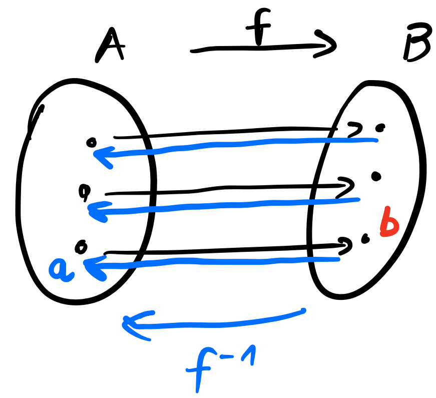


Decimos que $f : A \to B$ es biyectiva si

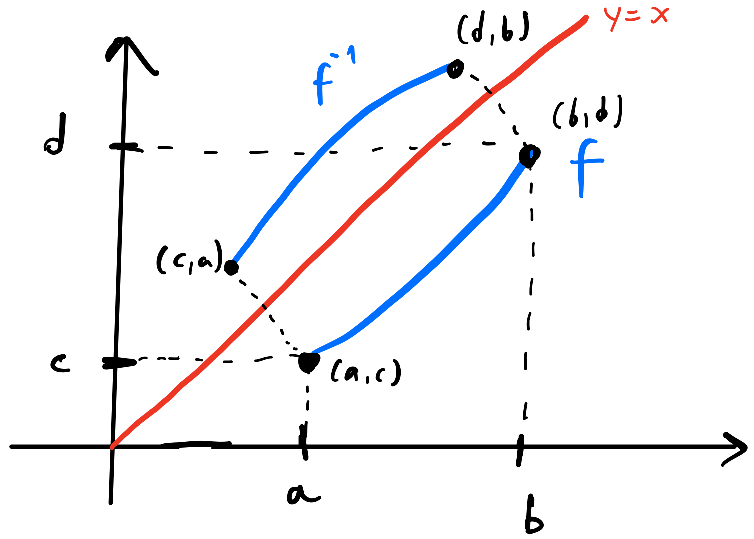
$\forall b \in B, \exists!\, a \in A \,/\, f(a) = b$

Si $f : A \to B$ es biyectiva podemos definir

$f^{-1} : B \longrightarrow A$ como $f^{-1}(b)$ es el único elemento de A que cuando le aplico f me da b

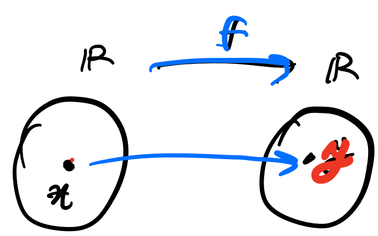
"En términos del "diagrama" la inversa se obtiene dando vuelta las flechas"

