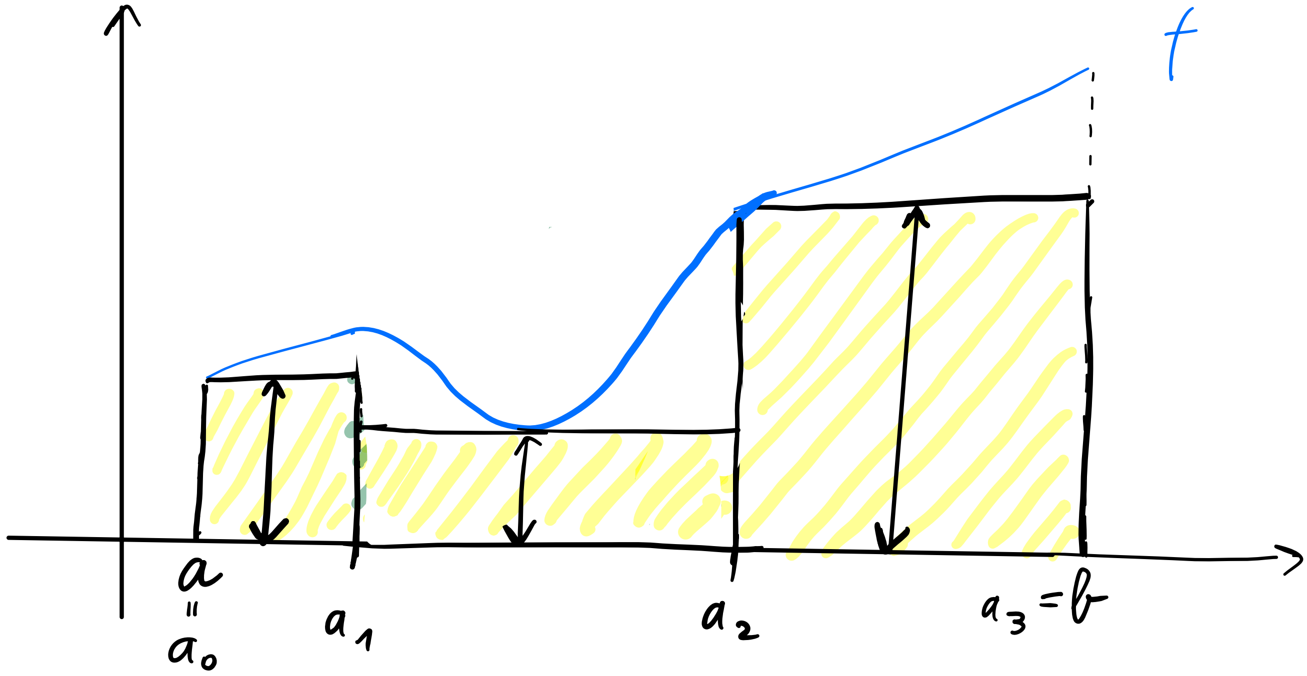


La suma inferior de f asociada a la partición

$P = \{a = a_0, a_1, a_2, a_3 = b\}$ es

$$\inf(f,[a_0,a_1])(a_1-a_0) + \inf(f,[a_1,a_2])(a_2-a_1) + \inf(f,[a_2,a_3])(a_3-a_2)$$

$$S_*(f,P) \sum_{i=0}^{i=2} \inf(f,[a_i,a_{i+1}]).(a_{i+1}-a_i)$$

En general si $P = \{a = a_0, \dots, a_n = b\}$ es

partición de $[a,b]$ y $f : [a,b] \longrightarrow \mathbb{R}$ función acotada

$Im(f)$ es un conjunto acotado

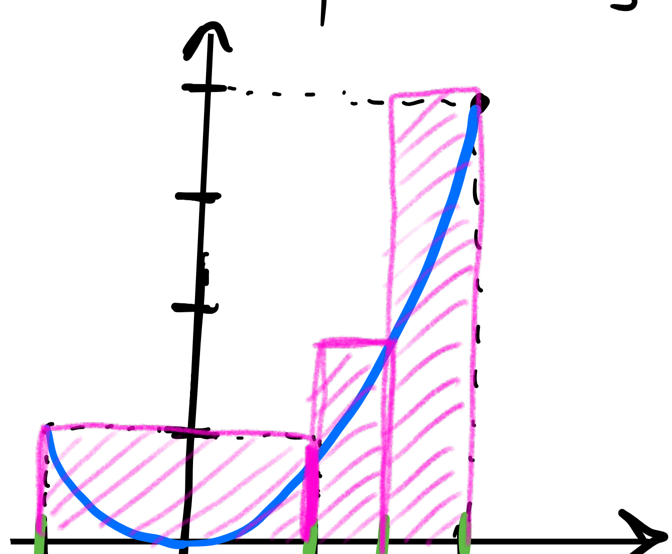
$$S_*(f, P) = \sum_{i=0}^{n-1} \inf(f, [a_i, a_{i+1}])(a_{i+1} - a_i)$$

