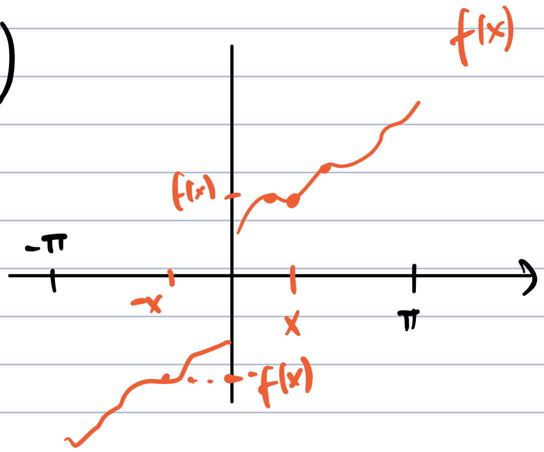
4.)



$$f(x) = \sum_{1}^{+\infty} a_n \operatorname{sen}(nx)$$

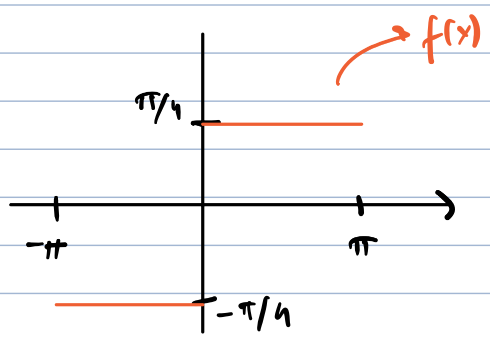
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

- f impar $\Rightarrow$ $f(x)\cos(nx)$ impar $\rightarrow \int = 0 = a_n$
- f par $\Rightarrow$ $f(x)\cos(nx)$ par $\rightarrow \int_{-\pi}^{\pi} = 2\int_{0}^{\pi}$

$$b_n = \frac{1}{\pi}\int f(x)\operatorname{sen}(nx)\,dx$$

- f impar $\Rightarrow$ $f(x)\cdot\operatorname{sen}(nx)$ par $\rightarrow \int_{-\pi}^{\pi} = 2\int_{0}^{\pi}$
- f par $\Rightarrow$ $f(x)\operatorname{sen}(nx)$ impar $\rightarrow \int = 0 = b_n$

En este caso queremos la expresión de la cte $\pi/4$ en términos de $\operatorname{sen}(nx)$



$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{\overset{\text{impar}}{f(x)}\cdot\overset{\text{par}}{\cos(nx)}}_{\text{impar}}\,dx = 0$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = 0.$$

Calculamos serie de Fourier de f:

$$b_n = \frac{1}{\pi}\left[\int_{0}^{\pi}\frac{\pi}{4}\cdot\operatorname{sen}(nx) + \int_{-\pi}^{0} -\frac{\pi}{4}\operatorname{sen}(nx)\right]$$

$$= \frac{1}{4}\left(\left.-\frac{\cos(nx)}{n}\right|_{0}^{\pi} + \left.\frac{\cos(nx)}{n}\right|_{-\pi}^{0}\right)$$

$$= \frac{1}{4}\left(\frac{-\cos(n\pi)+1}{n} + \frac{1-\cos(-n\pi)}{n}\right)$$

$$= \frac{1}{4}\left(-\frac{(-1)^n}{n} + \frac{1}{n} + \frac{1}{n} - \frac{(-1)^n}{n}\right) = \frac{1}{2}\left(\frac{1-(-1)^n}{n}\right)$$

$$= \begin{cases} \frac{1}{n} & \text{si } n \text{ impar} \\ 0 & \text{si } n \text{ par.} \end{cases}$$

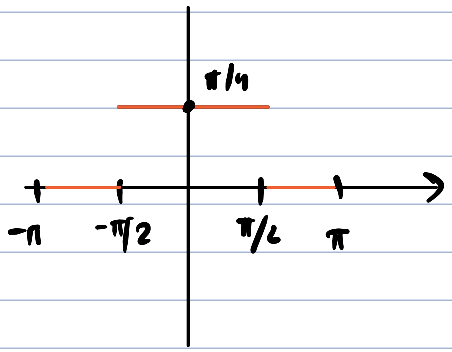
$$\Rightarrow \quad S_\infty f(x) = \sum_{0}^{+\infty} \frac{1}{2n+1}\operatorname{sen}((2n+1)x) = \operatorname{sen}(x) + \frac{\operatorname{sen}(3x)}{3} + \frac{\operatorname{sen}(5x)}{5} + \dots$$

Sabemos que, como f de clase C^1 en $(0,\pi)$, por teo. Dini

$$S_\infty f(x) = f(x) = \pi/4. \quad \forall x \in (0,\pi)$$

Por lo tanto $\forall x \in (0,\pi)$ $\quad \pi/4 = \operatorname{sen}(x) + \frac{\operatorname{sen}(3x)}{3} + \frac{\operatorname{sen}(5x)}{5} + \dots$

$$\left[\pi/4 = \frac{a_0}{2} + \sum_1^{+\infty} a_n \cdot \cos(nx)\right] \forall x \in (-\pi/2, \pi/2)$$



7) • ¿$\exists f$ tq $S_\infty f(x) = \sum_1^{+\infty} \frac{\operatorname{sen}(nx)}{\sqrt{n}}$?

$a_n = 0$

$b_n = 1/\sqrt{n}$.

Si f 2π-periódica, cont a trozos,

$$S_n f(x) = \frac{a_0}{2} + \sum_1^{+\infty} a_n \cos(nx) + \sum_1^{+\infty} b_n \operatorname{sen}(nx) \Rightarrow \sum_1^{+\infty} |a_n|^2 + |b_n|^2 \le \int_{-\pi}^{\pi} |f|^2 = \text{cte}.$$

En el caso del ejercicio tendríamos $\sum |b_n|^2 = \sum \frac{1}{n} = +\infty$.

Por lo tanto no existe tal f.