

# Linealidad de la integral

Si  $f, g: [a, b] \rightarrow \mathbb{R}$  son integrables.

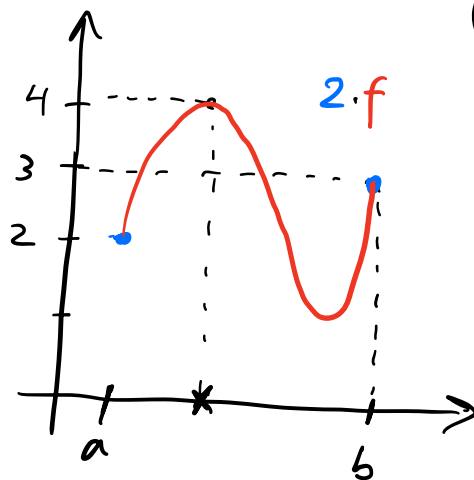
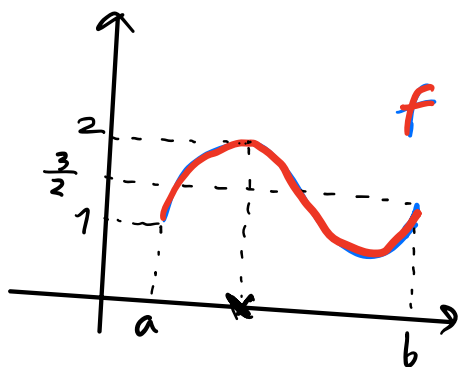
Entonces  $f+g$  es integrable

$$\int_a^b (f+g)(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

Además si  $\alpha \in \mathbb{R}$ , entonces  $\alpha f$  es integrable y

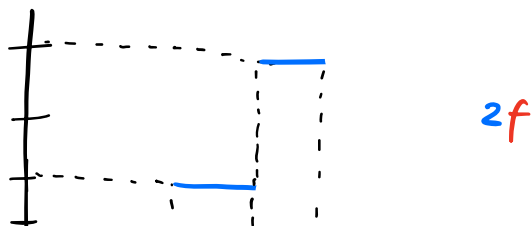
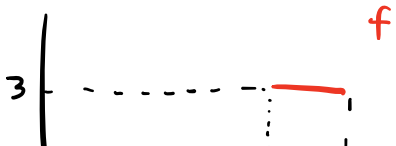
$$\int_a^b (\alpha f)(x) dx = \alpha \int_a^b f(x) dx$$

¿Porqué vale la linealidad?



$$(2f)(a) = 2f(a)$$

Ej sencillo:



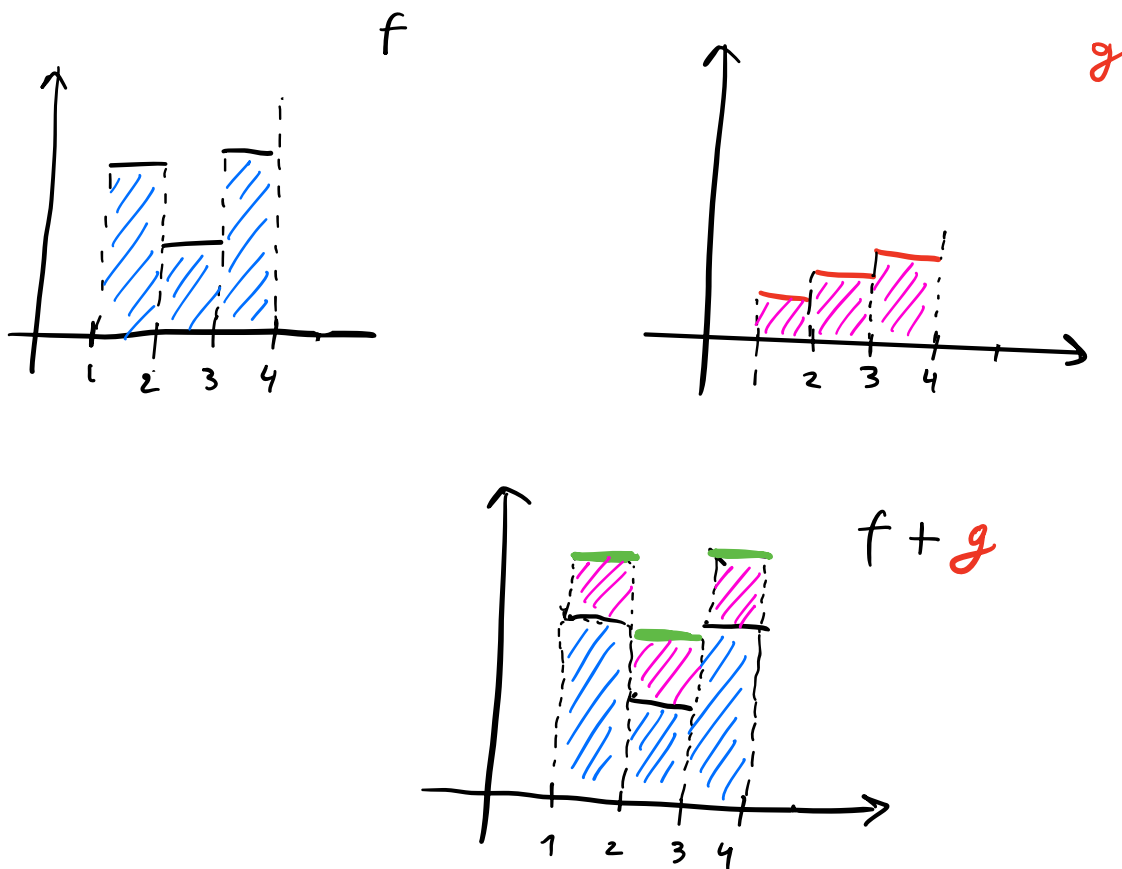


$$\int_1^4 f = 1 + 2 + 3 = 6$$

$$\int_1^4 2f = 2 + 4 + 6 = 2 \int f$$

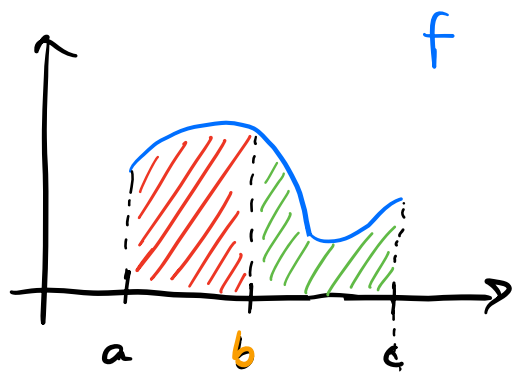


Para la suma, un ejemplo sencillo



$$\int_1^4 f(x) dx + \int_1^4 g(x) dx = \int_1^4 (f+g)(x) dx$$

Linealidad respecto del intervalo



Sean  $a < b < c$  números reales

Y  $f: [a, c] \rightarrow \mathbb{R}$  integrable,

Entonces  $f|_{[a, b]}$  y  $f|_{[b, c]}$  son integrables

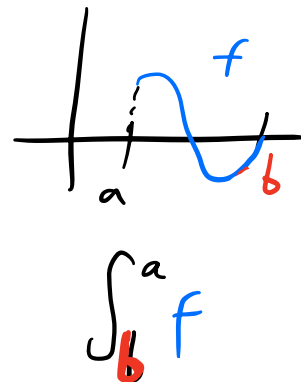
y se tiene

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

### Extensión de la definición de integral

Si  $a < b$ ; definimos

$$\int_b^a f(x) dx := - \int_a^b f(x) dx$$



Ej:

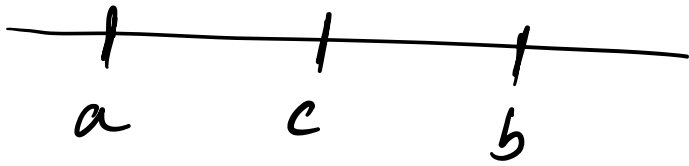
$$\text{Si } \int_1^4 f(x) dx = 6 \Rightarrow \int_4^1 f(x) dx = -6$$

$$\int_a^a f(x) dx = 0$$

Propiedad: si  $a, b, c \in \mathbb{R}$  (no necesariamente en orden creciente)

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

Deduzcamos la propiedad de la linealidad respecto del intervalo, en un caso.



