$\exists K>0 \,/$
$|f(x)|\leq K \ \forall x\in I$

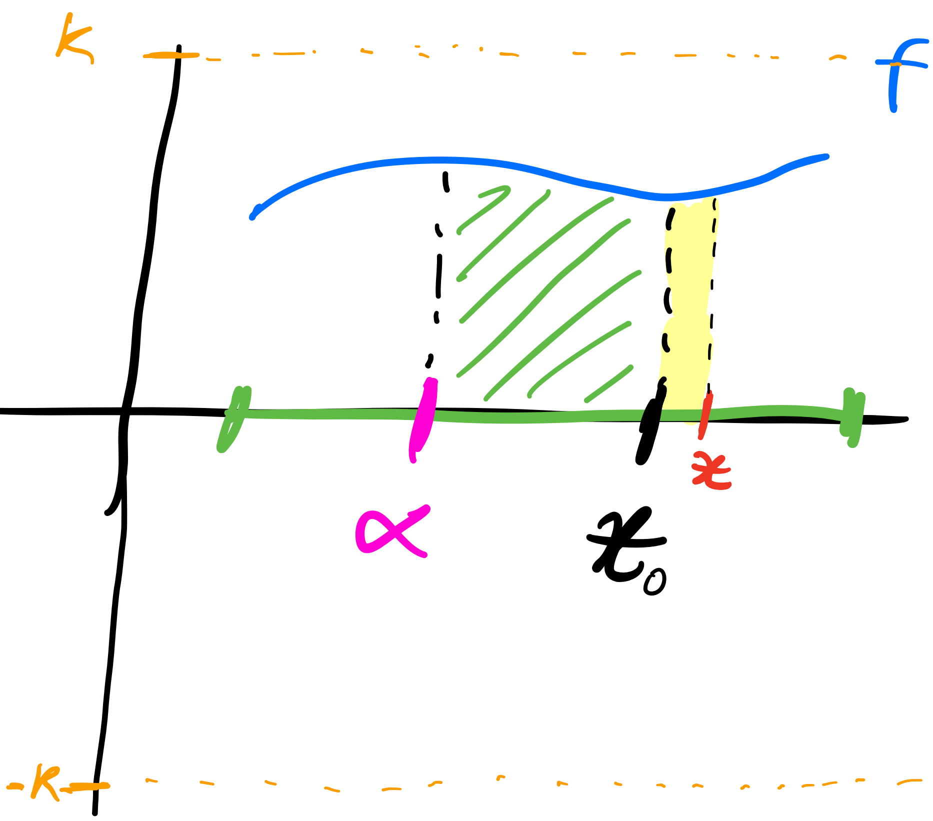
<u>Teorema</u>: Sea $f: I \longrightarrow \mathbb{R}$ acotada e integrable

Definimos $F: I \longrightarrow \mathbb{R}$

$$F(x) = \int_{\alpha}^{x} f(t)\,dt$$

Entonces, F es continua

<u>Idea de la prueba:</u>



$$\boxed{F(x) - F(x_0) = \int_{x_0}^{x} f(t)\,dt}$$

<u>Dem:</u> Vamos a ver la continuidad de F en un $x_0 \in I$.

$$F(x) - F(x_0) = \int^{x} f(t)\,dt - \int^{x_0} f(t)\,dt =$$

ADITIVIDAD RESPECTO DEL INTERVALO

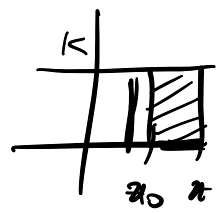
$$=\int_{x_0}^{\alpha} f(t)\,dt + \int_{\alpha}^{x} f(t)\,dt = \int_{x_0}^{x} f(t)\,dt$$

Como f es acotado, $\exists\ k>0 \,/\, |f(x)| < k \ \forall x \in I$

MONOTONÍA DE LA INTEGRAL

$$-k \le f(x) \le k$$

$$\Longrightarrow \underbrace{\int_{x_0}^{x} (-k)\,dt}_{(-k)(x-x_0)} \le \int_{x_0}^{x} f(t)\,dt \le \underbrace{\int_{x_0}^{x} k\,dt}_{= k(x-x_0)} \qquad (x > x_0)$$



En resumen:

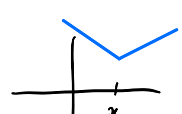
$$\boxed{-k(x-x_0) \le \int_{x_0}^{x} f(t)\,dt \le k(x-x_0)} \quad \text{si } x > x_0$$

