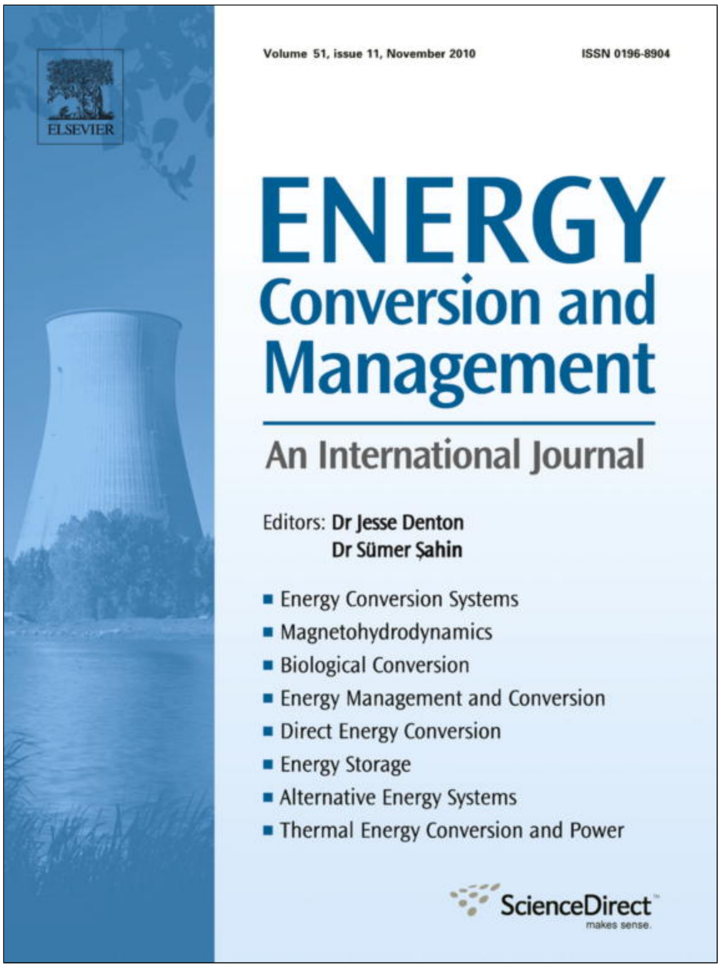
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Thermodynamic model and optimization of a multi-step irreversible Brayton cycle

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ABSTRACT

We present a general model for a multi-step regenerative irreversible Brayton cycle on thermodynamic basis. The model incorporates an arbitrary number or turbines and compressors with reheating and intercooling intermediate processes. We consider several internal and external irreversibility sources that include losses in the non-isentropic turbines and compressors, pressure drops in the heat input and heat release, irreversibilities in the regenerative heat exchanger, heat-leak through the plant to the ambient and non-ideal couplings with the external constant-temperature heat reservoirs. The general equations for power output and efficiency depend on a reasonable low number of parameters, with a clear physical meaning, that account for cycle design and geometry, and for the characterization of irreversibilities. From this general model several results found in the literature could be considered as particular or limit cases. Moreover, we explicitly compare our theoretical results with computer simulation results in the literature for particular plant arrangements where the number of turbines and compressors is not the same. Also, we analyze the maximum power and maximum efficiency working regimes in terms of internal and external irreversibilities.

1. Introduction

Thermodynamic efficiency and power output are essential parameters in the plant design and in the operating costs of gas power plants, especially attending to the actual limited resources of fuel. In the last years considerable efforts have been devoted to the improvement of gas turbine technologies with the objectives, among others, of improving cycle efficiency and power output. Nowadays, material sciences allow to build turbine blades operating at around 1500 °C and to obtain efficiencies around 40% [1,2].

From a theoretical viewpoint there are several ways to modify a simple Brayton cycle in order to increase performance. Some of these modifications have been discussed in the literature during the last years [3] and include regeneration [4–6], isothermal heat addition [7–9], intercooled compression [10,11], reheated expansion, and their combinations [12–14]. Moreover, significant advances were achieved in the model of different irreversibility sources always present in any real power gas turbine, that can come from the system itself (internal irreversibilities) and/or the coupling of the working fluid with its surroundings (external). Examples of both types of losses are pressure drops in the heat addition and release processes [15,16], irreversibilities in the turbines and compressors [17,18], irreversible couplings to the external reservoirs (with constant or variable temperatures) [4,5,15,11] and even heat-leak through the plant [19]. However, these studies lack of generality since none of them includes simultaneously regeneration, multiple compressors and turbines and all the feasible irreversibility sources. The main objective of this paper is to propose a general analytical model incorporating all those ingredients. So, our broad scheme is capable to recover most of the results included in previous partial works as particular cases and also to reproduce the efficiency or power output of real power plants.

Most of the works mentioned above were done within the frame of thermodynamic optimization (finite-time-thermodynamics or entropy generation minimization) [20,21]. Within this frame there exist three essential elements in the modeling of any heat engine: (a) to propose simple and macroscopic models for the different irreversibility contributions depending on a reduced set of parameters with a clear physical meaning, (b) to choose and optimize a function depending on the cycle parameters, and (c) to obtain the optimal working regime in accordance with the elected optimization procedure.

Among the diverse objective functions that have been analyzed for Brayton based heat engines are power and efficiency [4,17,22,23], power density [6,15,24], different ecological criteria [13,19,25], thermoeconomical functions [20,26] and other functions where a compromise between power output and energy

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consumption is the main ingredient [27]. In any case, as argued by Chen [28] the optimal working regime of any thermal engine should verify that the power output is between the power at maximum efficiency conditions and the maximum power. Moreover, efficiency should lay between that at maximum power conditions and the maximum one. In particular we shall analyze the maximum power and maximum efficiency regimes of the multi-step gas turbine considered.

Besides, other kind of analysis have been also developed during the last years in order to evaluate the performance of real complex gas power plants, the computational calculations based on the simulation of particular plant [2] configurations. The main disadvantage of these methods is, of course, that they provide information for a specific gas turbine, they do not give an idea about the generality of their predictions or about the physics governing the implied phenomena. The analytic studies are best suited for deriving this type of information, and computational results constitute an interesting test for these theoretical models.

In summary the objective of this work is to present a novel and unified theoretical model of a multi-step regenerative closed Brayton cycle that allows the consideration of an arbitrary number of intercooled compression steps and reheated expansion steps. We take into account the main irreversibility sources (external and internal) that contribute to the observed performance of real gas power plants. We shall consider non-ideal turbines and compressors, pressure drops during heat absorption and exhaust, losses in the counter-flow regenerator, heat-leak through the plant to the surroundings and non-ideal couplings of the working fluid to the external constant-temperature heat reservoirs. Moreover, we shall explicitly check our results with similar ones obtained from computational simulations of real power plants and compiled by Horlock [2].

In Section 2 we present the main elements of the regenerated multi-step thermodynamic model considered and obtain analytic general expressions for power output and thermal efficiency. Section 3 is devoted to analyze how our model can reproduce as particular or limit cases some previous results found in the literature. In Section 4 we present numerical results for power and efficiency taking realistic values for all the irreversibility parameters considered and analyze the related results. Finally, we briefly summarize the main conclusions of the paper.

