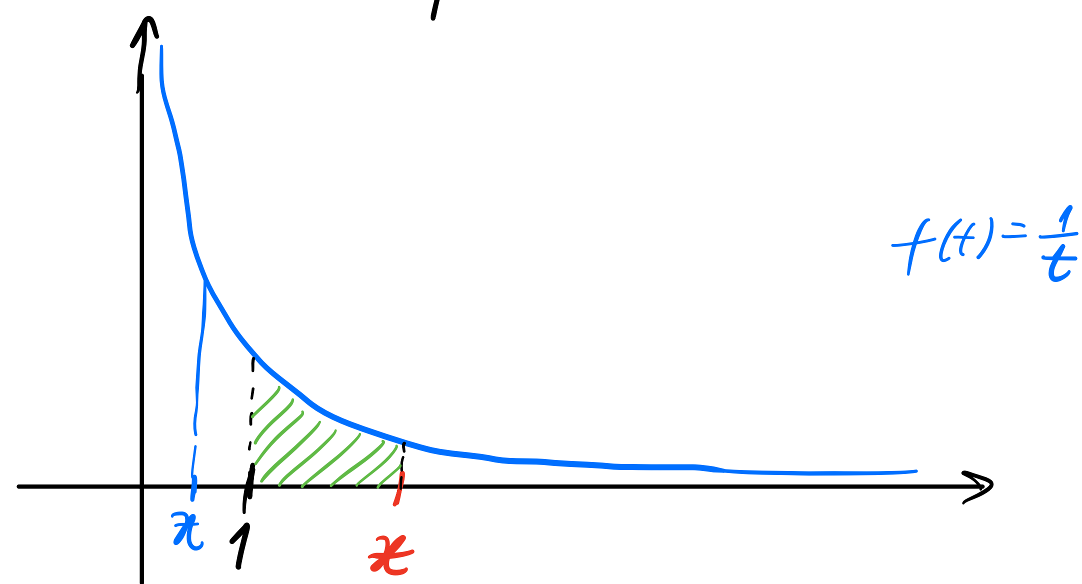
LA FUNCIÓN LOGARITMO

$$\log : \mathbb{R}^+ \longrightarrow \mathbb{R} \;;$$

$$\log(x) = \int_1^x \frac{1}{t}\,dt \qquad \forall x \in \mathbb{R}^+$$



$$\int_1^x \frac{1}{t}\,dt = -\overbrace{\boxed{\int_x^1 \frac{1}{t}\,dt}}^{1^{o}} < 0 \qquad \text{si} \quad 0 < x < 1$$

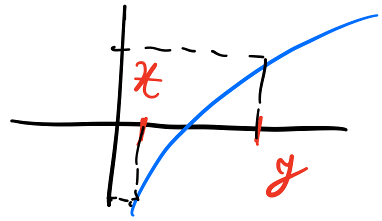
$$\int_1^x \frac{1}{t}\,dt \geq 0 \qquad \text{si} \quad x \geq 1$$

$$\log(1) = \int_1^1 \frac{1}{t}\,dt = 0$$

<u>log es una función estrictamente creciente</u>

Recordamos: $f:\mathbb{R}\to\mathbb{R}$ es estrictamente creciente

si $\quad x<y \Rightarrow f(x)<f(y)$



Para ver que log es estrictamente creciente tenemos que ver que si $0<x<y$

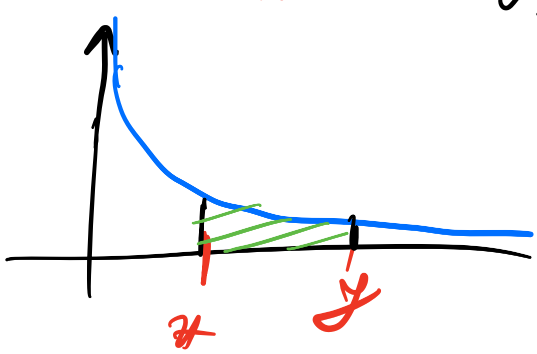
$$\Rightarrow \log(x)<\log(y)$$

Supongamos que $0<x<y$.