Se puede similarmente que

$$\boxed{k(x-x_0) \le \int_{x_0}^{x} f(t)\,dt \le -k(x-x_0)} \quad \text{si } x < x_0$$

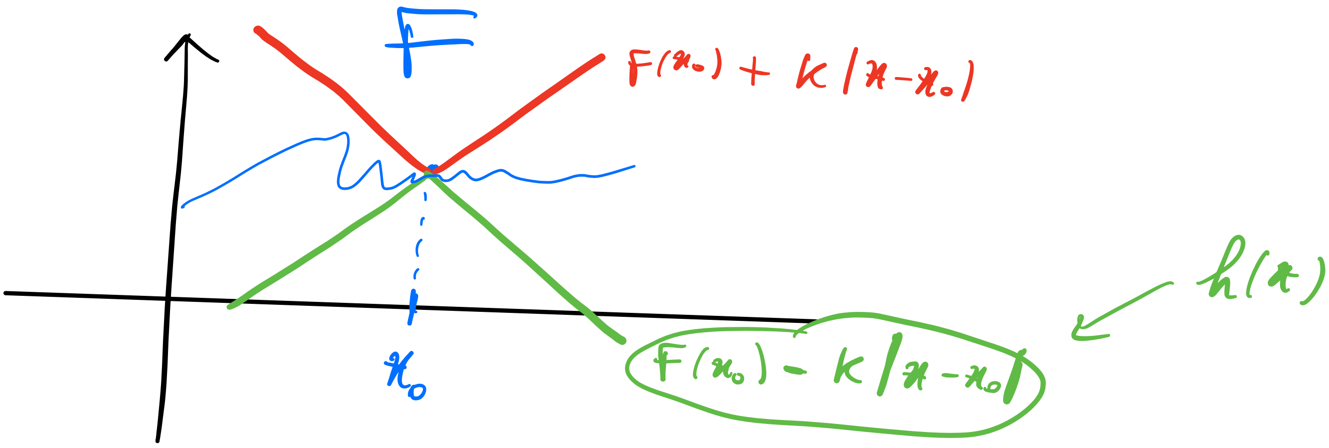
$$F(x_0) - k(x-x_0) \le \overbrace{\boxed{F(x_0) + \int_{x_0}^{x} f(t)\,dt}}^{F(x)} \le F(x_0) + k(x-x_0) \qquad x > x_0$$

$$F(x_0) + k(x-x_0) \le \underbrace{\boxed{F(x_0) + \int_{x_0}^{x} f(t)\,dt}}_{=} \le F(x_0) - k(x-x_0) \qquad x < x_0$$



$g(x)$ $\qquad$ $F(x)$

$$\overbrace{F(x_0)+k|x-x_0|}^{g(x)} = \begin{cases} F(x_0)+k(x-x_0) & \text{si } x > x_0 \\ F(x_0)-k(x-x_0) & \text{si } x < x_0 \end{cases}$$



Sabemos que $h(x) \leq F(x) \leq g(x) \quad \forall x \in I$

Por otro lado $\lim\limits_{x\to x_0} h(x) = \lim\limits_{x\to x_0} g(x) = F(x_0)$

Entonces, por el Teorema del sandwich,

$$\lim_{x\to x_0} F(x) = F(x_0)$$

Entonces F es continua en x_0.

Como consecuencia la función

logaritmo es continua.