2. Theoretical model

We consider a constant mass flow, $\dot{m}$, of an ideal gas with constant heat capacities, and adiabatic coefficient, γ, performing the gas turbine power cycle plotted in Fig. 1. This cycle is coupled to an infinite heat source at temperature, T_H, representing combustion chamber temperature and to an infinite heat sink at ambient temperature, T_L. So, the heat sources temperature ratio is $\tau = T_H/T_L > 1$. We also take into account a heat transfer rate that leaks directly through the machine, $|\dot{Q}_{HL}|$, representing, among others the heat transfer lost through the walls of the combustion chamber [19,29]. The main characteristics of the cycle can be summarized as follows:

(i) The working fluid is compressed from the initial state 1 at temperature T_1 by means of N_c compressors and $N_c - 1$ intercoolers. Compression processes are not necessarily adiabatic. Intercooling processes are considered isobaric by the reasons we shall detail later. Inlet temperature for each compression is always the same, T_1. We assume the same isentropic efficiency, ϵ_c, for all compressors as a measure of how real compressions deviate from ideal isentropic ones due to fluid friction losses [30],

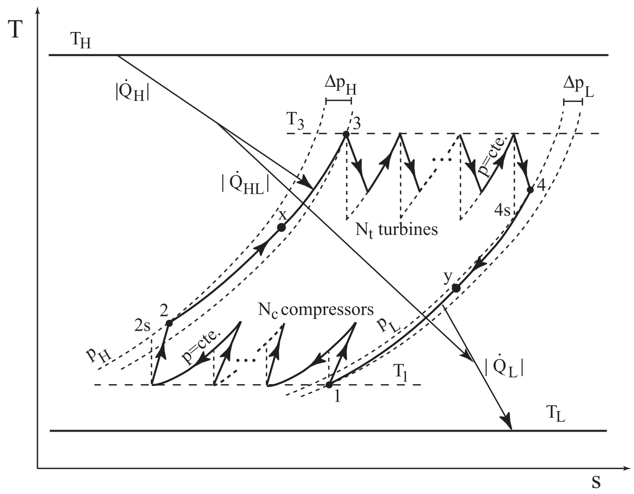

Fig. 1. T–S scheme of the considered multi-step irreversible Brayton cycle.

$$\epsilon_c = \frac{T_{2s} - T_1}{T_2 - T_1} \tag{1}$$

where T_{2s} is the temperature after the last adiabatic compression.

(ii) After compression to state 2, the system is pre-heated in a regenerative counterflow heat exchanger to state X. The efficiency of the regenerative heat exchanger is defined as the ratio of the actual heat transfer to the maximum one:

$$\epsilon_r = \frac{T_X - T_2}{T_4 - T_2} \tag{2}$$

For a non-regenerative cycle $T_X = T_2$, so $\epsilon_r = 0$, and for an ideal or limit regeneration, $\epsilon_r = 1$ because $T_X = T_4$.

In the process from X to 3 the fluid is heated up to the final maximum temperature, T_3. The per-unit-time heat transfer from the heat source at the combustion chamber temperature, T_H, is denoted by $|\dot{Q}_H|$. The global irreversibilities in this hot-end heat exchanger are accounted by [29,31]

$$\epsilon_H = \frac{T_X - T_3}{T_X - T_H} \tag{3}$$

We consider the global heating process as non-isobaric, with a total pressure drop, Δp_H, quantified by ρ_H, defined as [22,31]:

$$\rho_H = \left(\frac{p_3}{p_2}\right)^{(\gamma-1)/\gamma} = \left(\frac{p_H - \Delta p_H}{p_H}\right)^{(\gamma-1)/\gamma} \tag{4}$$

(iii) From state 3 to 4 the system is expanded by using N_t turbines combined with $N_t - 1$ reheaters. Expansion processes are not necessarily adiabatic. Inlet temperature, T_3, and isentropic efficiency of all turbines, ϵ_t, are considered identical,

$$\epsilon_t = \frac{T_3 - T_4}{T_3 - T_{4s}} \tag{5}$$

and reheating processes between turbines are considered without pressure drops. T_{4s} represents the temperature after the last turbine if eventually expansion is adiabatic.

(iv) In the exhaust process, between the last turbine and the first compressor the gas is first cooled from 4 to Y in the regenerative heat exchanger,

$$\epsilon_r = \frac{T_X - T_2}{T_4 - T_2} = \frac{T_Y - T_4}{T_2 - T_4} \tag{6}$$

and finally cooled again to its initial temperature, T_1. Thus, $\epsilon_r = 0$ corresponds to $T_Y = T_4$ and $\epsilon_r = 1$ to $T_Y = T_2$.

The irreversible heat transfer from the stream $Y \to 1$ to the ambient temperature, $|\dot{Q}_L|$, is associated to an effectiveness,

$$\epsilon_L = \frac{T_1 - T_Y}{T_L - T_Y} \tag{7}$$

A global pressure decay, Δp_L, is considered during the whole cooling process. This decay is evaluated by,

$$\rho_L = \left(\frac{p_1}{p_4}\right)^{(\gamma-1)/\gamma} = \left(\frac{p_L - \Delta p_L}{p_L}\right)^{(\gamma-1)/\gamma} \tag{8}$$

The isentropic compressor and turbine pressure ratios, a_c and a_t, respectively can be easily expressed through Eqs. (4) and (8) as,

$$a_c = \frac{T_{2s}}{T_1} = \left(\frac{p_H}{p_L - \Delta p_L}\right)^{(\gamma-1)/\gamma} \tag{9}$$

$$a_t = \frac{T_3}{T_{4s}} = \left(\frac{p_H - \Delta p_H}{p_L}\right)^{(\gamma-1)/\gamma} \tag{10}$$

which are related by $a_t = a_c \rho_H \rho_L$.

2.1. Heat input

Heat is provided to the system along the combustion path $2 \to 3$, although the existence of a regenerator reduces this process to $X \to 3$, and along the $N_t - 1$ reheating processes between turbines. If we denote C_w to the heat capacity rate of the working fluid,

$$|\dot{Q}_H| = C_w(T_3 - T_X) + C_w\epsilon_t \sum_{j=1}^{N_t-1}(T_3 - T_{js}) + |\dot{Q}_{HL}| \tag{11}$$

where the index j is associated to each reheating process and T_{js} is the final temperature after each turbine if it works perfectly adiabatic. $|\dot{Q}_{HL}|$ is the internal heat transfer rate (heat-leak) through the plant, that we consider simply linear, $|\dot{Q}_{HL}| = C_i(T_H - T_L) = C_i T_L(\tau - 1)$, where C_i is the internal conductance of the power plant [19,29].

We deal with the calculation of the second term on the r.h.s. of Eq. (11) in terms of the parameters of our model. Considering that T_3 is the same inlet temperature for all turbines.

$$\sum_{j=1}^{N_t-1}(T_3 - T_{js}) = (N_t - 1)T_3 - \sum_{j=1}^{N_t-1} T_{js} \tag{12}$$

To obtain $\sum_{j=1}^{N_t-1} T_{js}$, we calculate the pressure distribution in the turbines array by maximizing the total power output obtained from the N_t turbines respect to the intermediate pressures (see [12,32] for details) assuming that reheating processes are isobaric. This allows to find that the pressure ratio for each j turbine, $p_{t,j}/p_{t,j-1}$, is always the same, *i.e.*, j-independent and with value:

$$\frac{p_{t,j}}{p_{t,j-1}} = \left(\frac{p_L}{p_H - \Delta p_H}\right)^{1/N_t} \tag{13}$$

Thus,

$$\frac{T_{js}}{T_3} = \left(\frac{p_{t,j}}{p_{t,j-1}}\right)^{(\gamma-1)/\gamma} = a_t^{-1/N_t} \tag{14}$$

and

$$\sum_{j=1}^{N_t-1} T_{js} = T_3(N_t - 1)a_t^{-1/N_t} \tag{15}$$

and the second addend in Eq. (11) results:

$$C_w\epsilon_t \sum_{j=1}^{N_t-1}(T_3 - T_{js}) = C_w\epsilon_t(N_t - 1)\left(1 - a_t^{-1/N_t}\right)T_3 \tag{16}$$

