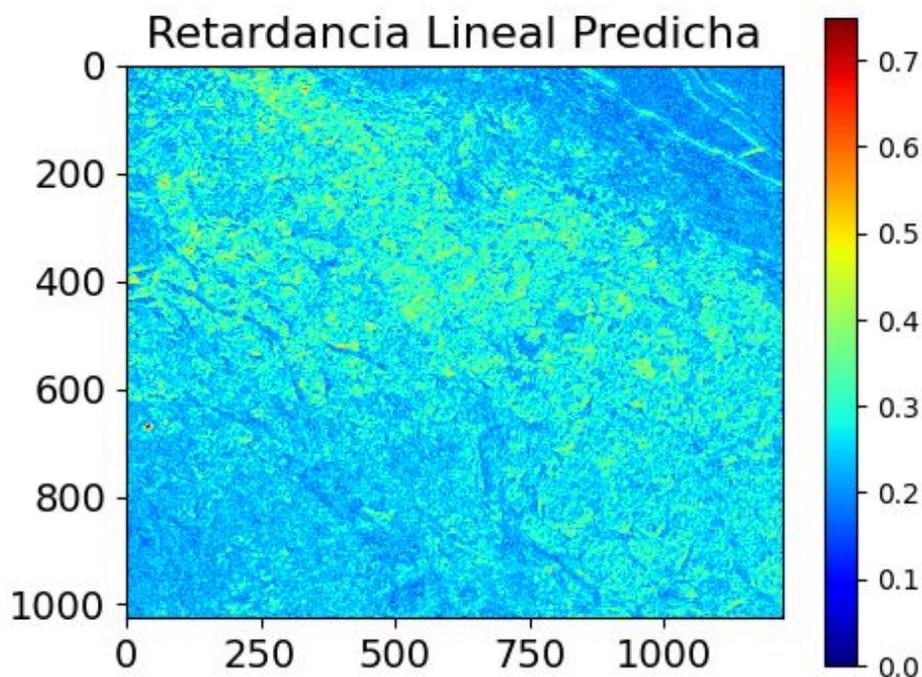
Piel Humana Nevo



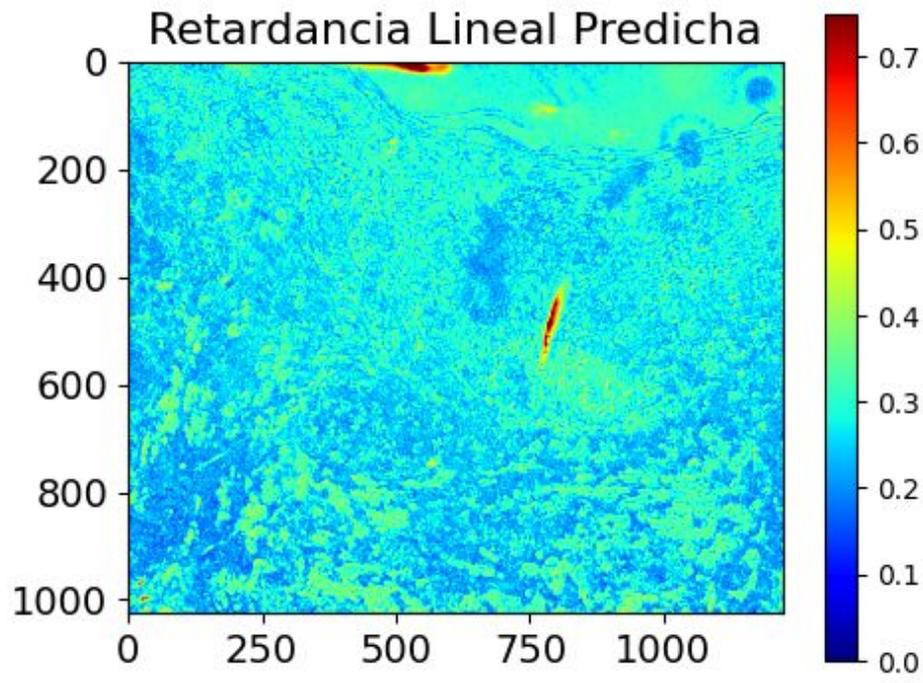