INVERSAS DE FUNCIONES REALES

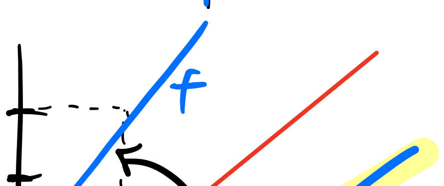


$f : [a,b] \to [c,d]$ biyectiva

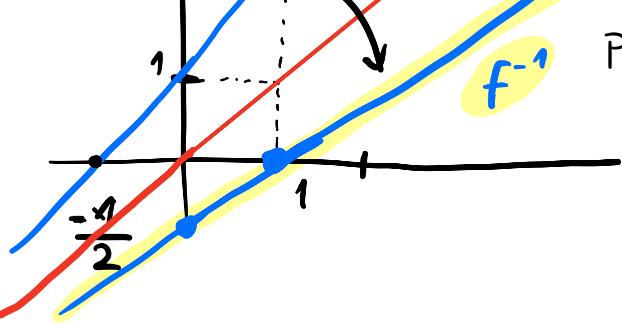
$f^{-1} : [c,d] \to [a,b]$



Ejemplo: $f : \mathbb{R} \to \mathbb{R}$; $f(x) = 2x + 1$; f es biyectiva



$f^{-1} : \mathbb{R} \to \mathbb{R}$



Para conocer $f^{-1}(y)$ debemos resolver

$$f(x) = y \iff 2x+1 = y$$

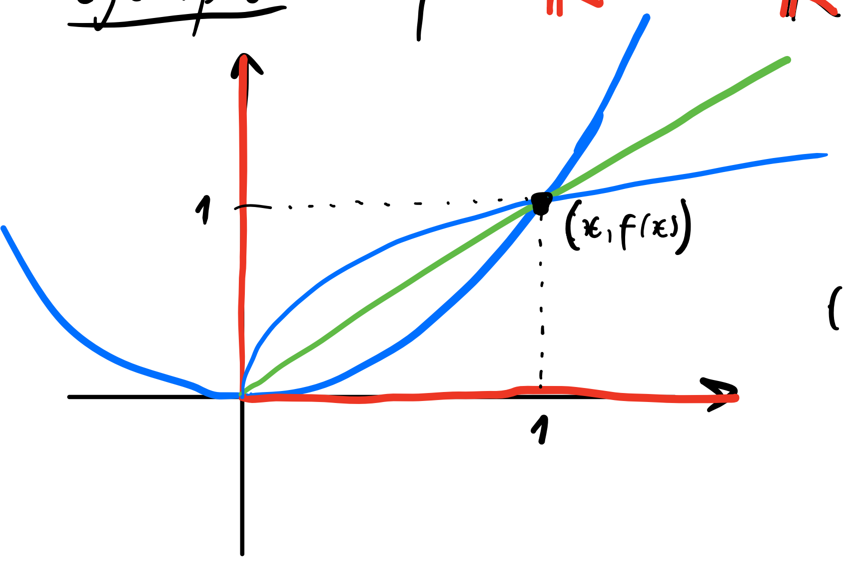
$$\Rightarrow x = \frac{y-1}{2} = \boxed{\frac{y}{2} - \frac{1}{2}}$$

$$f^{-1}(x) = \frac{x}{2} - \frac{1}{2}$$

$$f^{-1}(1) = \frac{1}{2} - \frac{1}{2} = 0$$

Proposición: Sea $f : [a,b] \longrightarrow [c,d]$ biyectiva y continua $\Rightarrow f^{-1} : [c,d] \longrightarrow [a,b]$ es continua