The first term in Eq. (11) is calculated in a similar way that in Ref. [16] to obtain an expression depending on the parameters accounting for irreversibilities we defined before, but considering N_t turbines. The result is the following:

$$\begin{aligned} C_w(T_3 - T_X) &= C_w\epsilon_H(T_H - T_X) \\ &= C_w\epsilon_H T_L\left[\tau - Z_c(1-\epsilon_r)\frac{T_1}{T_L} - \epsilon_r Z_t \frac{T_3}{T_L}\right] \end{aligned} \tag{17}$$

where

$$\begin{aligned} Z_c &= 1 + \frac{a_c^{1/N_c} - 1}{\epsilon_c} \\ Z_t &= 1 - \epsilon_t\left(1 - a_t^{-1/N_t}\right) \end{aligned} \tag{18}$$

and

$$\frac{T_1}{T_L} = \frac{\epsilon_L + (1-\epsilon_L)(1-\epsilon_r)Z_t\left(\frac{T_3}{T_L}\right)}{1 - (1-\epsilon_L)\epsilon_r Z_c} \tag{19}$$

$$\frac{T_3}{T_L} = \frac{\tau\epsilon_H[1 - (1-\epsilon_L)\epsilon_r Z_c] + \epsilon_L(1-\epsilon_H)(1-\epsilon_r)Z_c}{[1 - (1-\epsilon_L)\epsilon_r Z_c][1 - (1-\epsilon_H)\epsilon_r Z_t] - (1-\epsilon_H)(1-\epsilon_L)(1-\epsilon_r)^2 Z_t Z_c} \tag{20}$$

And finally, adding all the terms of Eq. (11), the per-unit-time heat input, $|\dot{Q}_H|$, can be written as:

$$\begin{aligned} |\dot{Q}_H| = C_w T_L \Big\{ &\epsilon_H\left[\tau - Z_c(1-\epsilon_r)\frac{T_1}{T_L} - \epsilon_r Z_t \frac{T_3}{T_L}\right] \\ &+ \epsilon_t(N_t - 1)\left(1 - a_t^{-1/N_t}\right)\frac{T_3}{T_L} + \xi(\tau - 1)\Big\} \end{aligned} \tag{21}$$

where ξ denotes the rate of the plant internal conductance respect to that of the working fluid, $\xi = C_i/C_w$.

2.2. Heat release

Due to the existence of an irreversible regenerator, effective heat release is associated to process $Y \to 1$ and to the heat transferred by the $N_c - 1$ intercoolers placed between the compressors to the heat sink at ambient temperature, T_L.

$$|\dot{Q}_L| = C_w(T_Y - T_1) + C_w\frac{1}{\epsilon_c}\sum_{k=1}^{N_c-1}(T_{ks} - T_1) + |\dot{Q}_{HL}| \tag{22}$$

where the index k stands for each cooling process and T_{ks} is the final temperature after each compressor if it works in a perfectly adiabatic way. In order to evaluate the second addend on the r.h.s., we consider the same inlet temperature for each compressor, so

$$\sum_{k=1}^{N_c-1}(T_{ks} - T_1) = -(N_c - 1)T_1 + \sum_{k=1}^{N_c-1} T_{ks} \tag{23}$$

To obtain $\sum_{k=1}^{N_c-1} T_{ks}$ we minimize the total input power respect to the $N_c - 1$ intermediate intercoolers pressure assuming isobaric coolings. This allows to obtain the same pressure ratios (see [12] for details) for each k-compressor:

$$\frac{p_{c,k}}{p_{c,k-1}} = \left(\frac{p_H}{p_L - \Delta p_L}\right)^{1/N_c} \tag{24}$$

Then,

$$\frac{T_{ks}}{T_1} = \left(\frac{p_{c,k}}{p_{c,k-1}}\right)^{(\gamma-1)/\gamma} = a_c^{1/N_c} \tag{25}$$

and

$$\sum_{k=1}^{N_c-1} T_{ks} = (N_c - 1)T_1 a_c^{1/N_c} \tag{26}$$

Thus, the second addend in Eq. (22) results:

$$C_w \frac{1}{\epsilon_c} \sum_{k=1}^{N_c-1} (T_{ks} - T_1) = C_w \frac{1}{\epsilon_c} (N_c - 1)(a_c^{1/N_c} - 1)T_1 \tag{27}$$

The first addend in Eq. (22) is calculated as in Ref. [16], and finally, the per-unit-time heat release is given by:

$$|\dot{Q}_L| = C_w T_L \left\{ \epsilon_L \left[-1 + Z_t(1-\epsilon_r)\frac{T_3}{T_L} + \epsilon_r Z_c \frac{T_1}{T_L} \right] + \frac{1}{\epsilon_c}(N_c - 1)(a_c^{1/N_c} - 1)\frac{T_1}{T_L} + \xi(\tau - 1) \right\} \tag{28}$$

where Z_c, Z_t, T_1/T_L and T_3/T_L are given by Eqs. (18)–(20).

As we shall see in the next section the equations obtained within our model for the heat input and heat release allow to recover as particular or limit cases several results found in the literature. Moreover, from the theoretical viewpoint, in spite of the consideration at the same time of several irreversibility sources they are analytical and not too intricate.

2.3. Cycle efficiency and net power output

Eqs. (21) and (28) show that the cycle efficiency, $\eta = 1 - |\dot{Q}_L|/|\dot{Q}_H|$ and the net power output, $P = |\dot{Q}_H| - |\dot{Q}_L|$ are both functions of some geometrical parameters, characterizing the shape and size of the cycle, and another ensemble of parameters that globally characterize the internal and external irreversibility sources considered. These parameters are the following.

- *Geometrical parameters:*
  (1) Extreme pressure and temperature ratios, a_c (note that a_t and a_c are linked by Eqs. (9) and (10)) and τ, respectively, and the adiabatic ideal gas coefficient, γ.
  (2) The number of compressors and turbines considered, N_c and N_t, respectively.
- *Parameters associated to internal irreversibilities:*
  (1) Isentropic efficiencies characterizing the non-ideal behavior of turbines and compressors, ϵ_t and ϵ_c.
  (2) The irreversibilities coming from the pressure drops in the heat input from the combustion chamber and heat release through exhaust, ρ_H and ρ_L, respectively.
  (3) The parameters accounting for the irreversibilities in the regenerator, ϵ_r, and the parameter ξ associated to the heat transfer through the plant to the surroundings.
- *Parameters associated to external irreversibilities:*
  (1) The isentropic efficiencies, ϵ_H y ϵ_L referred to the non-ideal couplings of the working fluid with the external heat reservoirs at temperatures T_H and T_L.

3. Particular and limit cases

3.1. External irreversibilities

In this case we consider that the turbines and compressors are ideal ($\epsilon_c = \epsilon_t = 1$), that the combustion and exhaust processes are isobaric ($\rho_H = \rho_L = 1$), that the regenerator does not exist $\epsilon_r = 0$ or works perfectly, $\epsilon_r = 1$, and the heat-leak is zero ($\xi = 0$). So, the only irreversibilities affecting the performance of the cycle are those coming from the external heat exchangers $\epsilon_H, \epsilon_L < 1$. Thus, this limit represents the so-called endoreversible model [33,34].

3.1.1. No regeneration, $\epsilon_r = 0$

For an arbitrary number of turbines N_t and compressors N_c, is easy to obtain analytical expressions for $|\dot{Q}_H|$ and $|\dot{Q}_L|$ from Eqs. (21) and (28) by considering that $a_t = a_c \equiv a$ because $\rho_H = \rho_L = 1$. Moreover, $a = (p_H/p_L)^{(\gamma-1)/\gamma} = r_p^{(\gamma-1)/\gamma}$. Thus,

$$|\dot{Q}_H| = C_w T_L \left[\epsilon_H \left(\tau - a^{1/N_c} \frac{T_1}{T_L} \right) + (N_t - 1)(1 - a^{-1/N_t}) \frac{T_3}{T_L} \right] \tag{29}$$

$$|\dot{Q}_L| = C_w T_L \left[\epsilon_L \left(-1 + a^{-1/N_t} \frac{T_3}{T_L} \right) + (N_c - 1)(a^{1/N_c} - 1) \frac{T_1}{T_L} \right] \tag{30}$$

where

$$\frac{T_1}{T_L} = \epsilon_L + (1 - \epsilon_L) a^{-1/N_t} \frac{T_3}{T_L} \tag{31}$$

$$\frac{T_3}{T_L} = \frac{\tau \epsilon_H + \epsilon_L (1 - \epsilon_H) a^{1/N_c}}{1 - (1 - \epsilon_H)(1 - \epsilon_L) a^{1/N_c - 1/N_t}} \tag{32}$$

In Fig. 2a we depict the evolution of efficiency in terms of the pressure ratio, r_p, for several values of ϵ_L and ϵ_H considered identical. Moreover, we also analyze the influence of the number of intercooling and reheating steps taking $N_t = N_c \equiv N$. In all cases extreme temperatures ratio, τ was taken as $\tau = 5$. On one side efficiency decreases with the increase of the external irreversibilities and on the other, for a fixed value of $\epsilon_L = \epsilon_H$, efficiency progressively decreases with the addition of more turbines and compressors, due to the increase of the irreversible heat transfers with the external heat sources. On the contrary, power output, for fixed external irreversibilities, increases with the number of steps, N (see Fig. 3a).

