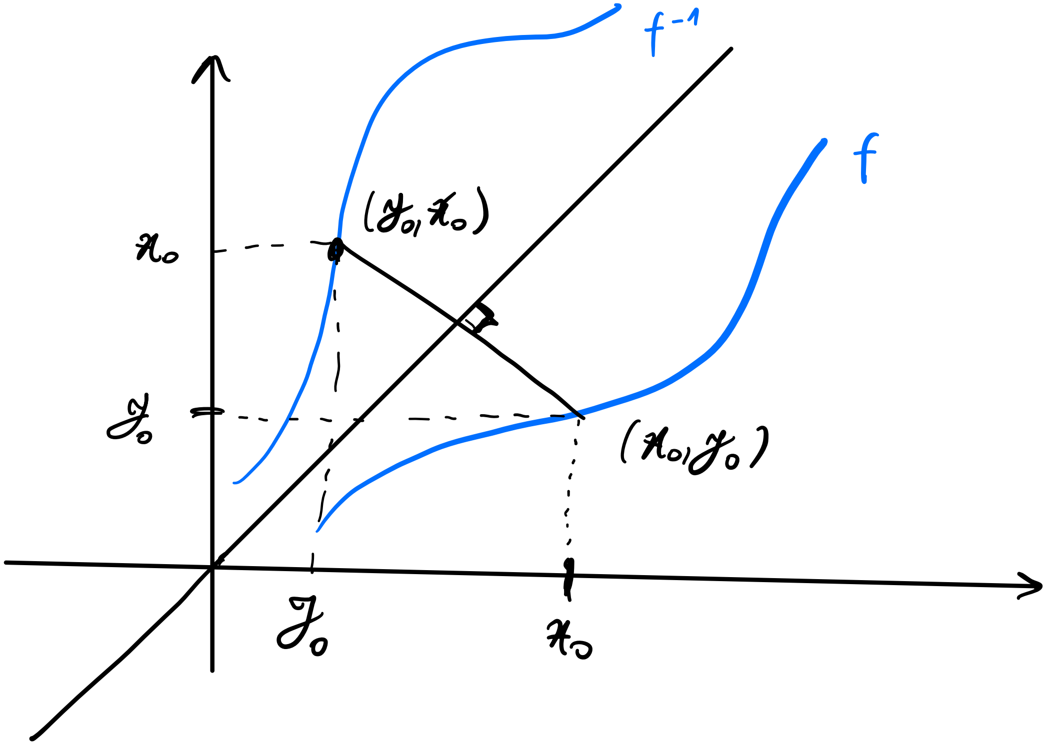
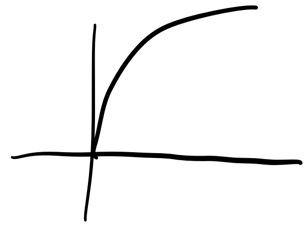
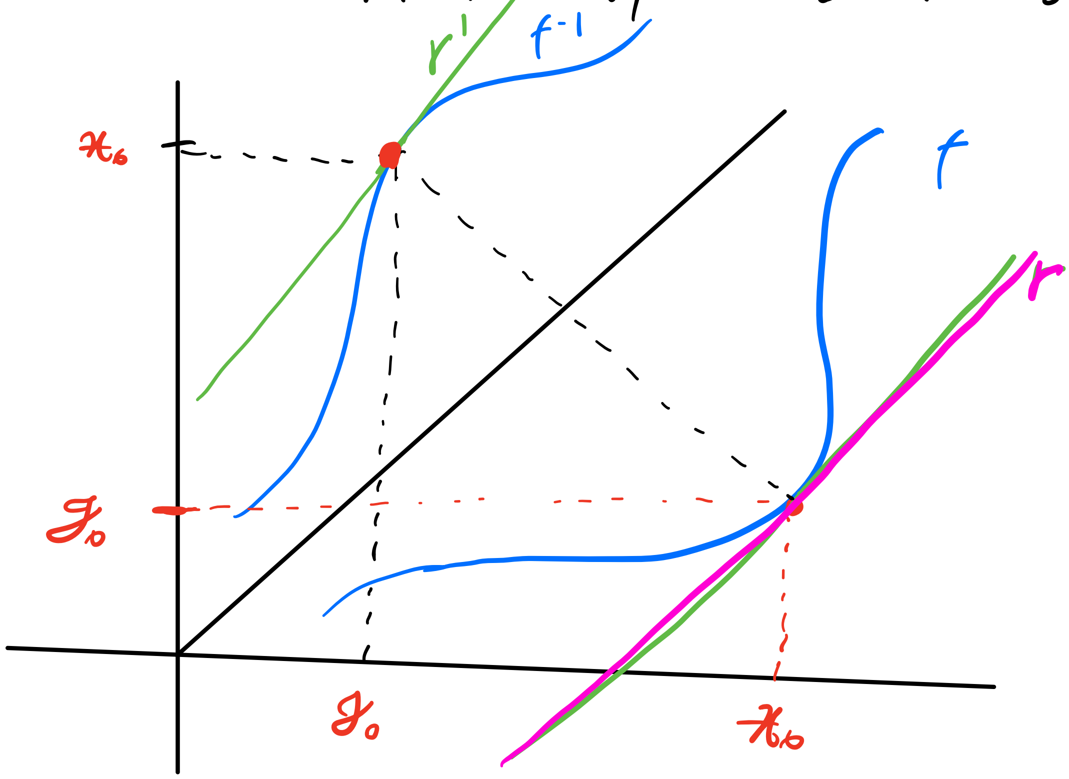