$$\boxed{\log(y)-\log(x)} = \int_1^y \frac{1}{t}\,dt - \int_1^x \frac{1}{t}\,dt =$$

$$= \int_1^y \frac{1}{t}\,dt - \left(-\int_x^1 \frac{1}{t}\,dt\right) = \int_x^1 \frac{1}{t}\,dt + \int_1^y \frac{1}{t}\,dt =$$

$$= \int_x^y \frac{1}{t}\,dt \boxed{>0}$$



ADITIVIDAD RESPECTO DEL INTERVALO

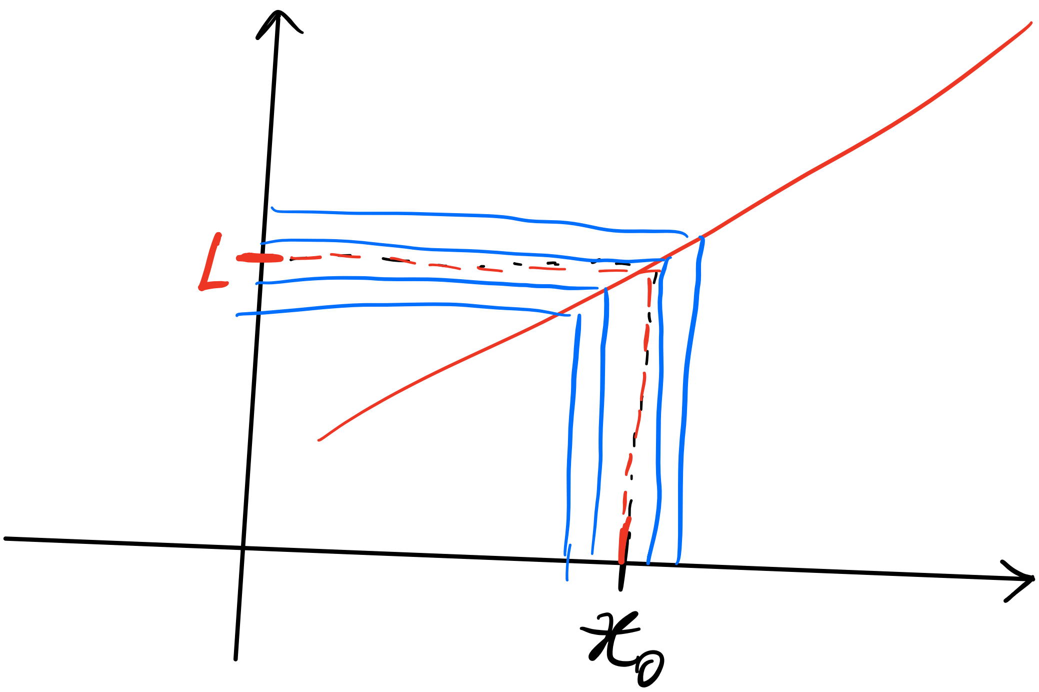
Por lo tanto, concluimos que

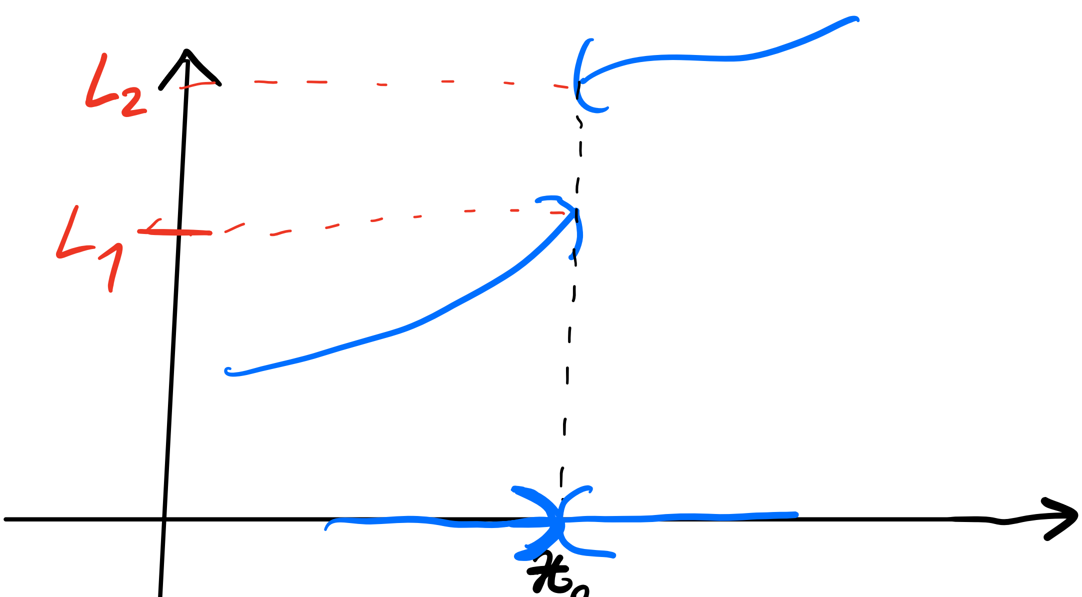
$$\log(y)-\log(x)>0 \Rightarrow \log(y)>\log(x)$$

Propiedad:

