- Regresión lineal simple.

$$L(\theta) = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - f(x_i,\theta)\right)^2 \qquad \theta = (\beta_0, \beta_1)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \beta_0 - \beta_1 x_i\right)^2$$

$$(\hat{\beta}_0, \hat{\beta}_1) = \underset{(\beta_0,\beta_1)}{\mathrm{Argmin}}\ L(\beta_0, \beta_1)$$

$$\hat{\theta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix} = (X'X)^{-1}X'Y$$

- Regresión lineal multiple

Idem $\hat{\theta} = (X'X)^{-1}X'Y$

- Enfoque verosimilitud $\qquad y_i \in \mathbb{R},\ x_i \in \mathbb{R}^p$

$$y_i = \underbrace{f(x_i,\theta)}_{\theta' x_i} + \varepsilon_i \qquad \varepsilon_i \sim N(0,\sigma^2)$$

la densidad de y_i es

$$p(y_i \mid x_i, \theta, \sigma^2) = N(\theta' x_i, \sigma^2)$$

Si tengo n datos me fijo en:

$$p(\underbrace{y_1, \dots, y_n}_{y} \mid \underbrace{x_1, x_2, \dots, x_n}_{x}, \theta, \sigma^2)$$

la verosimilitud es $L = p(y \mid x, \theta, \sigma^2) = \prod_{i=1}^{N} N(\theta' x_i, \sigma^2)$

El logaritmo de la verosimilitud es:

$$\ln L = \sum_{i=1}^{N} \ln \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(y_i - f(x_i,\theta))^2} \right]$$

$$= -\frac{N}{2}\ln 2\pi - N\ln\sigma - \frac{1}{2\sigma^2}\sum_{i=1}^{N}(y_i - \theta' x_i)^2$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{1}{\sigma^2}\frac{\partial}{\partial\theta}\left[\sum_{i=1}^{N}(y_i - \theta' x_i)^2\right] = 0 \text{ ; es un vector!}$$

$$= \frac{1}{\sigma^2}\frac{\partial}{\partial\theta}\|Y - X\theta\|^2 = 0$$

$$= \frac{1}{\sigma^2}\frac{\partial}{\partial\theta}\left[(Y - X\theta)'(Y - X\theta)\right] = 0$$

$$= \frac{1}{\sigma^2}\frac{\partial}{\partial\theta}\left[Y'Y - Y'X\theta - \theta'X'Y + (X\theta)'(X\theta)\right] = 0$$

$$= \frac{1}{\sigma^2}\frac{\partial}{\partial\theta}\left(-Y'X\theta - \theta'X'Y + \theta'X'X\theta\right) = 0$$

$$\frac{\partial x'Ax}{\partial x} = 2Ax \Rightarrow \frac{\partial\, \theta'X'X\theta}{\partial\theta} = 2X'X\theta$$

$$\frac{\partial Ax}{\partial x} = A' \Rightarrow \frac{\partial(-Y'X\theta)}{\partial\theta} = (-Y'X)' = -X'Y$$

$$\frac{\partial x'y}{\partial x} = y \Rightarrow \frac{\partial(-\theta'X'Y)}{\partial\theta} = -X'Y$$

$$\Rightarrow \frac{\partial \ln L}{\partial\theta} = 0 \Leftrightarrow -X'Y - X'Y + 2X'X\theta = 0$$

$$\Rightarrow \boxed{\hat{\theta} = (X'X)^{-1}X'Y}$$