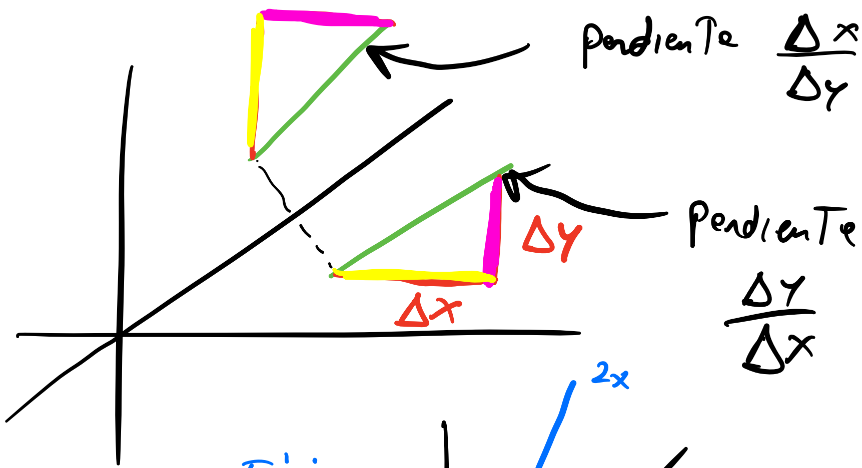


Recordar que el gráfico de f^{-1} se obtiene del gráfico de realizarle al gráfico de f una simetría axial respecto de la diagonal