In the case of a simple plant configuration with only one turbine and one compressor, $N_t = N_c = 1$ is straightforward to get that the power output and the efficiency are given by

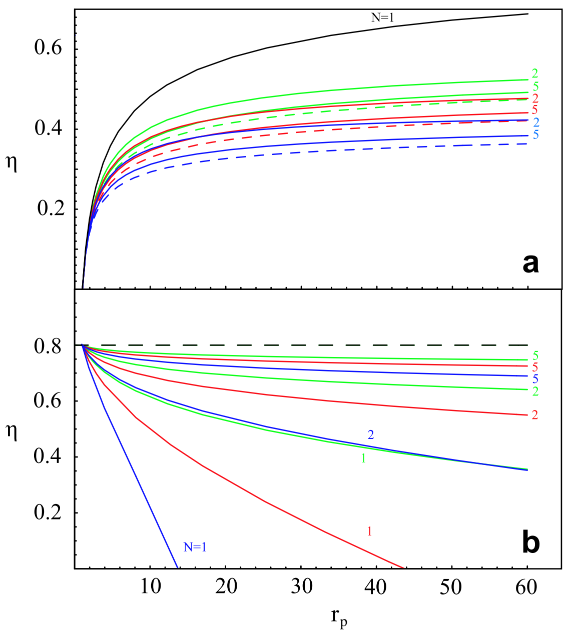

Fig. 2. Thermal efficiency, η, of the cycle as a function of the pressure ratio, r_p, in absence of internal irreversibilities and for an arbitrary number of turbines and compressors, $N_t = N_c \equiv N$ with values $N = 1, 2, 5$ for $\tau = 5$ and $\gamma = 1.4$. (a) No regeneration, $\epsilon_r = 0$. (b) Limit regeneration, $\epsilon_r = 1$. Green, $\epsilon_L = \epsilon_H = 1$ (ideal Brayton cycle); red, $\epsilon_L = \epsilon_H = 0.85$; and blue, $\epsilon_L = \epsilon_H = 0.7$. In each case the dashed line represents the limit $N \to \infty$. Black lines are the limit cases where efficiency does not depend on $\epsilon_L = \epsilon_H$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

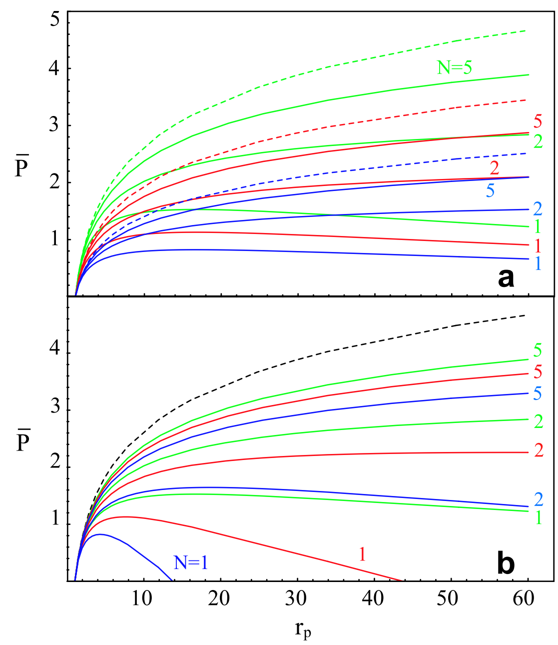

Fig. 3. Same that Fig. 2 but for the normalized power output, $\overline{P} = P/(C_wT_L)$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$P = C_wT_L\left[\frac{\epsilon_L\epsilon_H}{1-(1-\epsilon_H)(1-\epsilon_L)}\right]\left(1-\frac{1}{a}\right)(\tau - a) \tag{33}$$

$$\eta = 1 - \frac{1}{a} \tag{34}$$

These equations reproduce the results in Ref. [16]. Power output does depend on the irreversibilities caused by the heat exchangers, but efficiency corresponds to an ideal Brayton cycle without regeneration. Moreover, in the case $\epsilon_L = \epsilon_H = 1$, of course we recover the power output corresponding to the mentioned ideal case:

$$P = C_wT_L(a-1)\left(\frac{\tau}{a} - 1\right) \tag{35}$$

In the limit situation of infinite number of turbines and compressors, $N_t \to \infty$, $N_c \to \infty$, is also direct to obtain simple expressions for performance parameters through $|\dot{Q}_H|$ and $|\dot{Q}_L|$:

$$|\dot{Q}_H| = C_wT_L\left[\tau - \epsilon_L + \frac{\tau\epsilon_H + \epsilon_L(1-\epsilon_H)}{1-(1-\epsilon_H)(1-\epsilon_L)}\log a\right] \tag{36}$$

$$|\dot{Q}_L| = C_wT_L\left\{\epsilon_L\left[-1 + \frac{\tau\epsilon_H + \epsilon_L(1-\epsilon_H)}{1-(1-\epsilon_H)(1-\epsilon_L)}\right] + \epsilon_L \log a\right\} \tag{37}$$

Both power output and efficiency do depend on ϵ_L and ϵ_H. In the case of ideal heat transfer with the external reservoirs, $\epsilon_L = \epsilon_H = 1$, the cycle goes into a non-regenerative reversible cycle formed up by two isotherms and two isobars, resulting:

$$\eta = 1 - \frac{\log a - 1 + \tau}{\tau \log a - 1 + \tau} \tag{38}$$

$$P = C_wT_L(\tau - 1)\log a \tag{39}$$

Figs. 2a and 3a include these limit curves. It is remarkable that while the largest efficiency in reversible conditions appears in the case $N = 1$ (see upper curve of Fig. 2a), largest power output is obtained in the limit case of infinite number of intermediate steps.

3.1.2. Limit regeneration, $\epsilon_r = 1$

In the case of an arbitrary number of turbines N_t and compressors N_c, expressions for heat input and heat release become:

$$|\dot{Q}_H| = C_wT_L\left[\epsilon_H\left(\tau - a^{-1/N_t}\frac{T_3}{T_L}\right) + (N_t - 1)(1 - a^{-1/N_t})\frac{T_3}{T_L}\right] \tag{40}$$

$$|\dot{Q}_L| = C_wT_L\left[\epsilon_L\left(-1 + a^{1/N_c}\frac{T_1}{T_L}\right) + (N_c - 1)(a^{1/N_c} - 1)\frac{T_1}{T_L}\right] \tag{41}$$

where a is the same that in the case $\epsilon_r = 0$ but now (see Eqs. (19) and (20)),

$$\frac{T_1}{T_L} = \frac{\epsilon_L}{1-(1-\epsilon_L)a^{1/N_c}} \tag{42}$$

$$\frac{T_3}{T_L} = \frac{\tau\epsilon_H[1-(1-\epsilon_L)a^{1/N_c}]}{[1-(1-\epsilon_L)a^{1/N_c}][1-(1-\epsilon_H)a^{-1/N_t}]} \tag{43}$$

We depict in Figs. 2b and 3b the evolution of normalized power, $\overline{P} = P/(C_wT_L)$, and efficiency, for an arbitrary number of intermediate steps with the condition $N_t = N_c \equiv N$, in terms of the pressure ratio and analyze how external irreversibilities affect this evolution. Both, η and P, take larger values with the increase of N and with the decrease of the external irreversibilities, *i.e.*, as $\epsilon_L = \epsilon_H$ increases. In any case, η is a monotonic decreasing function, but P present a maximum at not too large values of r_p for low N.