Análogamente:

La suma superior de $f: [a, b] \longrightarrow \mathbb{R}$ asociada a $P = \{a = a_0, \dots, a_{n-1}, a_n = b\}$ es

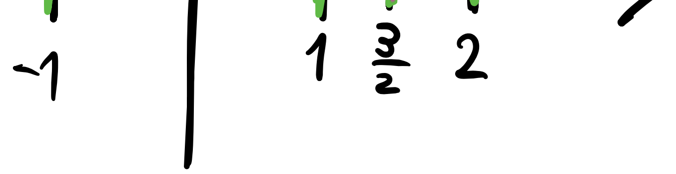
$$S^*(f, P) = \sum_{i=0}^{n-1} \sup(f, [a_i, a_{i+1}])(a_{i+1} - a_i)$$

<u>Ejemplo:</u> $f: [-1, 2] \longrightarrow \mathbb{R}$; $f(x) = x^2$



$P = \{-1, 1, \frac{3}{2}, 2\} =$

$= \{a_0, a_1, a_2, a_3\}$



$$S^*(f,P)=\sum_{i=0}^{2}\sup(f,[a_i,a_{i+1}])(a_{i+1}-a_i)$$

$$=\overbrace{\sup(f,[-1,1])}^{1}\cdot(1-(-1))+\overbrace{\sup(f,[1,\tfrac{3}{2}])}^{9/4}\left(\tfrac{3}{2}-1\right)+$$

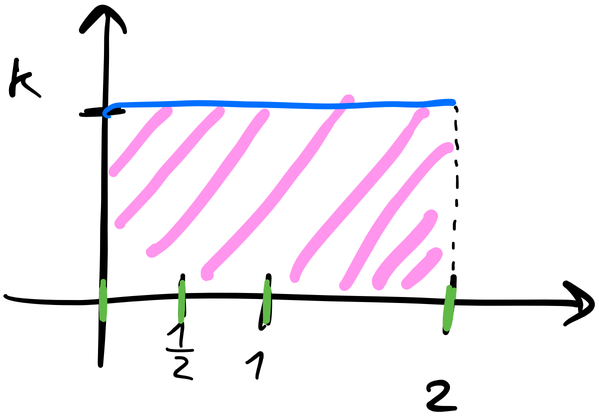
$$+\overbrace{\sup(f,[\tfrac{3}{2},2])}^{4}\left(2-\tfrac{3}{2}\right)=$$

$$=1\cdot 2+\frac{9}{4}\cdot\frac{1}{2}+4\cdot\frac{1}{2}=2+\frac{9}{8}+2=\frac{41}{8}$$

<u>Ejercicio</u>: Calcular $S_*(f,P)$

<u>Observación</u>: $\boxed{S_*(f,P)\leq S^*(f,P)}$

<u>Ej</u>: $f:[0,2]\longrightarrow\mathbb{R}$; $f(x)=k$ (cte)



$P = \{0, \frac{1}{2}, 1, 2\}$

$$S_*(f, P) = \overbrace{\inf\left(f, [0, \tfrac{1}{2}]\right)}^{k}\left(\tfrac{1}{2} - 0\right) + \overbrace{\inf\left(f, [\tfrac{1}{2}, 1]\right)}^{k}\left(1 - \tfrac{1}{2}\right) + \overbrace{\inf\left(f, [1, 2]\right)}^{k}(2-1) =$$

$$= k\,\underbrace{\left(\left(\cancel{\tfrac{1}{2}} - 0\right) + \left(\cancel{1} - \cancel{\tfrac{1}{2}}\right) + \left(2 - \cancel{1}\right)\right)}_{\overset{\shortparallel}{2}} = k \cdot 2$$

Si $f : [a, b] \longrightarrow \mathbb{R}$ es constante k, entonces, todas las sumas superiores y todas las sumas inferiores tienen el mismo valor, que es $\boxed{k \cdot (b-a)}$

Sea $f : [a, b] \longrightarrow \mathbb{R}$ acotada

$$\boxed{\mathcal{A}_*(f)} = \left\{ S_*(f, P) : P \text{ partición de } [a,b] \right\}$$

Conjunto de sumas inferiores

$$\boxed{\mathcal{A}^*(f)} = \left\{ S^*(f, P) : P \text{ partición de } [a,b] \right\}$$

Conjunto de sumas superiores

Vamos a ver que $\forall\, P, Q$ particiones

$$S_*(f, P) \leq S^*(f, Q)$$

Eso va a implicar que $\boxed{\sup \mathcal{A}_*(f) \leq \inf \mathcal{A}^*(f)}$