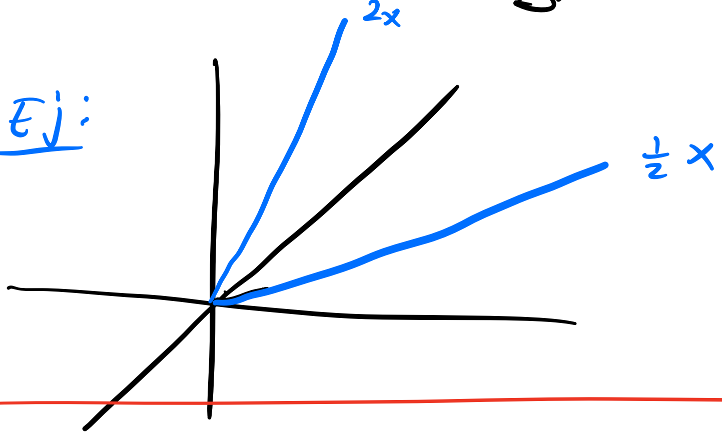


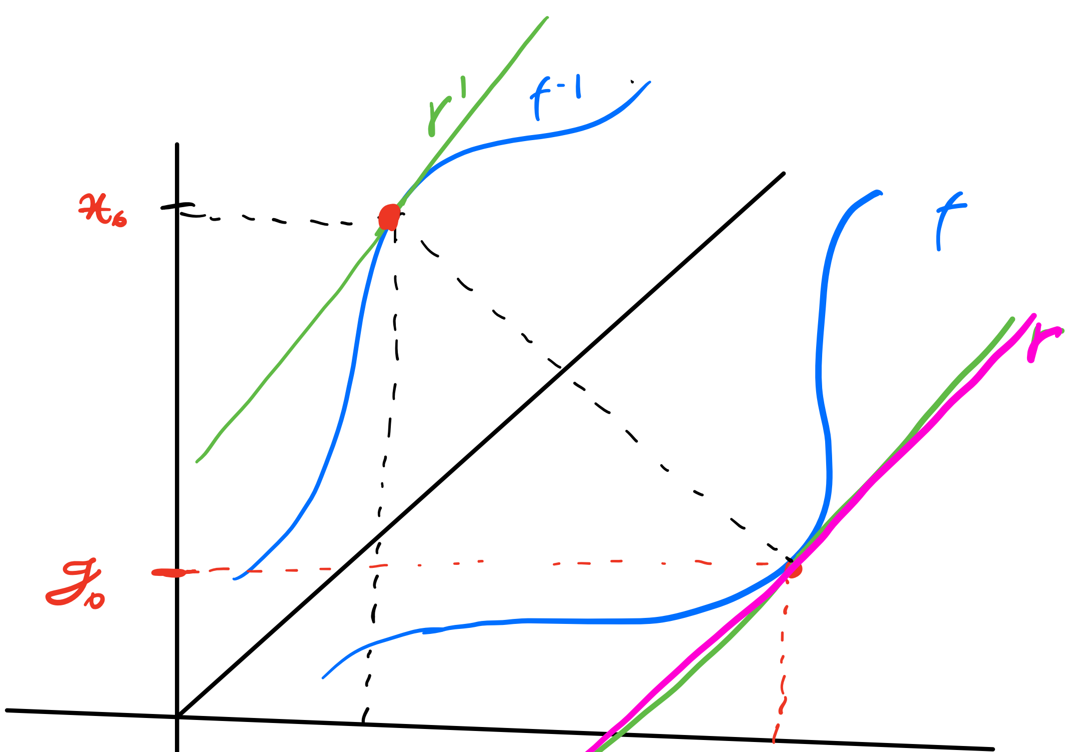
r' se obtiene al aplicar la simetría axial a la recta r.

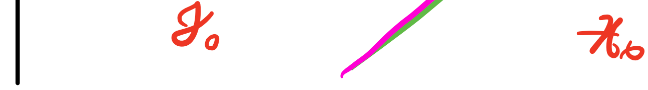
Cuando aplicamos una simetría axial respecto de la diagonal a una recta de pendiente α obtenemos otra recta cuya pendiente es $\frac{1}{\alpha}$



Ej:







$(f^{-1})'(y_0)$ es igual a la pendiente de r'

que es igual al inverso de la pendiente de r.

$$y_0 = f(x_0)$$
$$x_0 = f^{-1}(y_0)$$

$$\boxed{(f^{-1})'(y_0) = \frac{1}{f'(x_0)} = \frac{1}{f'(f^{-1}(y_0))}}$$

Otra forma de deducir la fórmula

$f: I \longrightarrow J$

$f^{-1}: J \longrightarrow I$

derivables

$f^{-1}\circ f: I \longrightarrow I$

$(f^{-1}\circ f)(x) = x$

$$(f^{-1}\circ f)' \underset{\text{Regla de la Cadena}}{=} (f^{-1})'\circ f \cdot f'$$

$$1 = (x)' = \left(f^{-1}(f(x))\right)' = (f^{-1})'(f(x)) \cdot f'(x)$$

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)} \xrightarrow[x = f^{-1}(x)]{} (f^{-1})'(\underbrace{f(f^{-1}(x))}_{x}) = \frac{1}{f'(f^{-1}(x))}$$