Ejemplo: $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$; $f(x) = x^2$

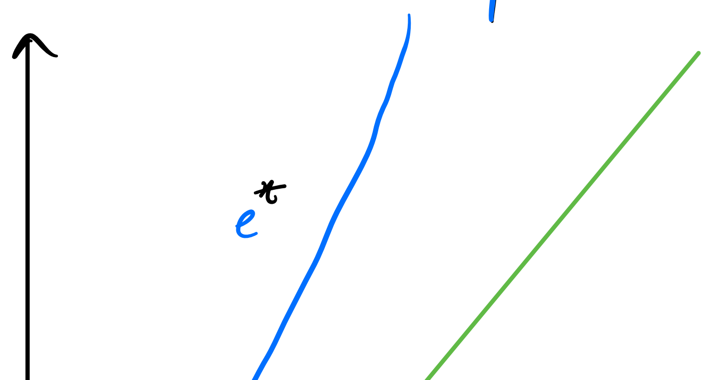


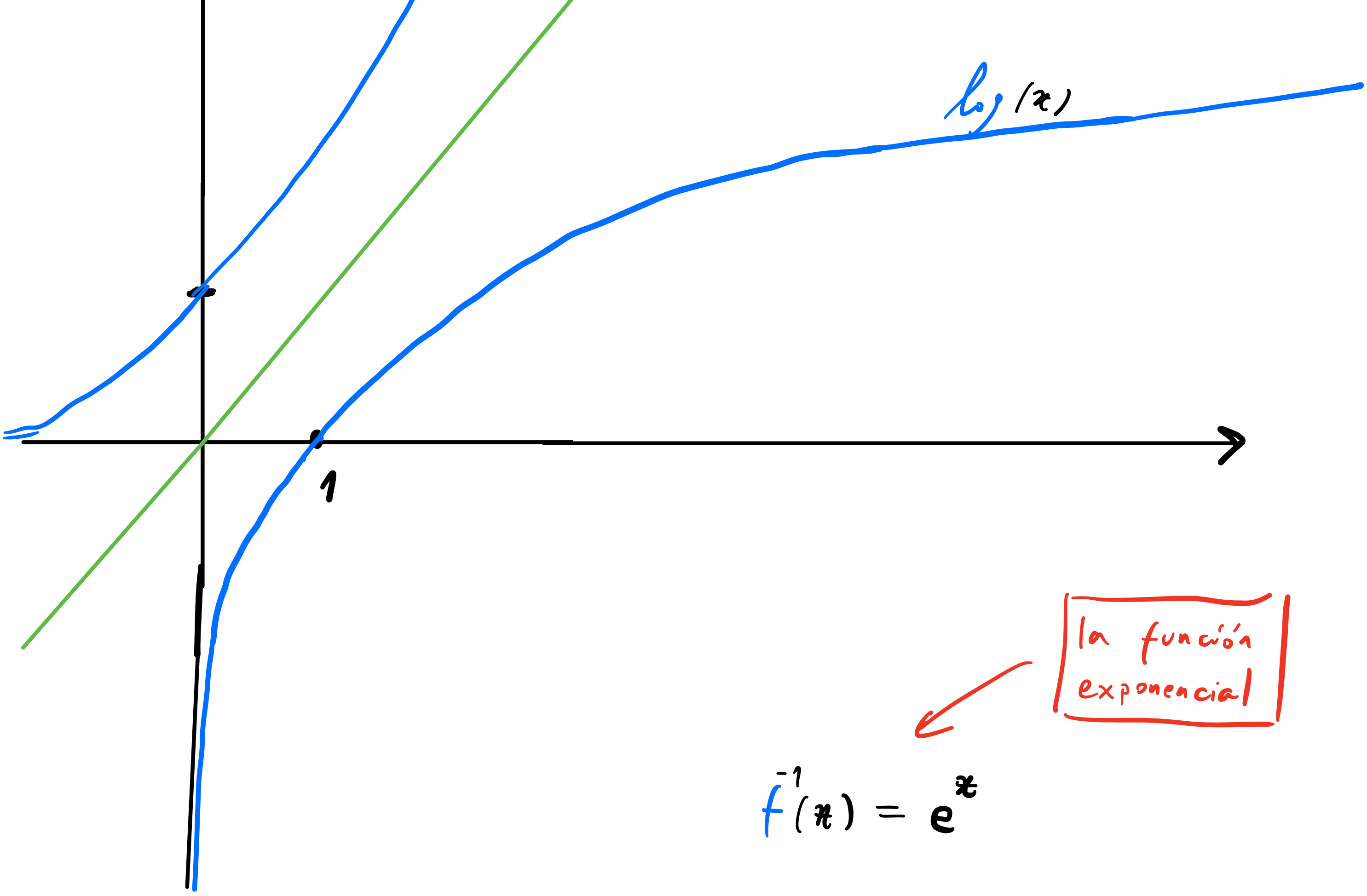
$f^{-1} : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ es $\boxed{f^{-1}(x) = \sqrt{x}}$