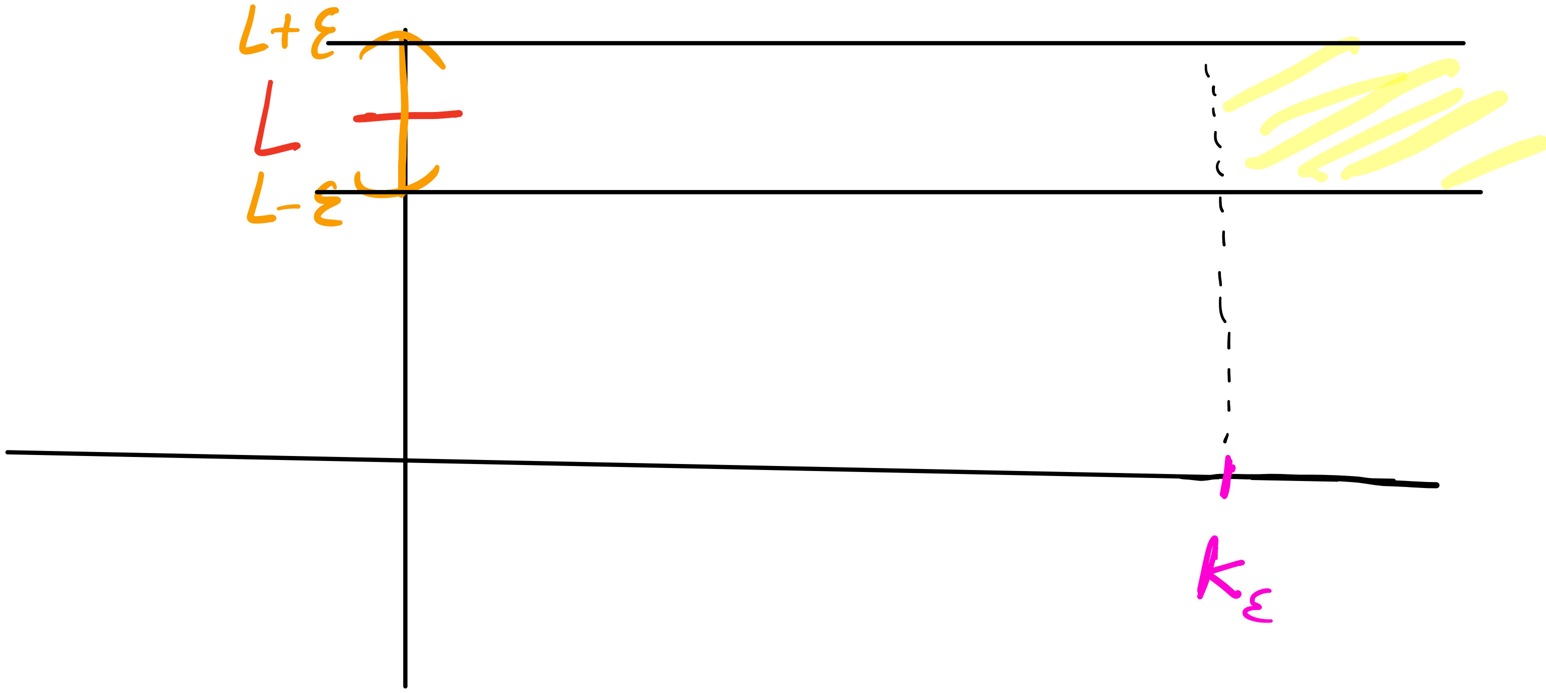
LÍMITES LATERALES E INFINITOS

$$\lim_{x \to \underbrace{x_0}_{\in \mathbb{R}}} f(x) = L \in \mathbb{R}$$

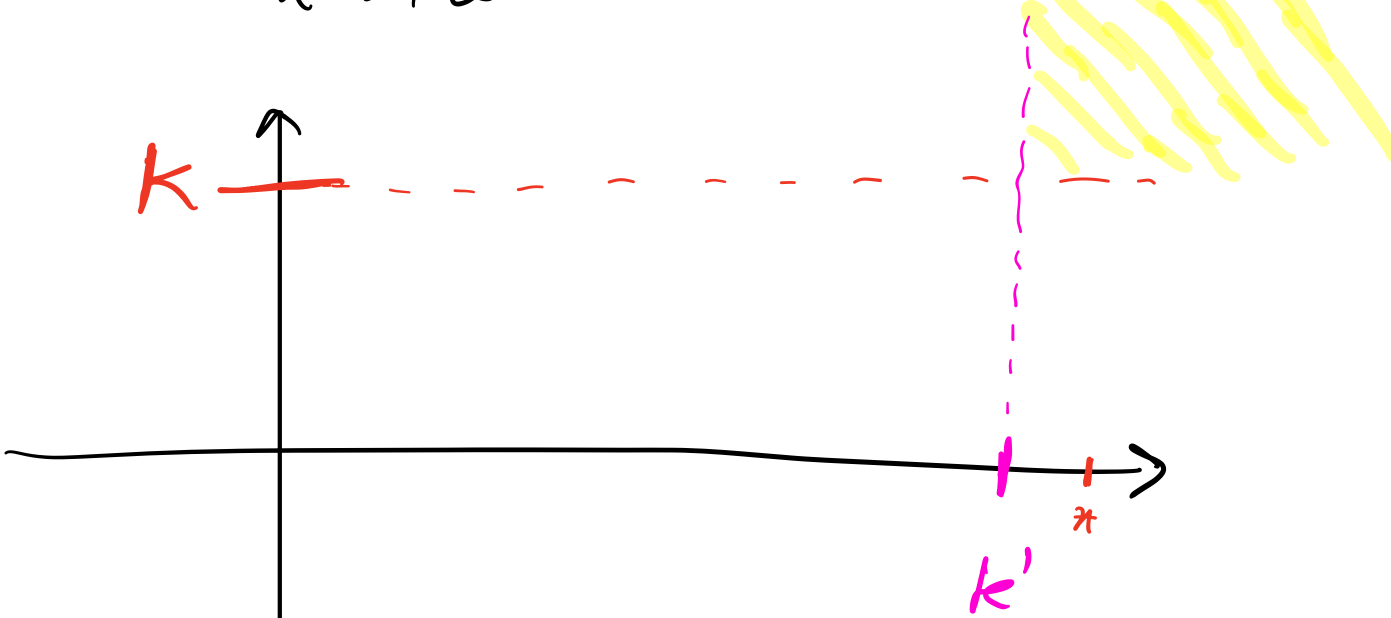
Def: Decimos que $f(x)$ tiende a $L \in \mathbb{R}$ cuando x tiende a $+\infty$ si:

$$\forall \varepsilon > 0, \ \exists K_\varepsilon > 0 \ /$$

si $x > K_\varepsilon \implies |f(x) - L| < \varepsilon$



Def : $\lim\limits_{x \to +\infty} f(x) = +\infty$ si



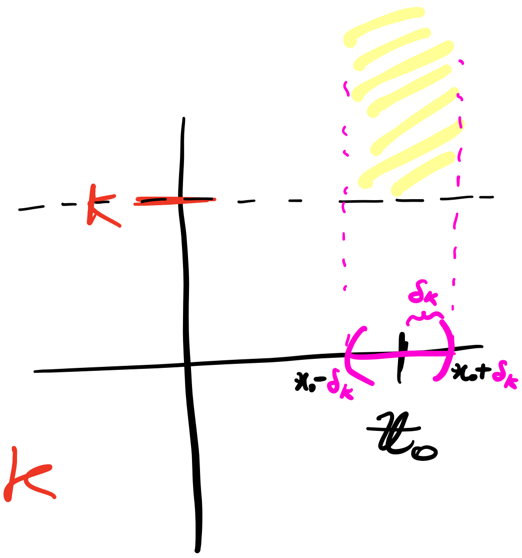
$\forall K > 0, \exists K' > 0 \ /$

si $x > K' \Rightarrow f(x) > K$

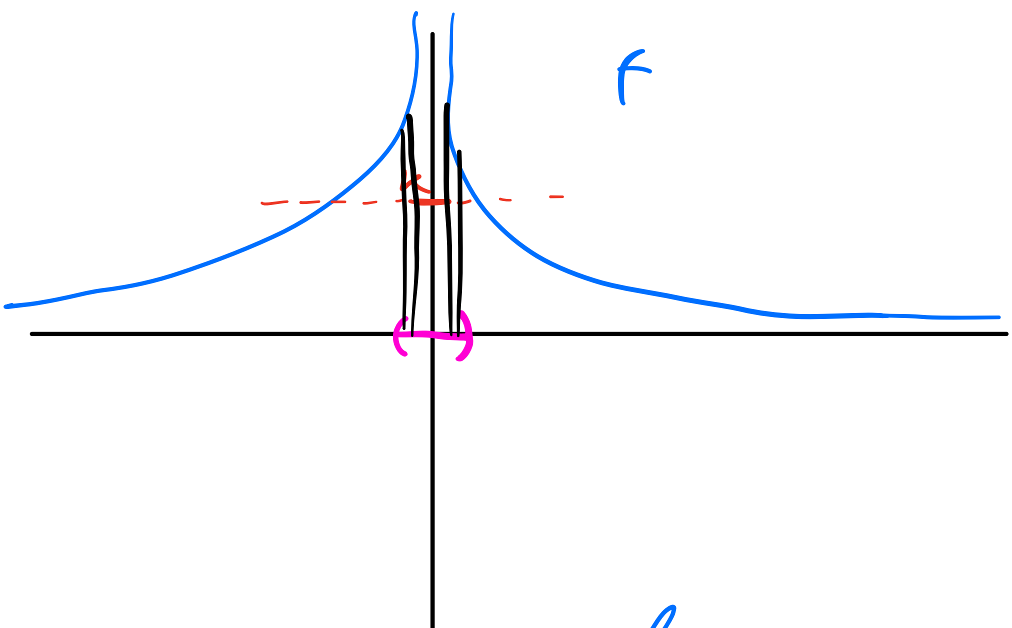
Def: $\lim_{x \to x_0} f(x) = +\infty$

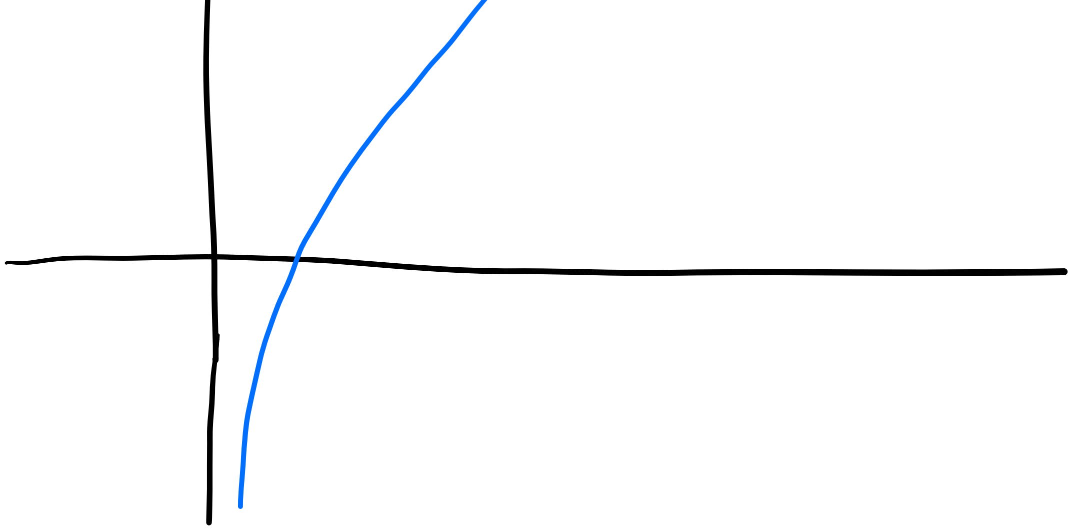
si $\forall K > 0, \exists \delta_K > 0 /$

si $0 < |x - x_0| < \delta_K \Rightarrow f(x) > K$



Ej: $f(x) = \frac{1}{|x|}$ tiene límite $+\infty$ en 0