$$\log(\cancel{x}\cancel{y}) = \log(\cancel{x}) + \log(\cancel{y})$$

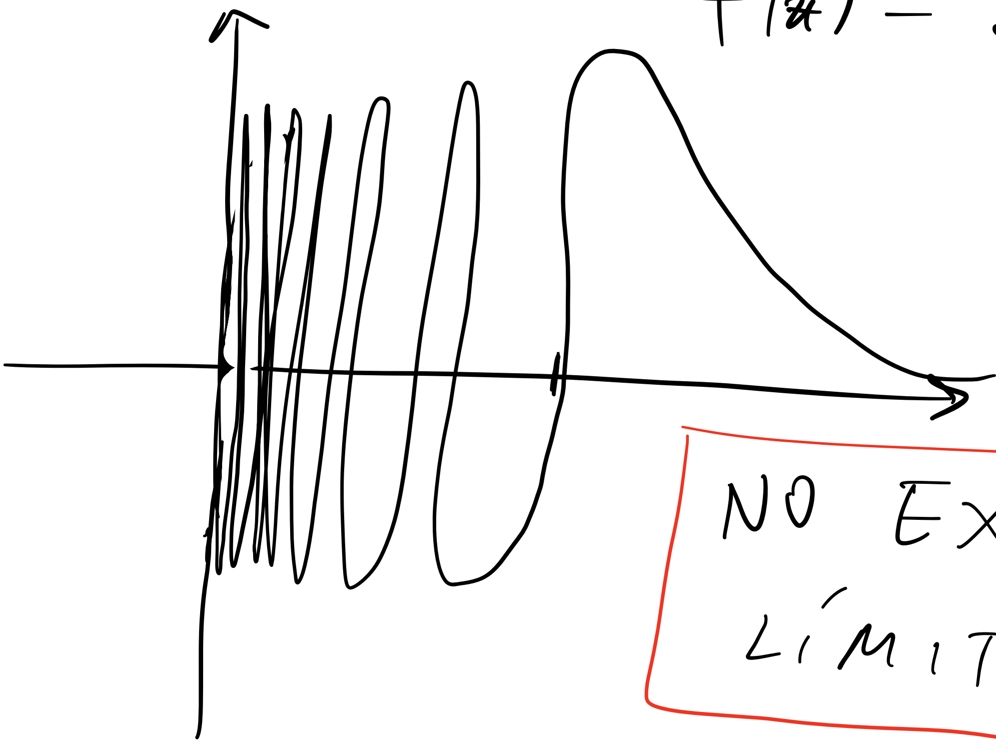
LÍMITES Y CONTINUIDAD





$f: \mathbb{R}^+ \longrightarrow \mathbb{R}$

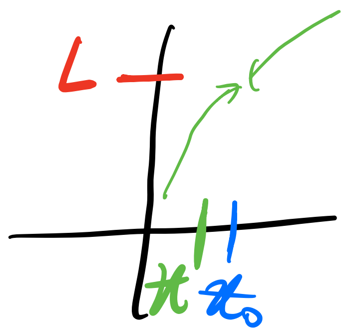
$f(x) = \operatorname{sen}\left(\frac{1}{x}\right)$



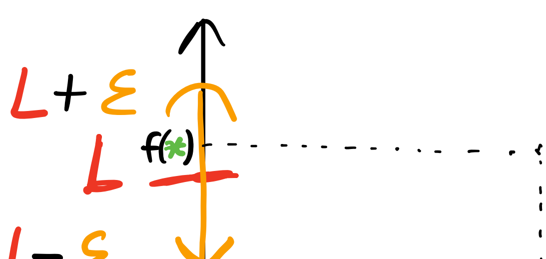
