REGLA DE LA CADENA

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

Ejemplos:

- $h(x) = \operatorname{sen}(3x^2+1)$

$h(x) = f(g(x))$ donde $g(x) = 3x^2+1$

$f(x) = \operatorname{sen}(x)$

Por la regla de la cadena:

$$h'(x) = \left(f(g(x))\right)' = f'(g(x)) \cdot g'(x) = \cos(3x^2+1) \cdot 6x$$

$f'(x) = \cos(x)$

$g'(x) = 6x$

- $h(x) = (x+1)^{100}$

$h(x) = f(g(x))$ donde $f(x) = x^{100}$

$g(x) = x+1$

$$h'(x) = \big(f(g(x))\big)' \underset{\substack{\uparrow \\ \text{Regla} \\ \text{de la} \\ \text{cadena}}}{=} f'(g(x)) \cdot g'(x) = 100\,(x+1)^{99} \cdot 1 =$$

$$= 100(x+1)^{99}$$

$f'(x) = 100 . x^{99}$

$g'(x) = 1$

$\circledast$ $\varphi(x) = \operatorname{sen}\big(\log(x^2+5)\big)$

$\varphi(x) = f(g(x))$ donde $f(x) = \operatorname{sen}(x)$

$g(x) = \log(x^2+5)$

$$\varphi'(x) = \big(f(g(x))\big)' = f'(g(x)) \cdot g'(x) = \boxed{\cos\big(\log(x^2+5)\big) \cdot \frac{2x}{x^2+5}}$$

$f'(x) = \cos(x)$

$$g'(x) = \big(\log(x^2+5)\big)' = \frac{2x}{x^2+5}$$

Calculemos $g'(x)$

$$g(x) = \log(x^2+5) = h(k(x)) \quad \text{donde} \quad h(x) = \log(x), \quad k(x) = x^2+5$$

$$\boxed{g'(x)} = h(k(x))' = h'(k(x)).\,k'(x) = \frac{1}{x^2+5}\cdot 2x = \boxed{\frac{2x}{x^2+5}}$$

Regla de la cadena

$$h'(x) = \frac{1}{x}$$

$$k'(x) = 2x$$

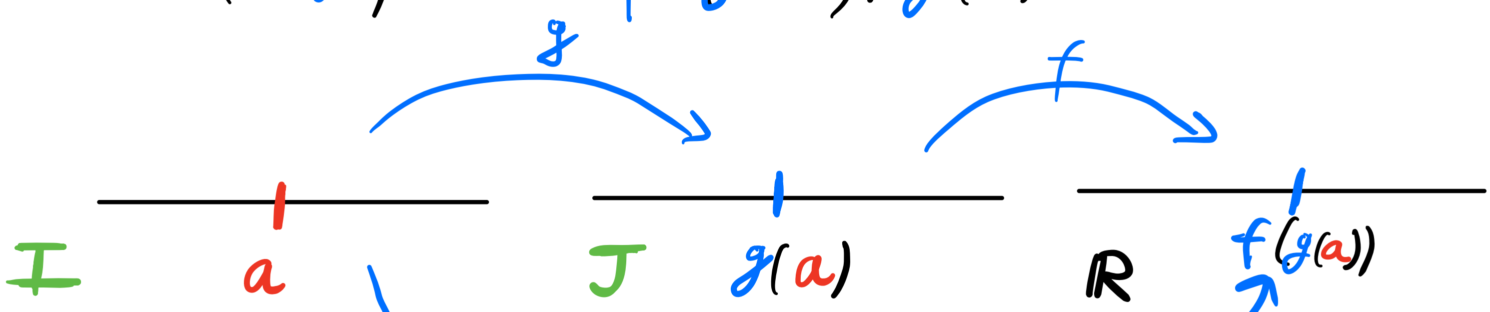
¿PORQUÉ ES CIERTA LA REGLA DE LA CADENA?