$$(x, x^2) = (x, x)$$

$$\Rightarrow x = x^2 \Rightarrow x = 1$$

$f : (0, +\infty) \longrightarrow \mathbb{R}$; $f(x) = \log(x)$

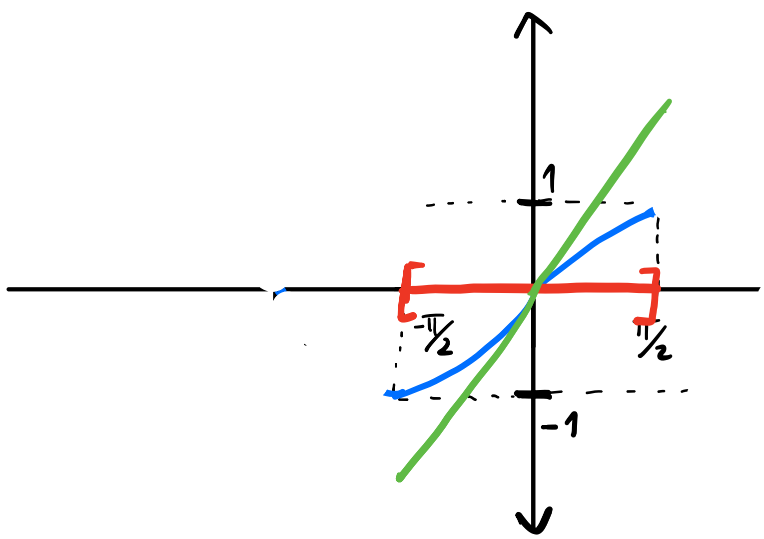


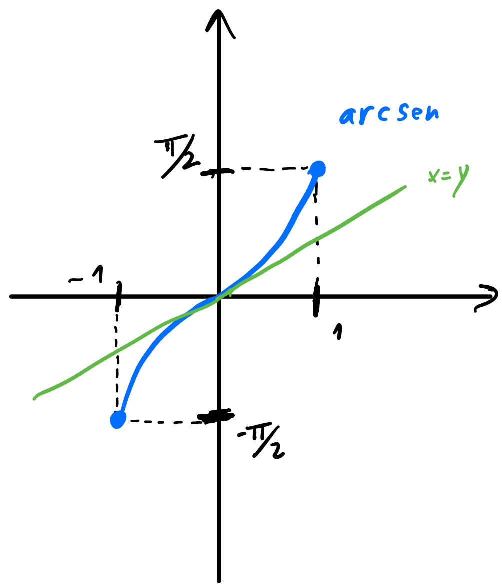


la función exponencial

$$f^{-1}(x) = e^{x}$$

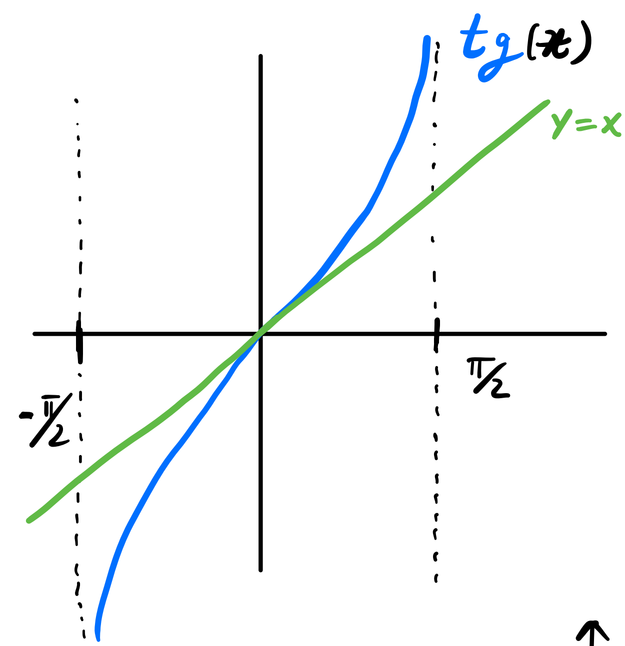
INVERSAS DE LAS FUNCIONES TRIGONOMÉTRICAS





$$\text{sen} : [-\pi/2, \pi/2] \longrightarrow [-1, 1]$$

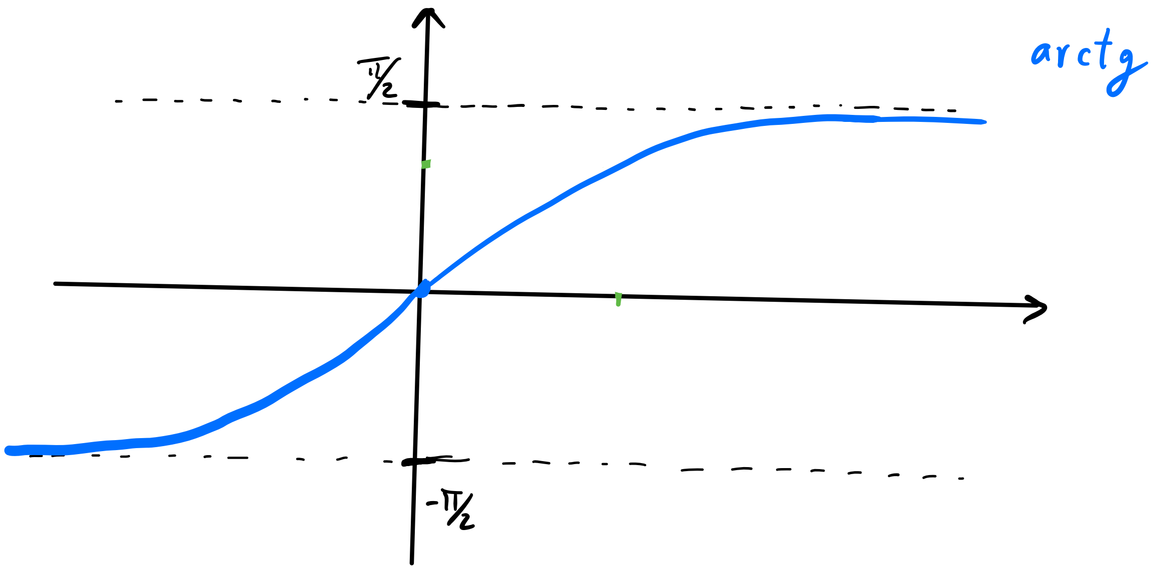
$$\text{sen}^{-1} : [-1, 1] \longrightarrow [-\pi/2, \pi/2] \quad ; \quad \text{sen}^{-1}(x) =: \text{arc sen}(x)$$



$tg : (-\pi/2, \pi/2) \longrightarrow \mathbb{R}$

es biyectiva

$tg^{-1} =: arctg : \mathbb{R} \longrightarrow (-\pi/2, \pi/2)$



$\cos : [0,\pi] \longrightarrow [-1,1]$

$\cos^{-1} =: \arccos : [-1,1] \longrightarrow [0,\pi]$