$$\longrightarrow \boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

__Teorema__: Sea $f: I \longrightarrow J$ derivable y biyectiva. Además $f'(x) > 0 \ \forall x \in I$

$\Longrightarrow f^{-1}: J \longrightarrow I$ es derivable y

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

LA EXPONENCIA COMO LA INVERSA DEL LOGARITMO

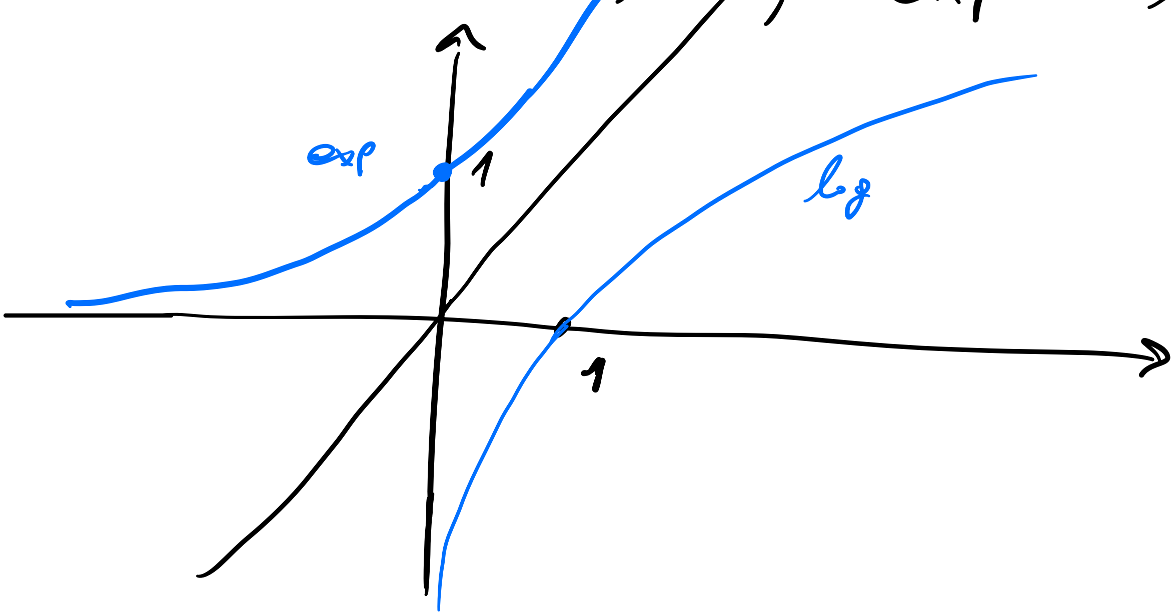
Recordamos: $\log : (0,+\infty) \longrightarrow \mathbb{R}$

la definimos como $\log(x) = \int_1^x \frac{1}{t}\,dt$

calculamos $\boxed{\log'(x) = \frac{1}{x}}$

Definimos la función exponencial como

$\exp : \mathbb{R} \longrightarrow (0,+\infty)$; $\exp := \log^{-1}$



$\exp(x) = e^x$

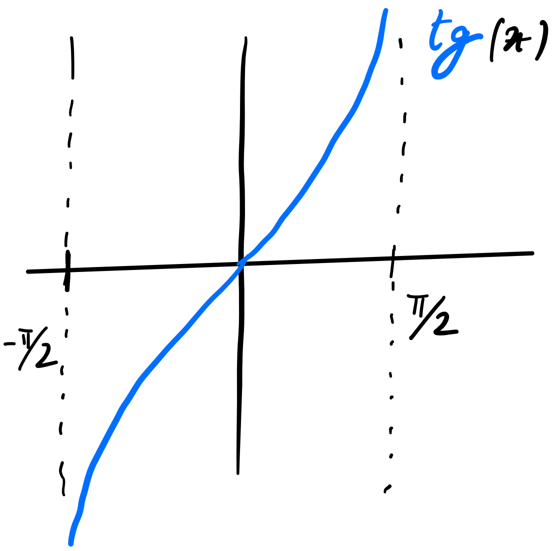
$(e^x)' = (\log^{-1}(x))' = \frac{1}{\log'(\log^{-1}(x))} =$