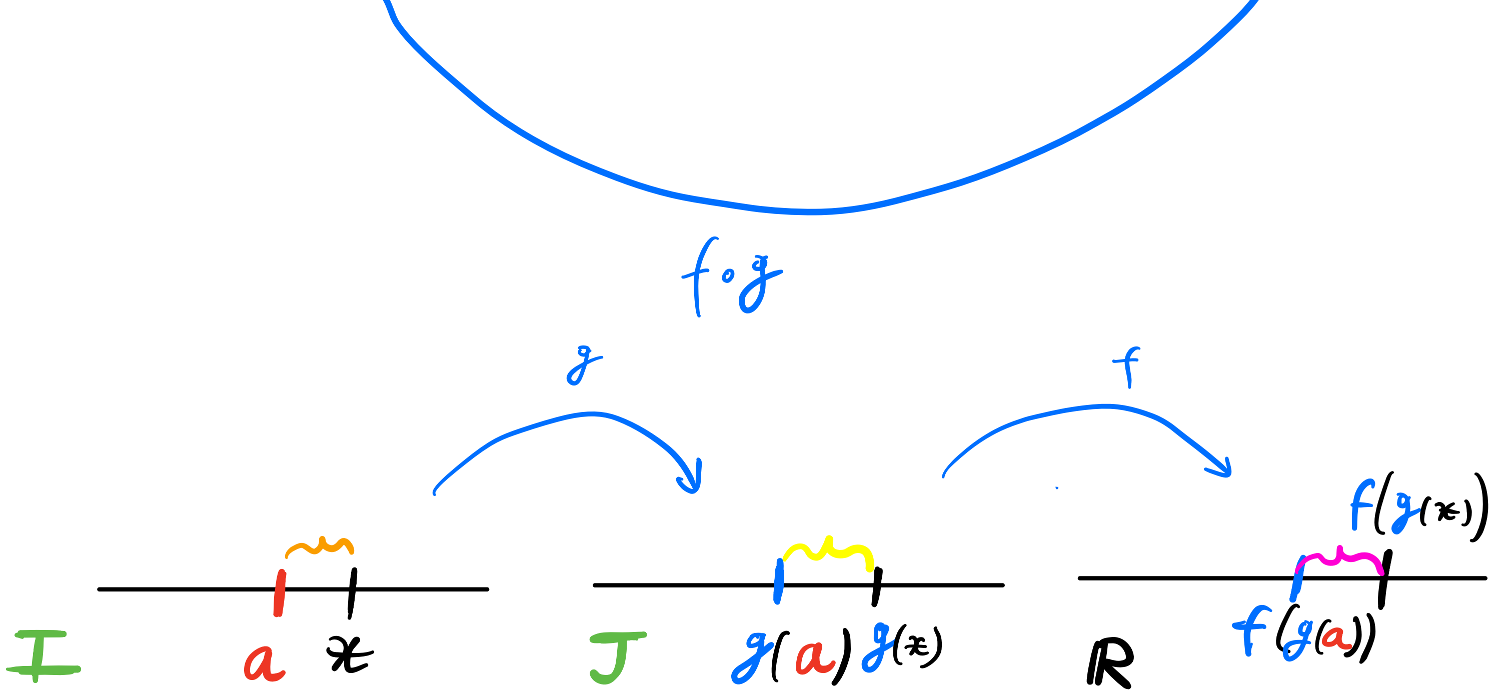
Enunciado formal

Proposición: Sean $g : I \to J$, $f : J \to \mathbb{R}$ funciones tales que: g es derivable en $a \in I$, f es derivable en $g(a)$.

Entonces, $f(g(x))$ es derivable en a y

$$(f \circ g)'(a) = f'(g(a)).\,g'(a)$$





$$(f\circ g)'(a) = \lim_{x\to a} \frac{f(g(x)) - f(g(a))}{x-a}$$

$$\frac{f(g(x)) - f(g(a))}{x-a} = \frac{f(g(x)) - f(g(a))}{g(x)-g(a)} \cdot \frac{g(x)-g(a)}{x-a}$$

"CASI DEMOSTRACIÓN DE LA REGLA CADENA"

$$\lim_{x\to a} \frac{f(g(x)) - f(g(a))}{x-a} =$$

$f'(g(a))$

$g'(a)$

$x \to a$

$$\lim_{x \to a} \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \cdot \frac{g(x) - g(a)}{x - a} = \boxed{f'(g(a)) \cdot g'(a)}$$

(*)

Limite del Producto

(*) $$\lim_{x \to a} \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} = \lim_{u \to g(a)} \frac{f(u) - f(g(a))}{u - g(a)} =$$

C.V

$u = g(x)$

$$= f'(g(a))$$

DERIVADA DEL COCIENTE

$$f(x) = \frac{1}{x}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} =$$

$$= \lim_{x \to a} \frac{\left(\left(\frac{1}{x}\right) - \left(\frac{1}{a}\right)\right)}{x - a} = \lim_{x \to a} \frac{\left(\frac{a}{ax} - \frac{x}{ax}\right)}{x - a} =$$

$$= \lim_{x \to a} \frac{\left(\frac{-(x-a)}{a \cdot x}\right)}{x - a} = \lim_{x \to a} \left(\frac{-\cancel{(x-a)}}{a \cdot x}\right) \cdot \frac{1}{\cancel{(x-a)}} =$$

$$= \lim_{x \to a} \frac{-1}{ax} = \boxed{\frac{-1}{a^2}}$$

a^2

$$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$\varphi(x) = \frac{1}{g(x)} = h(g(x)) \text{ donde } h(x) = \frac{1}{x}$$

$$h'(x) = -\frac{1}{x^2}$$

$$\varphi'(x) = \Big(h(g(x))\Big)' = h'(g(x))\, g'(x) = \frac{-1}{(g(x))^2} \cdot g'(x)$$

Regla de la cadena

$$\left(\frac{1}{g(x)}\right)' = \boxed{\frac{-g'(x)}{g^2(x)}}$$

Regla de Leibnitz

$$(f \cdot g)' = f' g + f \cdot g'$$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x) \cdot \frac{1}{g(x)}\right)' =$$

Regla de Leibnitz

$$= f'(x) \cdot \left(\frac{1}{g(x)}\right) + f(x) \boxed{\left(\frac{1}{g(x)}\right)'} =$$

$$= \frac{f'(x)}{g(x)} + f(x)\boxed{\frac{(-g'(x))}{g^2(x)}} = \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g^2(x)} =$$

Regla de la cadena

$$= \frac{f'(x)g(x)}{g^2(x)} - \frac{f(x)g'(x)}{g^2(x)} = \boxed{\frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}}$$