Piel Humana Nevo



Aplicación: Detección de Melanoma

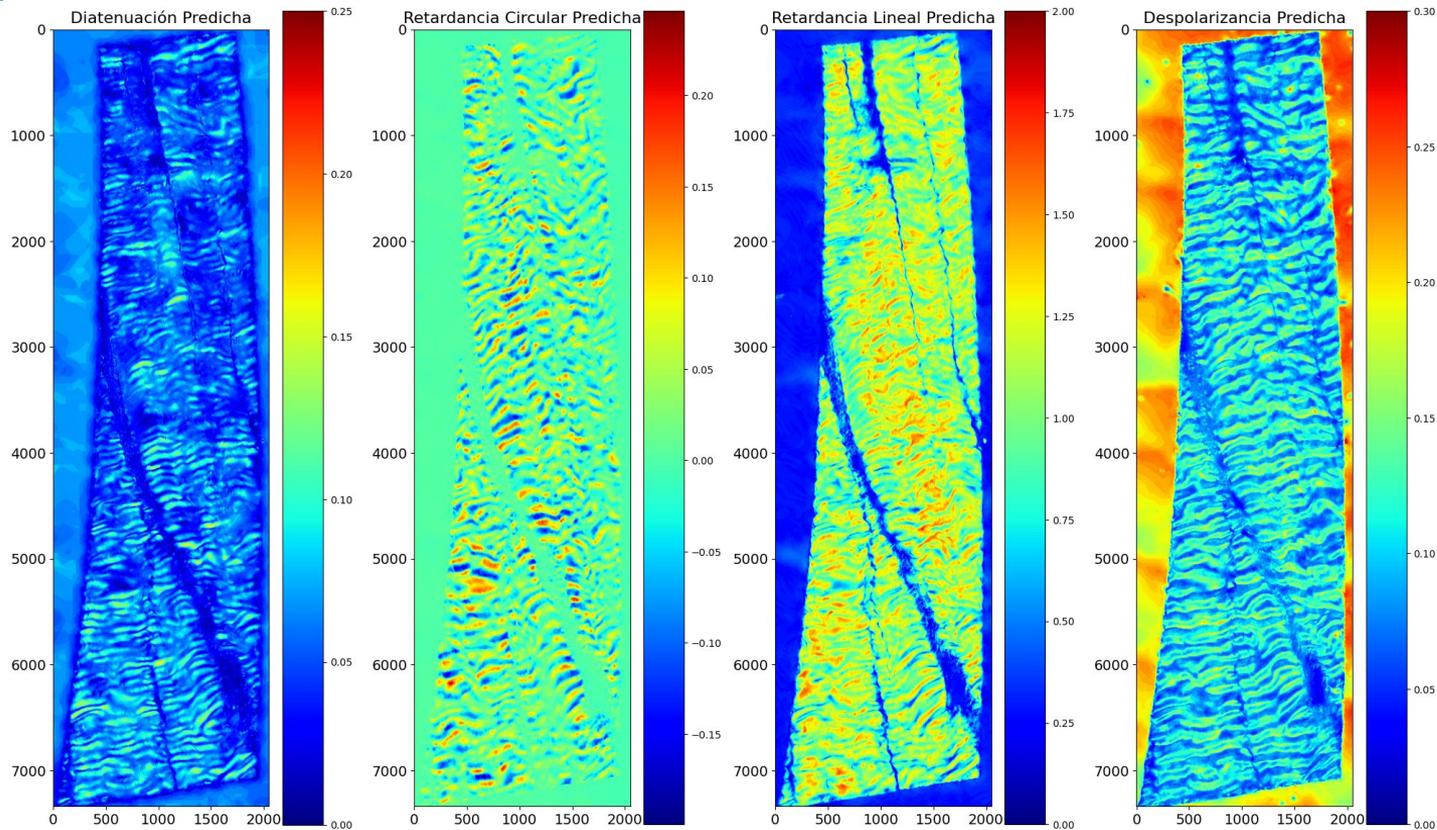