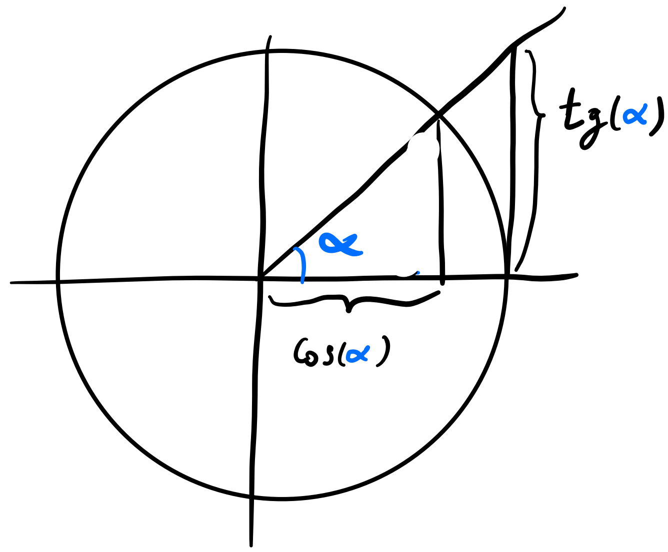
Derivada

de la
inversa

$$= \frac{1}{\log'(e^x)} = \frac{1}{\left(\frac{1}{e^x}\right)} = e^x$$

DERIVADA DE ARCTAN

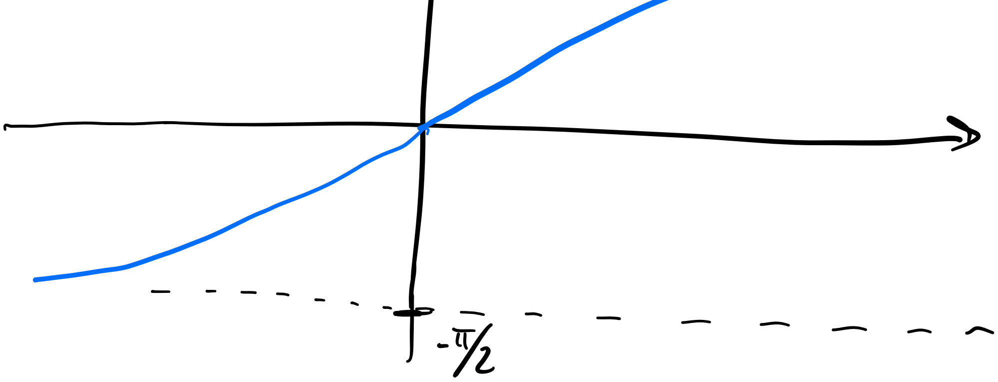




$$\left(tg(x)\right)' = \left(\frac{sen(x)}{cos(x)}\right)' = \frac{(sen(x))'cos(x) - sen(x)(cos(x))'}{cos^2(x)}$$

$$= \frac{cos^2(x) + sen^2(x)}{cos^2(x)} = \frac{1}{cos^2(x)}$$

$$arctg : \mathbb{R} \longrightarrow (-\pi/2, \pi/2)$$



$$(\operatorname{arctg})'(x) = \frac{1}{tg'(\operatorname{arctg}(x))} = \frac{1}{1+tg^2(\operatorname{arctg}(x))} =$$