NO EXISTE EL LÍMITE EN 0

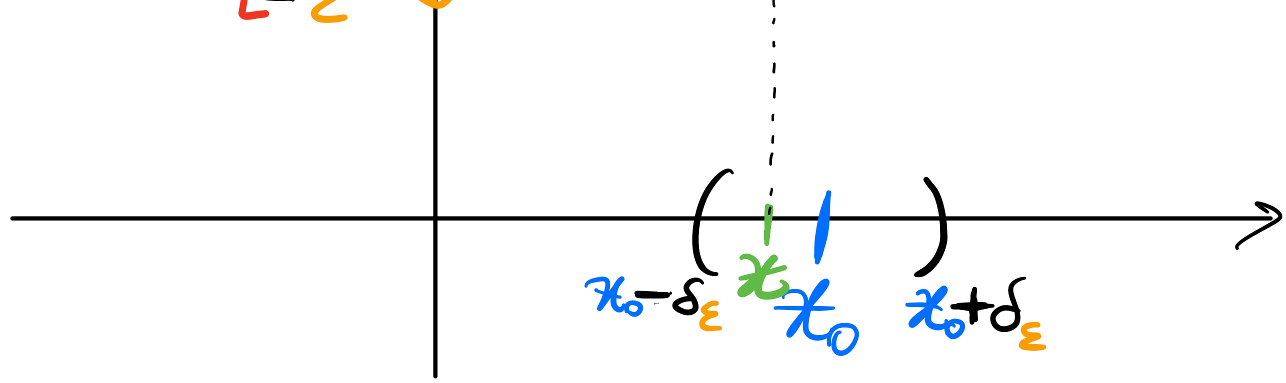
Hacia la definición formal de límite

"f tiene límite L en x_0



si las imágenes $f(x)$ están tan cerca de L como queramos con la condición de que x esté suficientemente cerca de x_0"



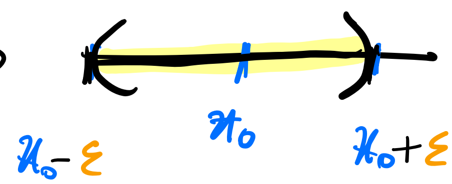


Terminología:

El entorno de centro x_0 y radio $\varepsilon>0$ es

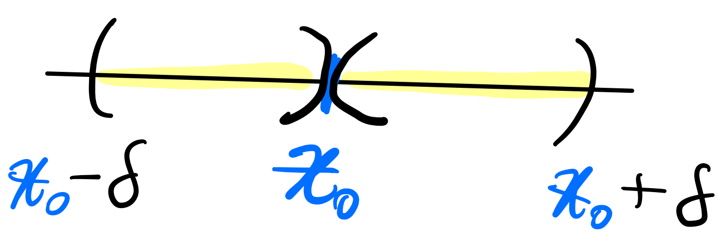
$$E(x_0,\varepsilon)=\{x\in\mathbb{R} : \underbrace{|x-x_0|<\varepsilon}\}$$

x está a distancia menor a ε de de x_0



El entorno reducido de centro x_0 y radio $\delta>0$ es

$$E^*(x_0,\delta)=\{x\in\mathbb{R} : 0<|x-x_0|<\delta\}$$

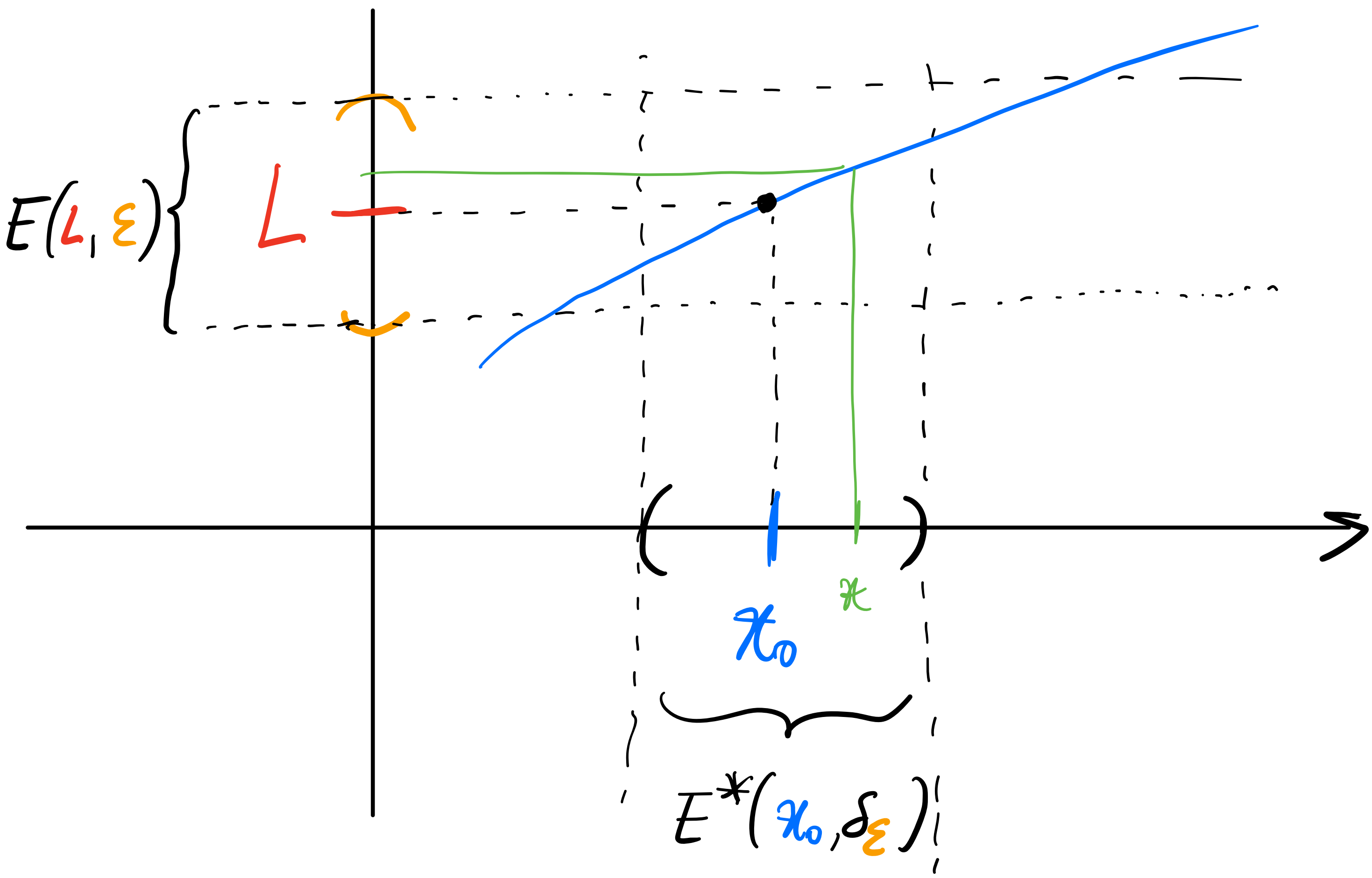


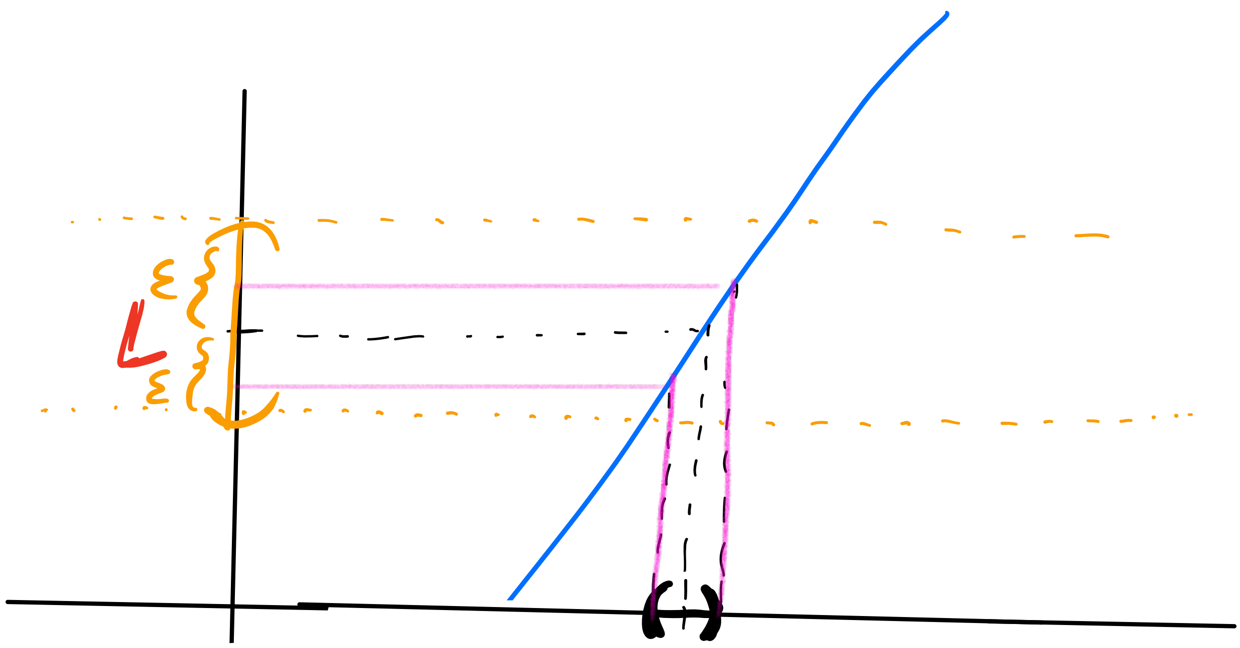
Definición: Decimos que f tiene límite L en x_0 si:

$\forall\varepsilon>0\ \exists\,\delta>0\ /$

$\forall \varepsilon > 0, \exists \delta_\varepsilon > 0$

$f(x) \in E(L, \varepsilon) \quad \forall x \in E^*(x_0, \delta)$





$\mathcal{X}_0$