For a simple plant formed by only one turbine and one compressor, $N_t = N_c = 1$, at difference with the case of no regeneration power and efficiency do depend on the irreversibilities of the heat exchanges with the external reservoirs, through ϵ_L and ϵ_H. Their analytic expressions are:

$$P = C_wT_L\left[\epsilon_H\frac{\tau(a-1)}{a+\epsilon_H-1} - \epsilon_L\frac{a-1}{1+a(\epsilon_L-1)}\right] \tag{44}$$

$$\eta = 1 - \frac{\epsilon_L(a+\epsilon_H-1)}{\epsilon_H\tau[1+a(\epsilon_L-1)]} \tag{45}$$

These equations recover as particular cases the results of Roco et al. [16]. In the reversible limit, $\epsilon_L = \epsilon_H = 1$, the expressions for a limit regeneration simple Brayton cycle are recovered. Power is as in the non-regenerative case, Eq. (35), and efficiency is $\eta = 1 - a/\tau$.

Now we consider an infinite number of turbines and compressors, $N_t \to \infty$, $N_c \to \infty$. In this situation neither efficiency nor power output depend on ϵ_L or ϵ_H. The limit efficiency in this case corresponds to the Carnot limit, $\eta_C = 1 - 1/\tau$, that arises as the efficiency limit for a large number of reheating and cooling intermediate steps, *i.e.*, a cycle formed by two isotherms and two adiabatics: the Ericsson cycle with perfect regeneration. Power in this limit coincides with that of the corresponding non-regenerative case, Eq. (39). Figs. 2b and 3b include also this limit.

3.2. Internal irreversibilities

We consider here our Brayton cycle model without irreversibilities coming from the heat exchanges with the external heat reservoirs, $\epsilon_L = \epsilon_H = 1$. Thus, we only take into account internal irreversibilities arising from the pressure drops in the combustion chamber and in the exhaust, from the non-ideal turbines and compressors, from the heat-leak between the reservoirs, and from the regenerator.

For an arbitrary number of turbines N_t and compressors N_c, $T_L = T_1$, and, $T_H = T_3$. Efficiency, $\eta = P/|\dot{Q}_H|$, can be obtained from power output, P, and heat input, $|\dot{Q}_H|$:

$$P = C_wT_L\Big[1 + \tau(1 - Z_t) - Z_c + \epsilon_t(N_t - 1)(1 - a_t^{-1/N_t})\tau - \frac{1}{\epsilon_c}(N_c - 1)(a^{1/N_c} - 1)\Big] \tag{46}$$

$$|\dot{Q}_H| = C_wT_L\Big[\tau - Z_c(1 - \epsilon_r) - \epsilon_r Z_t \tau + \epsilon_t(N_t - 1)(1 - a_t^{-1/N_t})\tau + \xi(\tau - 1)\Big] \tag{47}$$

where Z_c and Z_t are given by Eq. (18). Power output does not depend on the efficiency of the regenerator, ϵ_r, neither on the heat-leak between heat reservoirs, ξ.

In the case of only one turbine and one compressor, $N_t = N_c = 1$, by substituting in the equations above we obtain:

$$P = C_w T_L \left[\tau \epsilon_t \left(1 - \frac{1}{\rho_H \rho_L a_c} \right) - \left(\frac{a_c - 1}{\epsilon_c} \right) \right] \tag{48}$$

$$|\dot{Q}_H| = C_w T_L [\tau - Z_c(1 - \epsilon_r) - \epsilon_r Z_t \tau + \xi(\tau - 1)] \tag{49}$$

where now:

$$Z_c = 1 + \frac{a_c - 1}{\epsilon_c}; \quad Z_t = 1 - \epsilon_t \left(1 - \frac{1}{a_t} \right) \tag{50}$$

These equations recover those obtained by Roco et al. [16] for $\xi = 0$. Furthermore, if we particularize to the non-regenerative case, $\epsilon_r = 0$, it is also obtained as a particular case the one considered by Gordon and Huleihil [30].

$$\eta = \frac{\left(\epsilon_t - \frac{a}{\tau \epsilon_c} \right) \left(1 - \frac{1}{a} \right)}{1 - \frac{1}{\tau} - \frac{1}{\tau \epsilon_c}(a - 1)} \tag{51}$$

By taking the limit $N_t \to \infty$, $N_c \to \infty$ in Eqs. (46) and (47) it is straightforward to obtain:

$$P = C_w T_L \left(\epsilon_t \tau \log a_t - \frac{1}{\epsilon_c} \log a_c \right) \tag{52}$$

$$|\dot{Q}_H| = C_w T_L [\tau(1 - \epsilon_r + \epsilon_t \log a_t) - (1 - \epsilon_r) + \xi(\tau - 1)] \tag{53}$$

From these equations is again easy to get the reversible limits commented in the previous section.

Figs. 4 and 5 represent the evolution of efficiency and power output respect to the pressure ratio for particular values of τ, $\rho_H = \rho_L$, and $\epsilon_c = \epsilon_t$. In Fig. 4a we plot $\eta = \eta(r_p)$ in absence of heat-leak, $\xi = 0$, for several values of ϵ_r, $\epsilon_r = 0, 0.8, 1$ and for $N_t = N_c \equiv N$. For no regeneration, efficiency slightly decreases with an increasing N, but the presence of a heat regenerator, reverses this fact and thermal efficiency increases with the number of reheating–intercooling steps. For low N, efficiency presents a maximum as a function of r_p (for low r_p values), but this evolution turns to monotonic when $N \gtrsim 2 - 3$.

Of course, when heat-leak is considered (Fig. 4b), globally efficiency decreases, especially for $\epsilon_r \neq 0$, making all the curves be closer, but behaviors with N or ϵ_r are similar. Fig. 5 represents P as a function of the pressure ratio. Only a group of curves is shown because power output does not depend on ϵ_r or on ξ.

An interesting comparison of thermal efficiency and power for several plant configurations for the case $\epsilon_L = \epsilon_H = 1$ is depicted in Fig. 6. First, it is compared the efficiency of a simple irreversible plant with one turbine and one compressor with ($N_t = N_c = 1$) with or without regeneration. For the sake of conciseness we shall use Horlock's notation [2] that denotes CICIC…TBTB…X a plant

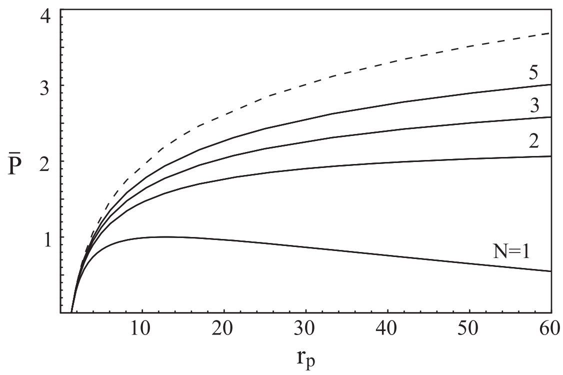

Fig. 5. Power output, $\overline{P}(r_p)$, in absence of external irreversibilities, $\epsilon_L = \epsilon_H = 1$ for different number of intermediate steps, N. In this case power does not depend on ϵ_r nor ξ. Dashed line represents the limit $N \to \infty$ and we take $\tau = 5$, $\rho_L = \rho_H = 0.97$, and $\epsilon_c = \epsilon_t = 0.90$.