hay que ver que

$$\int_a^c f(x) dx = \left( \int_a^b f(x) dx \right) + \left( \int_b^c f(x) dx \right)$$

Por linealidad respecto del intervalo; como  $a < c < b$ ;

$$\left( \int_a^b f(x) dx \right) = \left( \int_a^c f(x) dx + \int_c^b f(x) dx \right)$$

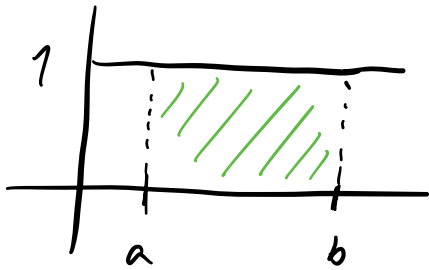
Por otro lado,  $\left( \int_b^c f(x) dx \right) = \left( - \int_c^b f(x) dx \right)$

Entonces

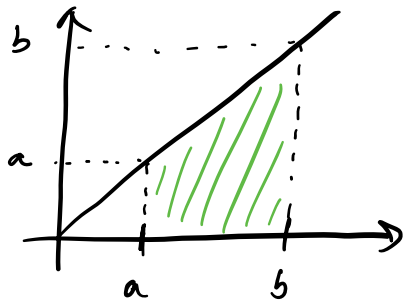
$$\left( \int_a^b f \right) + \int_b^c f = \left( \int_a^c f + \int_c^b f \right) + \left( - \int_c^b f \right) = \int_a^c f$$

The diagram shows the derivation of the property. On the left, the expression  $\left( \int_a^b f \right) + \int_b^c f$  is circled in black. This is equal to  $\left( \int_a^c f + \int_c^b f \right) + \left( - \int_c^b f \right)$ . The terms  $\int_c^b f$  and  $-\int_c^b f$  are crossed out with blue diagonal lines. The final result is  $\int_a^c f$ , which is also circled in black. A long arrow points from the first circled expression to the second circled expression, with an equals sign at the bottom.

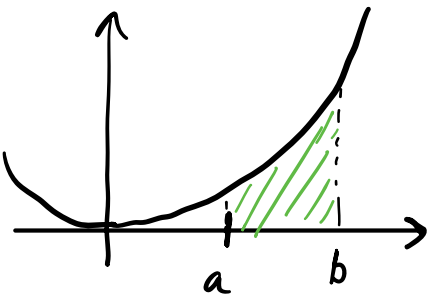
# Spoiler:



$$\int_a^b 1 dx = b - a$$



$$\int_a^b x dx = \frac{b^2 - a^2}{2}$$



$$\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$$

$$\int_a^b x^n dx = \frac{b^{n+1} - a^{n+1}}{n+1}$$

después  
del parcial

$$\int_1^3 3x^2 + 5x - 7 dx = \int_1^3 3x^2 dx + \int_1^3 5x dx + \int_1^3 (-7) dx =$$

linealidad  
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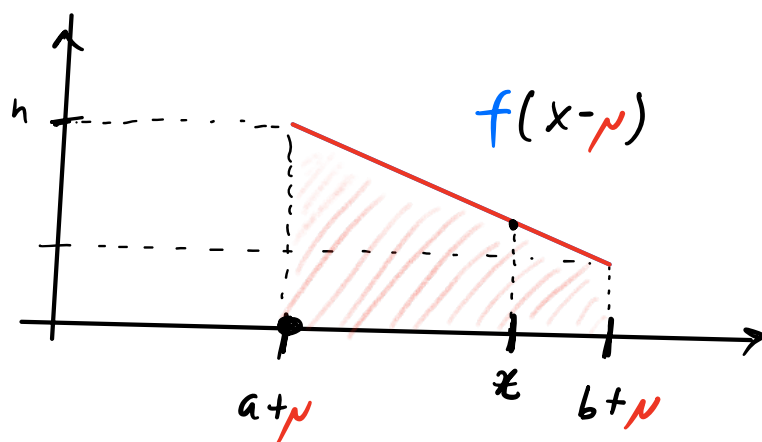
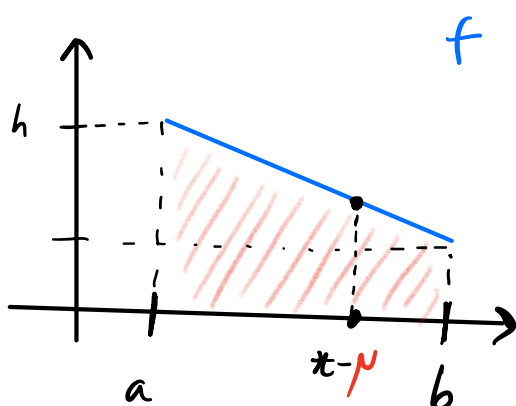
$$\Rightarrow 3 \int_1^3 x^2 dx + 5 \int_1^3 x dx + (-7) \int_1^3 1 dx =$$