$$= \frac{1}{1+x^2}$$

$$tg' = 1 + tg^2$$

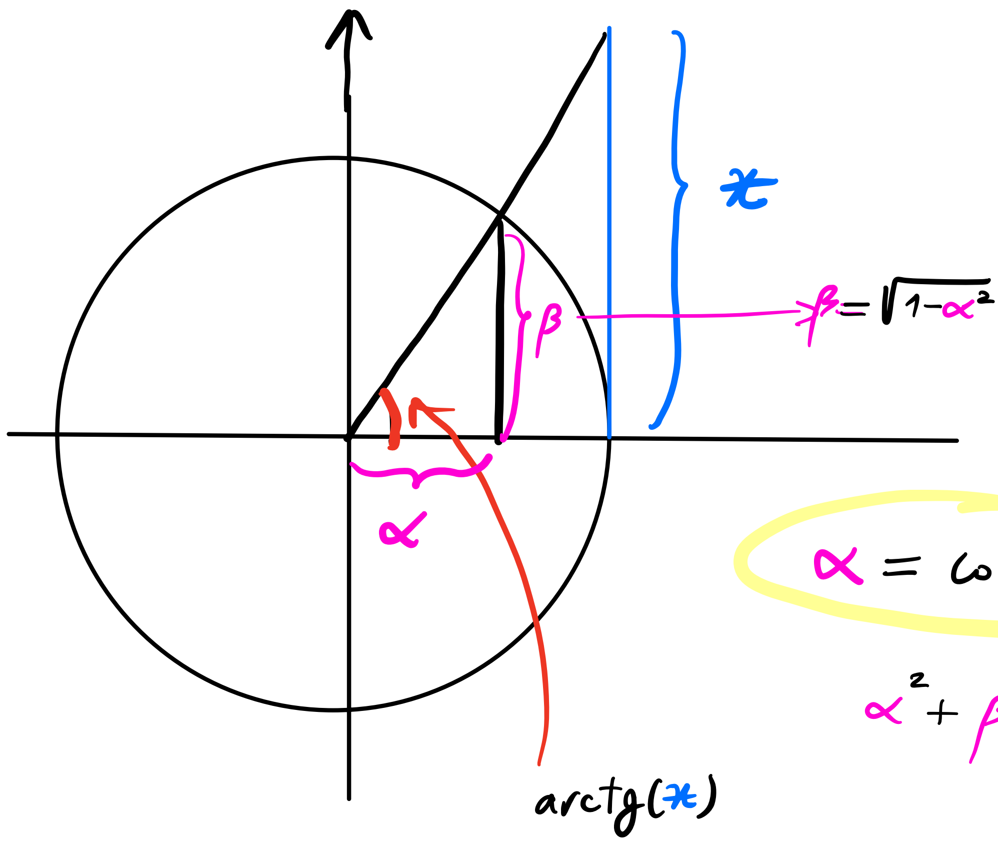
OTRA MANERA QUE TAMBIÉN SIRVE PARA CALCULAR (ARCSEN)'

$\text{Y} \quad (\text{ARCCOS})'$

$$\operatorname{arctg}'(x) = (tg^{-1})'(x) = \frac{1}{tg'(\operatorname{arctg}(x))} =$$

Derivada de la inversa

$$= \frac{1}{\left(\frac{1}{\cos^2(\operatorname{arctg}(x))}\right)} = \cos^2(\operatorname{arctg}(x))$$



$$\boxed{\alpha = \cos(\operatorname{arctg}(x))}$$

$$\alpha^2 + \beta^2 = 1 \Rightarrow \beta = \sqrt{1-\alpha^2}$$

$$\frac{x}{1} = \frac{\beta}{\alpha} = \frac{\sqrt{1-\alpha^2}}{\alpha} \Rightarrow x = \frac{\sqrt{1-\alpha^2}}{\alpha} \Rightarrow$$

$$x\alpha = \sqrt{1-\alpha^2} \Longrightarrow x^2\alpha^2 = 1-\alpha^2 \Longrightarrow \alpha^2(x^2+1) = 1$$

$$\Longrightarrow \alpha^2 = \frac{1}{x^2+1}$$

$$\Longrightarrow$$

$$(\text{arctg})'(x) \overset{(*)}{=} \cos^2(\text{arctg}(x)) = \alpha^2 = \frac{1}{x^2+1}$$

$$\frac{\cos^2(x) + \text{sen}^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$\parallel$$

$$1 + \left(\frac{\text{sen}(x)}{\cos(x)}\right)^2 = 1 + \text{tg}^2(x) = \text{tg}'(x)$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$