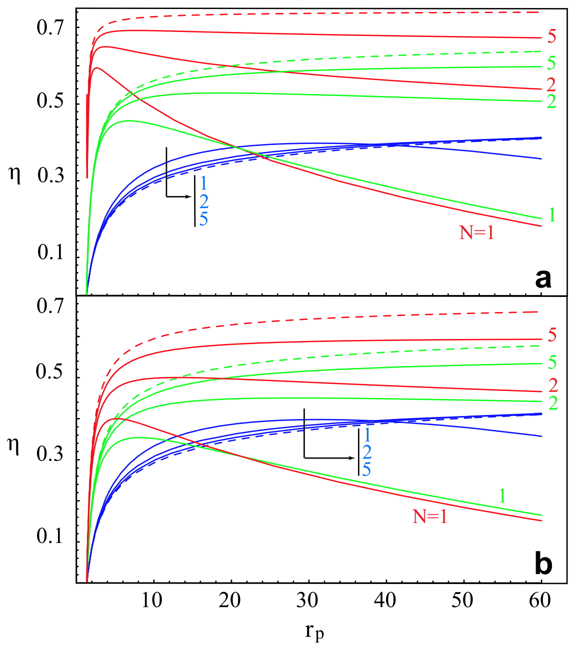

Fig. 4. Thermal efficiency, $\eta = \eta(r_p)$, of the cycle without external irreversibilities, $\epsilon_L = \epsilon_H = 1$ and for an arbitrary number of compressors and turbines, $N = 1, 2, 5$. (a) No heat-leak. (b) Large heat-leak, $\xi = 0.15$. Blue, no regeneration, $\epsilon_r = 0$; red, limit regeneration, $\epsilon_r = 1$, and green, $\epsilon_r = 0.8$. In all cases the dashed line represents the limit $N \to \infty$ and we take: $\tau = 5$, $\rho_L = \rho_H = 0.97$, and $\epsilon_c = \epsilon_t = 0.9$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

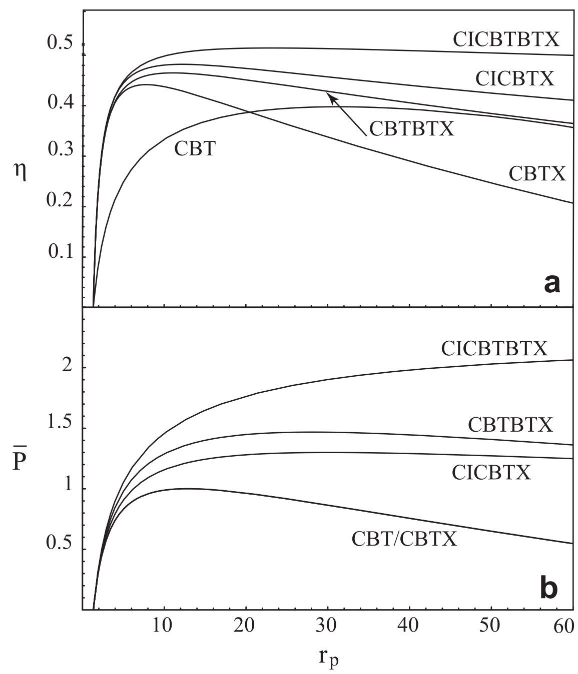

Fig. 6. Efficiency (a) and power (b) for several particular configurations following Horlock's notation (see text) in absence of external irrevesibilities ($\epsilon_L = \epsilon_H = 1$) and heat-leak ($\xi = 0$). For all of them we take $\tau = 5$, $\rho_L = \rho_H = 0.97$, $\epsilon_c = \epsilon_t = 0.9$, and $\epsilon_r = 0.75$.

with several compressors (C) and intercoolers (I), several turbines (T) and intermediate reheaters (B) and regeneration (X). So, CBT represents a simple turbine without regeneration and CBTX with added regeneration. Regeneration was always fixed at $\epsilon_r = 0.75$.

The efficiency when regeneration is not present (CBT) has a maximum around $r_p = 32$ and for greater r_p decreases to zero as the compressor back work finally absorbs all the turbine output. When regeneration is switched on (CBTX), maximum value of efficiency considerably increases (around 10 %), but improvement becomes less important as pressure ratio increases and finally regeneration is useless for $r_p > 20$, where the compressor delivery temperature eventually equals the exhaust temperature.

When we add another single turbine (CBTBTX) or another compressor (CICBTX), maximum efficiency increases respect to the simple regenerative $N_t = N_c = 1$ case (0.45 at $r_p = 8$). When a turbine is added maximum efficiency reaches around 0.47 at $r_p = 11$, and when a single compressor is added, maximum efficiency is greater, 0.49 at $r_p = 12$. Moreover, now efficiency decreases much slower when pressure ratio increases.

A way, of both increase efficiency and keep its value almost constant at its maximum value is by considering two turbines and two compressors (CICBTBTX). For the same regeneration, now maximum efficiency is around 0.52 at $r_p = 24$. All these results reproduce to a great extent those obtained by Wood (see Fig. 1.43 in [35]) and Horlock (see Fig. 3.15 in [2]) from numerical simulations including real gas effects.

It is possible to perform the same study for power output (see Fig. 6b). Now results are independent of regeneration. Power output is increased when we add one turbine or one compressor over the basic $N_t = N_c = 1$ configuration. Opposite to efficiency, maximum power is better improved with the inclusion of another turbine CBTBTX. The power obtained for CICBTX is slightly lower. Of course, for double turbine and double compressor CICBTBTX, power significatively increases, although it does not reach a maximum for realistic values of r_p.

We have also plotted (Fig. 7), for the mentioned configurations, power–efficiency, $\eta = \eta(P)$, parametric curves by eliminating r_p, around the maximum power–maximum efficiency region. It is clear how regeneration (CBTX) increases efficiency keeping power output unaltered over the basic CBT configuration. It is interesting that these curves are built in opposite senses. If $\epsilon_r = 0$, when increasing r_p, first the maximum power condition is reached and after that maximum efficiency for a larger r_p. This is evident from the comparison of both maxima positions in Fig. 6a and b. This means that $\eta(P)$ curve is constructed counterclockwise. On the contrary, when regeneration is considered $\eta(P)$ curve is constructed clockwise because pressure ratio at maximum efficiency is lower than at maximum power. This behavior is maintained

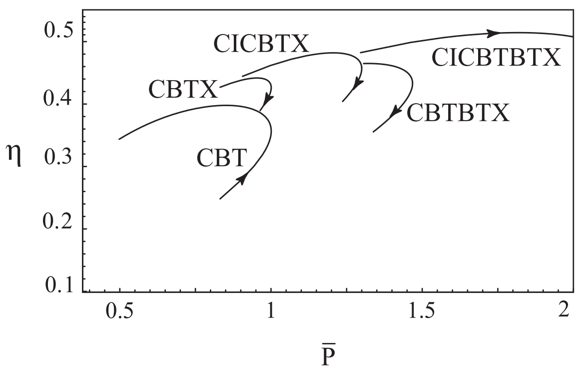

Fig. 7. Parametric η–$\overline{P}$ curves obtained by eliminating r_p for several particular plant arrangements. Notation and values of parameters are like in Fig. 6.

Table 1
Comparison of our calculations with Horlock's results (in parenthesis) [2] when only internal irreversibilities are considered ($\tau = 5$, $\rho_L = \rho_H = 0.97$, $\epsilon_c = \epsilon_t = 0.9$, $\epsilon_r = 0.75$, $\xi = 0$, and $\epsilon_L = \epsilon_H = 1$) for maximum efficiency, $\eta_{\max}$, pressure ratio at maximum efficiency, $r_{p,\max\eta}$, maximum power, $P_{\max}$, and pressure ratio at maximum power conditions, $r_{p,\max P}$.

	$r_{p,\max\eta}$	$\eta_{\max}$	$r_{p,\max P}$	$P_{\max}$
CBT	45(32)	0.44(0.43)	16(12)	–(1.0)
CBTX	9(7)	0.46(0.46)	16(12)	–(1.0)
CBTBTX	16(11)	0.45(0.48)	30(28)	–(1.5)
CICBTX	16(10)	0.50(0.49)	35(27)	–(1.3)
CICBTBTX	36(43)	0.50(0.51)	–	–

for any N_t, N_c values, when regeneration exists. This result is in perfect accordance with that of Horlock (see Fig. 3.16 in [2]) for numerical simulations of similar plant configurations that include non-ideal gas effects.

