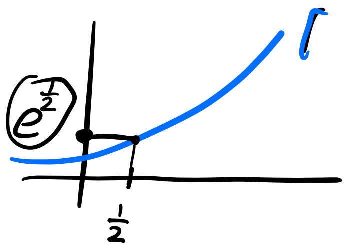
$\left(|\beta - \alpha| < \frac{1}{100}\right)$

$\beta = 1{,}231...$

$\rightsquigarrow \alpha = 1{,}2???$

$\beta = 1{,}29.....$

En este caso $\alpha = 1{,}???$



Queremos conocer $e^{\frac{1}{2}}$ hasta el primer dígito después de la coma

Calculemos entonces $e^{\frac{1}{2}}$ con un error menor a $\frac{1}{100}$

Recordamos que $P_n(e^x, 0) = 1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!}$

$e^x = P_n(e^x, 0) + R_n(e^x, 0)$;

$e^{\frac{1}{2}} = P_n(e^x, 0)(\frac{1}{2}) + R_n(e^x, 0)(\frac{1}{2})$; queremos entonces que $|R_n(e^x, 0)(\frac{1}{2})| < \frac{1}{100}$

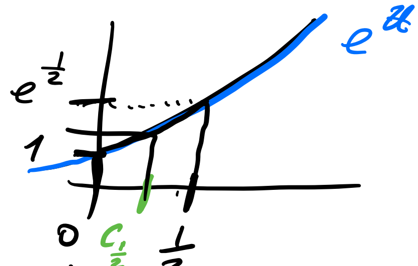
La fórmula del resto de Lagrange:

$$R_n(f, 0)(x) = \frac{f^{(n+1)}(c_x)\, x^{n+1}}{(n+1)!}$$

c_x está entre 0 y x.

En nuestro caso $f = e^x \Rightarrow f^{(n+1)} = e^x$

$$R_n(e^x, 0)(\tfrac{1}{2}) = \frac{e^{c_{\frac{1}{2}}} \cdot (\frac{1}{2})^{n+1}}{(n+1)!}$$



Como $c_x \in (0, \frac{1}{2}) \Rightarrow e^{c_{\frac{1}{2}}} \in (1, e^{\frac{1}{2}}) \Rightarrow |e^{c_{\frac{1}{2}}}| < e^{\frac{1}{2}} < 2$

$e <$ [illegible]

$$\Rightarrow |e^{\frac{c_1}{2}}| < 2 \qquad \boxed{e^{\frac{1}{2}} < 2}$$

Resumiendo:

$$\left|R_n(e^x, 0)\left(\frac{1}{2}\right)\right| \leq \frac{2 \cdot \left(\frac{1}{2}\right)^{n+1}}{(n+1)!} = \frac{\left(\frac{1}{2}\right)^n}{(n+1)!} = \frac{1}{2^n \cdot (n+1)!}$$

$$n = 3 \Rightarrow \frac{1}{2^3 \cdot (3+1)!} = \frac{1}{8 \cdot 24} = \frac{1}{192} < \frac{1}{100}$$

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Entonces:

$$\left|R_3(e^x, 0)\left(\frac{1}{2}\right)\right| < \frac{1}{100}$$

$$P_3(e^x, 0)\left(\frac{1}{2}\right) = 1 + \left(\frac{1}{2}\right) + \frac{\left(\frac{1}{2}\right)^2}{2} + \frac{\left(\frac{1}{2}\right)^3}{3!} =$$

$$= 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{8 \cdot 6} = 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{48} =$$

$$= 1{,}6458 \ldots$$

Entonces sabemos que los primeros tres dígitos pueden ser 1,65 ; 1,64 o 1,63.
Eso permite asegurar que los primeros dos dígitos son 1,6

$$\left|R_n(\text{sen}, 0)(x)\right| = \left|\frac{f^{(n+1)}(c_x)(x)^{n+1}}{(n+1)!}\right| \leq \frac{x^{n+1}}{(n+1)!}$$

$$f^{(n+1)} \in \left\{ \begin{matrix} \text{sen} \\ \cos \\ -\text{sen} \\ -\cos \end{matrix} \right. \Rightarrow \left| f^{(n+1)}(c_x) \right| \leq 1$$

$$\int_c^{x^2} f(t)\,dt = \frac{1}{1+x^2} - c$$

$c \geq 0$

f continua

Hallar c y f.

sustituyendo x por $\sqrt{c}$:

$$0 = \int_c^{\overbrace{(\sqrt{c})^2}^{c}} f(t)\,dt = \frac{1}{1+(\sqrt{c})^2} - c = \frac{1}{1+c} - c$$

$$\Rightarrow \frac{1}{1+c} - \frac{c(1+c)}{(1+c)} = \frac{1-c-c^2}{1+c}$$

c es raíz de $-x^2 - x + 1$

$1 \pm \sqrt{1-4(1)}$

$$\Rightarrow c = \frac{1 \pm \sqrt{(-1)^2 - 4(-1)}}{-2} = \frac{1 \pm \sqrt{1+4}}{-2} = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

Como $c > 0$ por letra $\Rightarrow$ $c = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

$$\int^{x^2} f(t)\,dt \quad \frac{1}{\ } - c$$

$$\int_c f(t)\,dt = \frac{}{1+x^2}$$

$$\left(\int_c^{x^2} f(t)\,dt\right)' = \left(F(x^2)\right)' \quad \text{donde} \quad F = \int_c^{x} f(t)\,dt$$

$$\underset{T.F.C}{\Longrightarrow} F' = f(t)$$

Entonces $\left(F(x^2)\right)' \underset{\text{Regla de la cadena}}{=} F'(x^2).2x = f(x^2).2x$

Por otro lado $\left(\frac{1}{1+x^2} - c\right)' = \frac{-2x}{(1+x^2)^2}$

Igualando las derivadas obtenemos

$$f(x^2).2x = -2x \cdot \frac{1}{(1+x^2)^2} \Longrightarrow f(x^2) = \frac{-1}{(1+x^2)^2}$$

$$\Longrightarrow f(x) = \frac{-1}{(1+x)^2}$$