$$\lim_{x \to +\infty} \log(x) = +\infty$$

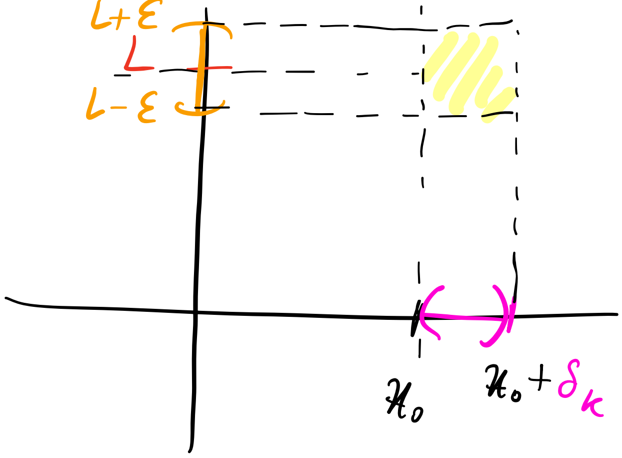
$$\boxed{\lim_{x \to 0^+} \log(x) = -\infty}$$

<u>Def</u>: $\lim_{x \to x_0^+} f(x) = L$ si :

$$\forall \varepsilon > 0, \exists \delta_\varepsilon > 0 /$$

si $0 < x - x_0 < \delta_\varepsilon \implies |f(x) - L| < \varepsilon$

$\boxed{x > x_0}$



<u>Ejercicio</u>: Escribir definición de

$$\lim_{x \to x_0^-} f(x) = L$$

<u>Def</u>: $\lim_{x \to x_0^+} f(x) = -\infty$ si

$\forall K < 0, \; \exists \, \delta_K > 0$

si $x_0 < x < x_0 + \delta_K \Longrightarrow$

$f(x) < K$