Table 1 contains a comparison of the r_p locations of maximum power and maximum efficiency points. Also maximum values of efficiencies obtained in our model compare very satisfactorily with Horlock's results [2]. Maximum power values are not included in the table because in Horlock's work are presented in real specific units and we obtain normalized mass independent power.

Finally, it is remarkable from Fig. 7 is that the configuration CBTBTX is most favorable respect to efficiency that CICBTX, but the opposite is true respect to power output. Moreover, when the number of turbines and compressors is duplicated from the simplest arrangement, *i.e.*, when passing from CBTX to CICBTBTX power output increases almost 50% but efficiency improves approximately 13%.

4. Numerical results including all irreversibility sources

We now study the simultaneous influence of all the irreversibility sources we consider in our theoretical model. It is worth mentioning that it is difficult to find in the literature all the parameters of real power plants required to compare the efficiency and power output from our model with those of recent real power plants. The most elevated efficiencies known nowadays, between 40% and 50% [36–38], are achievable within our model by electing realistic irreversibility parameters. As an example we have considered very recent simulation results by Herranz et al. [38] for nuclear high temperature gas-cooled reactors. For a power plant similar to our CICBTX arrangement, considering He as working fluid ($c_p = 5.193$ kJ/kg and $\gamma = 1.67$) and taking the parameters similar to those given by the authors ($\tau = 3.936$, $\eta_c = 0.9$, $\eta_t = 0.93$, $\eta_r = 0.90$, $\rho_H = \rho_L = 0.97$, $\epsilon_L = \epsilon_H = 0.98$) simulation results lead to an efficiency of 46.9% at a pressure ratio of 2.55 and our theoretical results give a maximum efficiency of 46.1% at $r_p = 3.29$. For the same parameters, but for a configuration with two compressors and three turbines Herranz et al. [38] obtain a maximum efficiency of 50.7% at $r_p = 2.90$ and we get $\eta = 50.9\%$ at $r_p = 4.97$.

In Fig. 8a we represent the evolution of the thermal efficiency for several plant configurations and realistic irreversibility parameters. When heat-leak is included in calculations the shape of the efficiency curves does not substantially change, only their numerical value diminishes around 2–6% for $\xi = 0.02$ depending on the configuration.

It is interesting to compare this picture with Fig. 6a. The only difference between the irreversibility parameters taken to build up both graphs are ϵ_L and ϵ_H, the measure of the irreversibilities of the coupling with the thermal reservoirs. When ϵ_L and ϵ_H are different from unity (Fig. 8a) the pressure ratio giving maximum efficiency, $r_{p,\max\eta}$, decreases, maximum efficiency, $\eta_{\max}$, also decreases

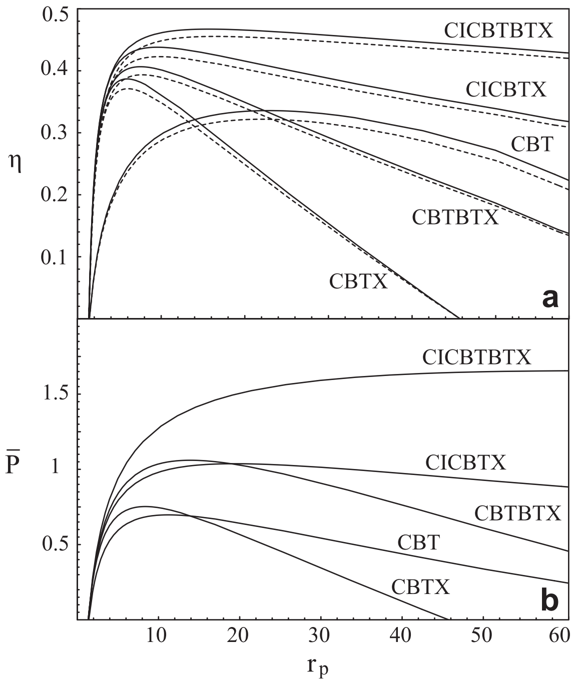

Fig. 8. Thermal efficiency, η, and normalized power output, $\overline{P}$, for several arrangements of turbines and compressors when all irreversibility sources are considered. In the case of efficiency (a) the results are displayed with heat-leak ($\xi = 0.02$, dashed) and without heat-leak ($\xi = 0$, solid). The other parameters are: $\tau = 5,\ \rho_L = \rho_H = 0.97,\ \epsilon_c = \epsilon_t = 0.9,\ \epsilon_L = \epsilon_H = 0.9$, and $\epsilon_r = 0.75$.

Table 2
Results for maximum efficiency, $\eta_{\max}$, pressure ratio at maximum efficiency, $r_{p,\max\eta}$, maximum power, $P_{\max}$, and pressure ratio at maximum power conditions, $r_{p,\max P}$, when external and internal irreversibilities are considered ($\tau = 5,\ \rho_L = \rho_H = 0.97,\ \epsilon_c = \epsilon_t = 0.9,\ \epsilon_r = 0.75,\ \xi = 0$, and $\epsilon_L = \epsilon_H = 0.9$). In parenthesis are also included the results for efficiency considering heat-leak ($\xi = 0.02$).

	$r_{p,\max\eta}$	$\eta_{\max}$	$r_{p,\max P}$	$P_{\max}$
CBT	23(23)	0.34 (0.32)	11.5	0.71
CBTX	5.5(6)	0.39 (0.37)	8.3	0.76
CBTBTX	7.5(8)	0.41 (0.40)	13.7	1.1
CICBTX	9.3(10)	0.44 (0.43)	19.3	1.05
CICBTBTX	15.5(17)	0.47 (0.46)	–	–

and curves trend to zero faster. Numerical changes on $r_{p,\max\eta}$ and $\eta_{\max}$ are displayed in Table 2. The most interesting is that the slope of the curves after $\eta_{\max}$ significatively depends on the plant configuration. Decrease is much important for CBTX and CBTBTX configurations and less important for the rest. This fact makes to get better efficiencies with the simplest CBT arrangement for $r_p > 15$ when comparing with CBTX and for $r_p > 23$ when we compare with CBTBTX. When $\epsilon_L = \epsilon_H = 1$ (Fig. 6a), CBTBTX efficiency is always over CBT efficiency for all r_p range and only for $r_p > 22$ greater efficiency was obtained from CBT respect to CBTX. The consideration of slightly larger regenerator efficiencies ($\epsilon_r \simeq 0.9$) does not modify the aspect of these curves, only the numerical values of efficiencies become smoothly larger.

This effect is also substantial in the curves $P(r_p)$ (compare Fig. 8b with Fig. 6b). First, note that the consideration of $\epsilon_L,\ \epsilon_H \neq 1$ makes power output depend on the efficiency of the regenerator and power curves for CBT and CBTX split (in Fig. 6a they were overlapped). The monotonic decay to zero of the power output after its maximum value is much faster for CBTX and CBTBTX when irreversibility affecting external heat exchangers is considered. Here the decay of CBTX is so rapid that for r_p over 14, power output is larger with the simplest CBT arrangement.

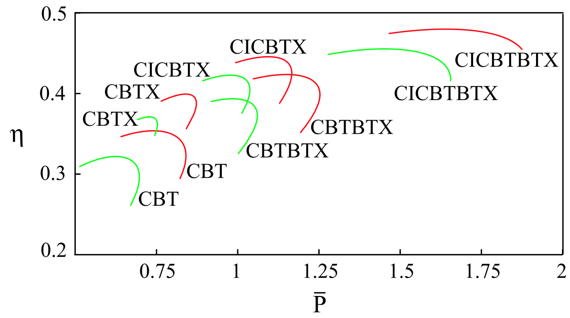

Fig. 9. Curves $\eta = \eta(\overline{P})$ obtained when all irreversibility sources are simultaneously considered for two different irreversible couplings to the external reservoirs: red, $\epsilon_L = \epsilon_H = 0.95$ and green, $\epsilon_L = \epsilon_H = 0.9$. In both cases $\xi = 0.02$. The other parameters are like in Fig. 8. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Moreover, when $\epsilon_L = \epsilon_H = 1$ power obtained from the configuration CBTBTX was always over that obtained from CICBTX, but now, for $\epsilon_L = \epsilon_H = 0.9$ more power output is obtained from CBTBTX only for $r_p < 20$.