Nevo

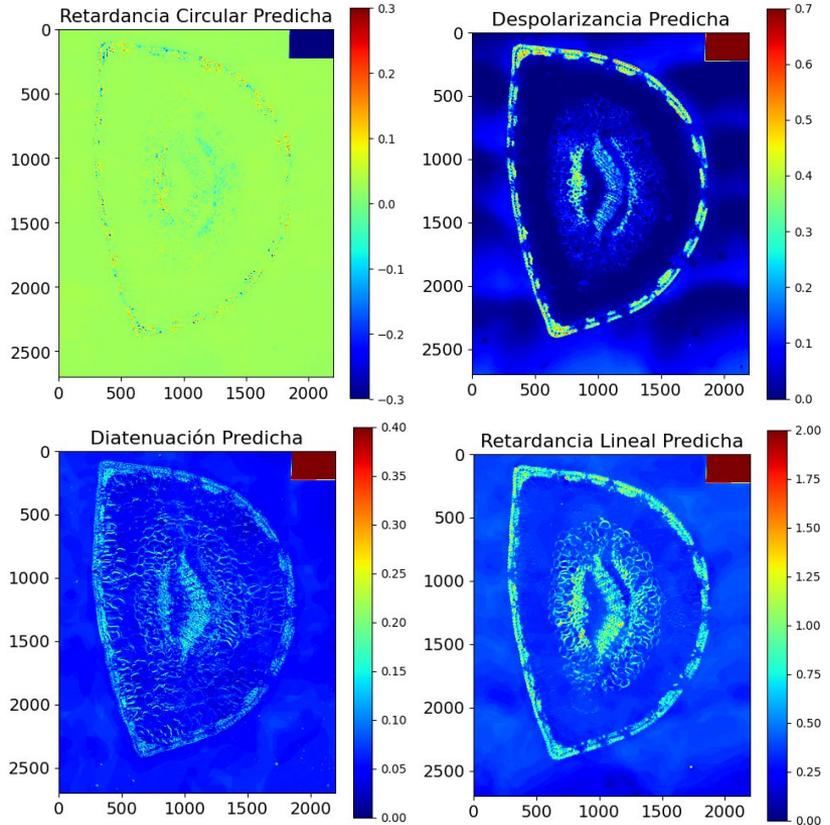
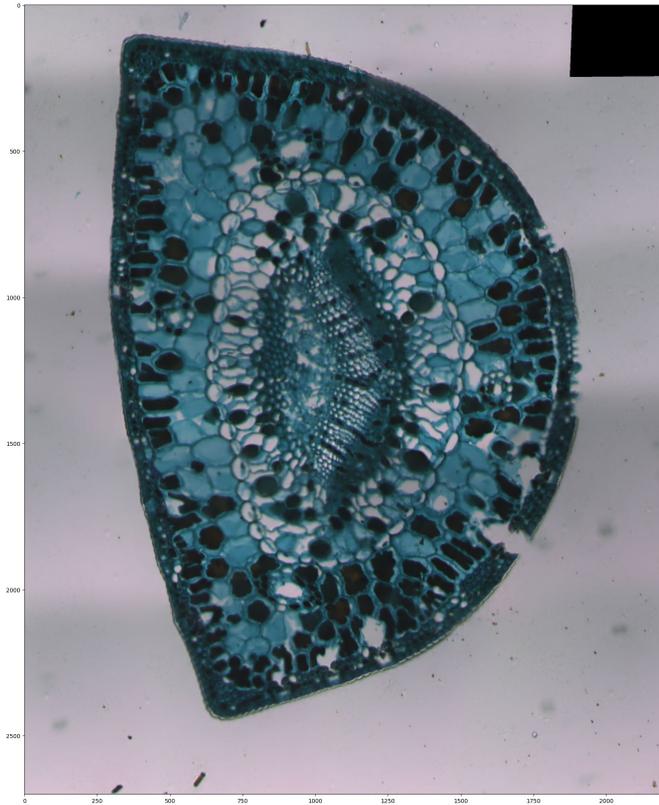