linealidad  
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integral

$$= 3\left(\frac{3^3-1^3}{3}\right) + 5\left(\frac{3^2-1^2}{2}\right) + (-7)(3-1) = \frac{3 \cdot 26}{3} + 5 \frac{8}{2} - 14$$

→  
SUSTITUIAMOS  
spoiler

## Fórmulas de cambio de variable lineal

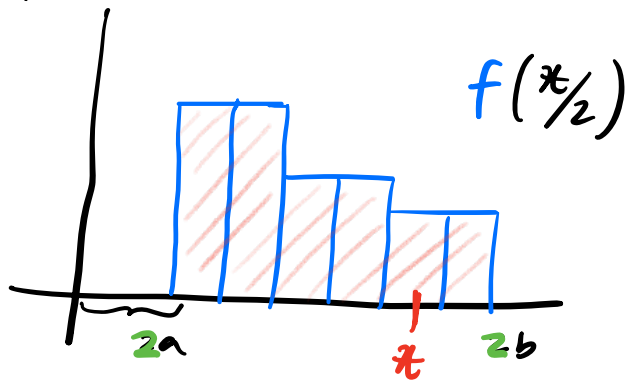
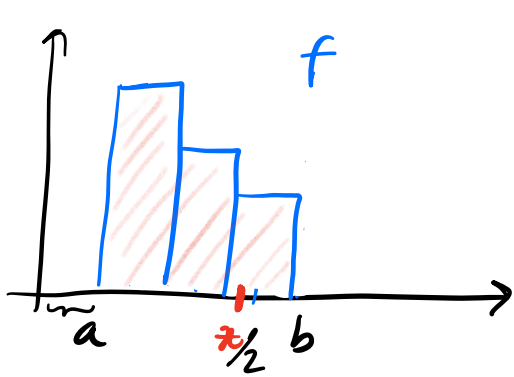


$$\int_a^b f(x) dx = \int_{a+\mu}^{b+\mu} f(x-\mu) dx$$

Ej:  $\int_3^5 (x-7)^2 dx = \int_{3-7}^{5-7} x^2 dx = \int_{-4}^{-2} x^2 dx =$

$$= \frac{(-2)^3 - (-4)^3}{3}$$

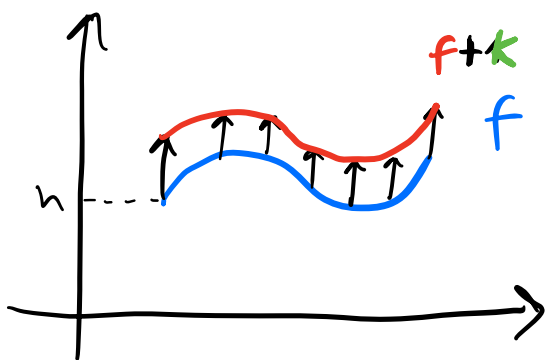
Cambio de variable por una dilatación



$$\lambda \int_a^b f(x) dx = \int_{\lambda a}^{\lambda b} f\left(\frac{x}{\lambda}\right) dx$$

$$\lambda \int_a^b f(x) dx = \int_{\lambda a + \mu}^{\lambda b + \mu} f\left(\frac{x - \mu}{\lambda}\right) dx$$

¿Qué pasa si trasladamos el gráfico en la dirección del eje y?



$$\int_a^b f + k dx = \int_a^b f dx + \int_a^b k dx =$$

$$= \int_a^b f(x) dx + K(b-a)$$