In Fig. 9 we plot the η–P curves of several configurations around the region of maximum efficiency and maximum power. Two different values of $\epsilon_L = \epsilon_H$ were considered. We see again that the influence of this irreversibility source does not affect all the configurations in the same manner. For instance, when $\epsilon_L = \epsilon_H = 0.90$, maximum power of CICBTX and CBTBTX is similar, but when external irreversibilities decrease and $\epsilon_L = \epsilon_H = 0.95$, CBTBTX returns almost an 8% more power than CICBTX. Another clear effect is that for $\epsilon_L = \epsilon_H = 0.95$ maximum power increases less than for $\epsilon_L = \epsilon_H = 0.90$, by adding a regenerator to CBT.

In order to get more insight of the influence of the external irreversibilities on maximum efficiency and maximum power output we have plotted on Figs. 10 and 11 the evolution of $\eta_{\max}$, and $P_{\max}$ and their locations, $r_{p,\max\eta}$ and $r_{p,\max P}$. Fig. 10a shows that

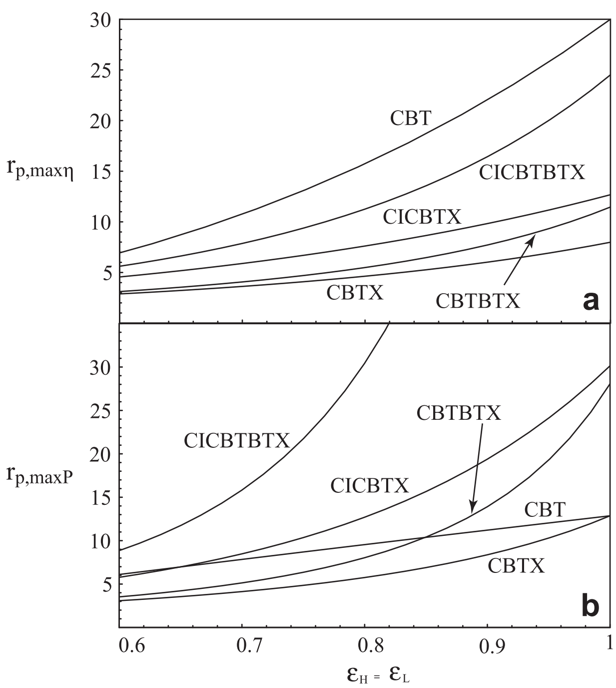

Fig. 10. (a) Pressure ratio giving maximum efficiency ($r_{p,\max\eta}$) and (b) maximum power ($r_{p,\max P}$) as functions of $\epsilon_L = \epsilon_H$ for several particular configurations. Parameters used are: $\tau = 5,\ \rho_L = \rho_H = 0.97,\ \epsilon_c = \epsilon_t = 0.9,\ \epsilon_r = 0.75$, and $\xi = 0.02$.

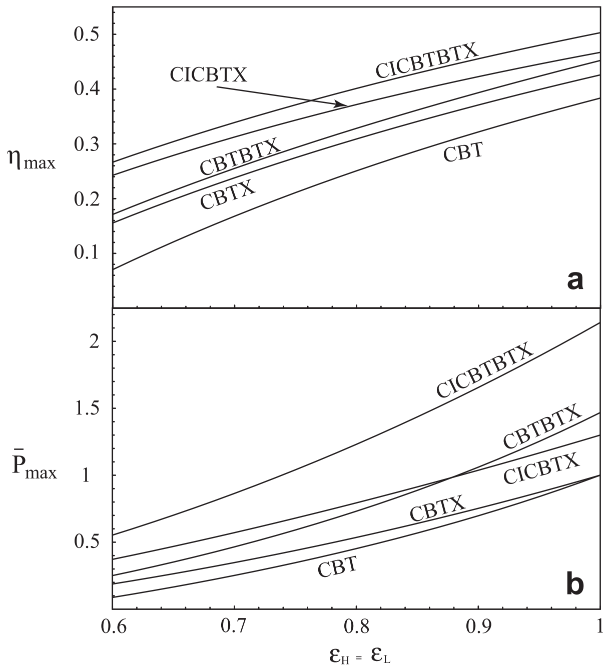

Fig. 11. Maximum values of efficiency (a) and normalized power (b) for different plant arrangements as functions of the parameters characterizing coupling to the external heat reservoirs. Parameters are like in Fig. 10.

the increase of $r_{p,\max\eta}$ when external irreversibilities decrease up to reach $\epsilon_L = \epsilon_H = 1$ is similar for all configurations. The behavior is similar for $\eta_{\max}$. Nevertheless the evolution of the parameters associated to power output is richer. In Fig. 10b we see that $r_{p,\max P}$ of CBT and CBTX collapses to an identical value, since for $\epsilon_L = \epsilon_H = 1$ power is independent of regeneration efficiency. Moreover, for CBTBTX, $r_{p,\max P}$ increases rapidly, crossing over CBT for $\epsilon_L = \epsilon_H = 0.85$. Interestingly, a similar fact occurs for $P_{\max}$. Here, CBTBTX crosses over CICBTX for $\epsilon_L = \epsilon_H > 0.88$ and when external irreversibilities are very small maximum power is obtained from CBTBTX configuration. At a glance of Fig. 11, note that for maximum efficiency, always CICBTX configuration is more favorable than CBTBTX.

As a summary is important to stress that the external irreversibilities associated to the coupling to the external heat reservoirs are source of a rich behavior of thermal efficiency and power output, making essential a good knowledge of $\epsilon_L = \epsilon_H$ to determine which plant configuration could be more interesting. Of course, the case $\epsilon_L \neq \epsilon_H$ would give even a more diverse arrangement of patterns.

5. Summary and conclusions

We have presented a generalized theoretical and analytical model for an irreversible regenerative multi-step Brayton cycle. One of the major achievements of this work is that our formalism allows for the theoretical analysis of any plant configuration including an arbitrary number of compressors and turbines with the corresponding intercooling and reheating intermediate processes and also the main irreversibility sources affecting this kind of facilities. The equations giving the performance of the cycle depend on a low number of parameters with a clear physical meaning accounting for cycle geometry and design, and the irreversibility sources considered. The last include losses in the non-isentropic turbines and compressors, pressure drops in the heat input and heat release, irreversibilities in the regenerator, heat-leak through the plant to the ambient and non-ideal couplings with the external constant-temperature heat reservoirs. We have analyzed the particular cases where either external or internal irreversibilities are treated separately in order to recover several results found in previous works. We have considered as limit cases that of a simple plant with only one turbine and one compressor and that with an infinite number of reheating and intercooling steps (Ericsson type cycles).

Moreover, we have analyzed apart from the curves of power output and efficiency in the case of identical number of turbines and compressors, several particular real arrangements where these numbers are different. In the last case we have reproduced results obtained from numerical simulations of particular gas turbine power plants. Power–efficiency curves obtained by eliminating the pressure ratio give an interesting picture of the evolution of maximum power and maximum efficiency with the number of compressors and turbines.

Finally, we have obtained the evolution of power and efficiency when all irreversibility sources are considered at the same time, taking realistic values for all the parameters involved. It is specially interesting the dependence of maximum power and maximum efficiency and the corresponding pressure ratios on the parameters accounting for the irreversibilities coming from the couplings of the working fluid with the external heat reservoirs. It is remarkable that in particular due to crossing effects between maximum power output curves, $\overline{P}_{\max} = \overline{P}_{\max}(\epsilon_H = \epsilon_L)$ (Fig. 11) a precise knowledge of these losses is desirable to choose a particular plant arrangement in order to get the required power output. Respect to thermal efficiency is also interesting that the efficiency curves, $\eta = \eta(r_p)$, after their maxima decrease much faster when irreversible coupling to the external heat reservoirs is considered (compare Fig. 6a with Fig. 8a). In conclusion, we believe that these results can be helpful in the design and analysis of the optimal working regime of real modern gas power plants.

Acknowledgements

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