Melanoma

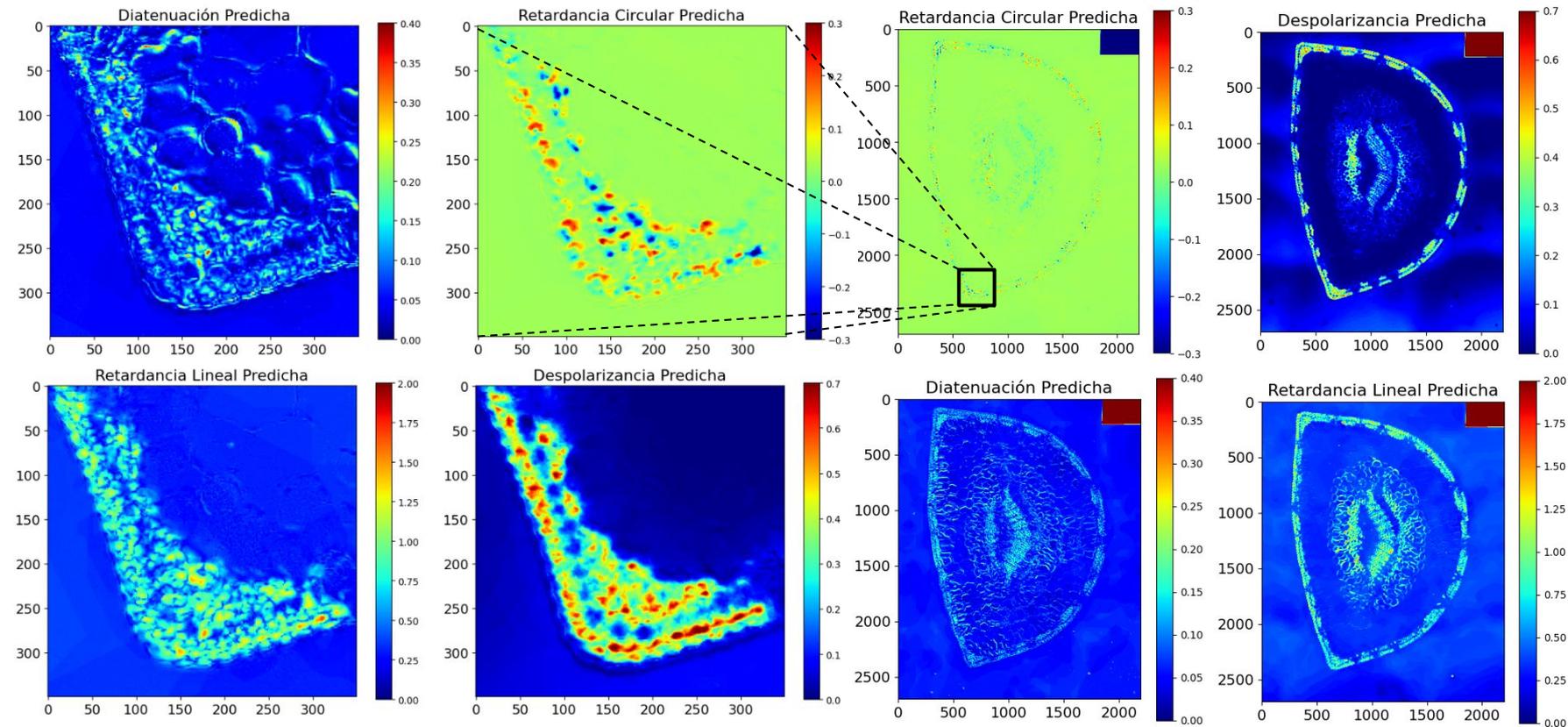
Aplicación: Whole Slide Imaging



Aplicación: Whole Slide Imaging



Whole Slide Imaging



Resultados SSIM

Muestra	Diatenuación	Retardo Lineal	Retardo Circular	Despolarizancia
Denso Conectivo	0.997	0.999	0.999	0.997
Tallo de Pino	0.993	0.996	0.997	0.913
Piel Humana	0.995	0.999	0.995	0.746

Conclusiones y Trabajo a Futuro

- **Agiliza el cálculo de Lu-Chipman:**
 - **Más rápidas**
 - **Menos memoria**
- **Dónde se entrenan, son precisas**
- **Futuro: Agilizar cálculo de Mueller**