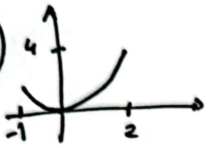
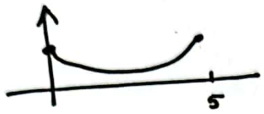
RESOLUCIÓN - V1

V o F:

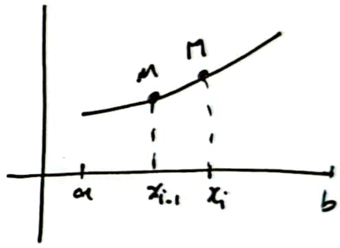
Af 1: V $f(x)=x$ es integrable $\Longrightarrow \forall\, \varepsilon>0\ \exists\, P$ tal que $S^*(f,P)-S_*(f,P)<\varepsilon$. En particular, tomando $\varepsilon=\frac{1}{2}$.

Af 2: V  Sobre pero no inyectiva.

Af 3: F  No tiene raíz.

Af 4: V Sea $P_n=\{a=x_0<x_1<\dots<x_n=b\}$ una partición de $[a,b]$ con

$x_i-x_{i-1}=\frac{b-a}{n}\ \forall i \Longrightarrow$



$$S^*(f,P_n)=\frac{(b-a)}{n}\sum_{i=1}^{n} f(x_i)$$

$$S_*(f,P_n)=\frac{(b-a)}{n}\sum_{i=1}^{n} f(x_{i-1})$$

$$\Longrightarrow S^*(f,P_n)-S_*(f,P_n)=\frac{(b-a)}{n}\sum_{i=1}^{n}\left(f(x_i)-f(x_{i-1})\right)=$$

$$=\frac{(b-a)}{n}\left[\cancel{f(x_1)}-f(x_0)+\cancel{f(x_2)}-\cancel{f(x_1)}+\dots+f(x_n)-\cancel{f(x_{n-1})}\right]$$

$$=\frac{(b-a)}{n}\left(f(\overset{b}{\overset{\shortparallel}{x_n}})-f(\overset{a}{\overset{\shortparallel}{x_0}})\right)<\varepsilon \text{ si tomamos}$$

$$n>\frac{(b-a)\left(f(b)-f(a)\right)}{\varepsilon}.$$

Af 5: F continua no implica derivable (y menos que $f'(a)=0$).

Ej1

- $(0,10] \subseteq A$ porque $10 \in A$ y $\forall y \in (0,+\infty)$ con $y<10$ se cumple que $y \in A$ $\Longrightarrow$ $\inf(A)=0$ y no puede ser mínimo porque $A \subseteq (0,+\infty)$
- $[10,+\infty) \subseteq B$ porque $10 \in B$ y $\forall y \in (0,+\infty)$ con $y>10$ se cumple que $y \in B$ $\longrightarrow$ * B no tiene supremo (no está acotado sup.)

  * B está acotado inf. por 0 $\Longrightarrow$ $\exists \inf(B)$ pero no sabemos quién es $\inf(B)$ ni si es mínimo

<u>Conclusión</u>: $\boxed{\text{A no tiene mínimo y B no tiene supremo}}$

Ej2:

$f(x)=\dfrac{e^{\alpha x^2}-1}{x^2}$ es continua en $(-\infty,0)$ y $f(x)=(\alpha x+3)^2$ es continua en $(0,+\infty)$, así que basta con estudiar la cont. de f en $x=0$.

- $f(0)=9$
- $$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{e^{\alpha x^2}-1}{x^2} \underset{\uparrow \text{ l'Hopital}}{\overset{\frac{0}{0}}{=}} \lim_{x\to 0^-} \frac{e^{\alpha x^2}\cdot \alpha \cancel{2x}}{\cancel{2x}} = \alpha$$
- $$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (\alpha x+3)^2 = 9$$

$\longrightarrow$ tiene que ser $\boxed{\alpha = 9}$.

Ej3:

$f(x) = \text{Arctg}(\cos(x^2))$

$f'(x) \underset{\uparrow\ R.C.}{=} (\text{Arctg})'(\cos(x^2)) \times \underbrace{(\cos(x^2))'}_{\overset{R.C.}{=}\ -\text{sen}(x^2) \times 2x}$ $\underset{\uparrow}{=} \dfrac{1}{1+(\cos(x^2))^2} \times (-\text{sen}(x^2)) \times 2x$

$(\text{Arctg}(u))' = \dfrac{1}{1+u^2}$

$\Rightarrow \boxed{f'\left(\sqrt{\tfrac{\pi}{6}}\right)} = \dfrac{1}{1+\Big(\underbrace{\cos\left(\tfrac{\pi}{6}\right)}_{=\frac{\sqrt{3}}{2}}\Big)^2} \times (-1) \times \overbrace{\text{sen}\left(\tfrac{\pi}{6}\right)}^{\frac{1}{\cancel{2}}} \times \cancel{2}\sqrt{\tfrac{\pi}{6}}$

$= \dfrac{-1}{\underbrace{1+\frac{3}{4}}_{=\frac{7}{4}}} \times \sqrt{\tfrac{\pi}{6}} = \boxed{-\dfrac{4}{7} \cdot \sqrt{\tfrac{\pi}{6}}}$.

Ej4: Basta con estudiar el signo de $f'(x)$:

$f'(x) = e^x(2x^2+x-8+4x+1) = \overset{>0}{e^x}\overset{?}{(2x^2+5x-7)}$

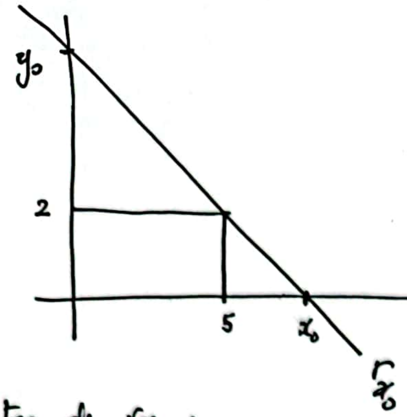
$2x^2+5x-7 = 0 \Leftrightarrow x = \dfrac{-5 \pm \sqrt{25-4.2.(-7)}}{4} = \dfrac{-5 \pm \sqrt{25+56}}{4} = \dfrac{-5 \pm \sqrt{81}}{4}$

$= \dfrac{-5 \pm 9}{4}$ → $x = 1$, $x = -\frac{7}{2}$

		$-\frac{7}{2}$		0		1	
$\text{sgn}(f')$	$+\ +$		$-\ -\ -$		$-\ -$		$+\ +\ +\ +$
	↗		↘		↘		↗

f es ↗ en $(1, +\infty)$

Ej5:



Sean x_0 = base del triángulo ($x_0 \geq 5$)

y_0 = altura "

r_{x_0} = recta que pasa por $(x_0, 0)$ y $(5,2)$, como se muestra en la figura.

Ec. de r_{x_0}:

r_{x_0}: $y = ax+b$ con $\begin{cases} 0 = ax_0 + b \to b = -ax_0 \\ 2 = a.5 + b \to 2 = 5a - ax_0 \to \end{cases}$

$$2 = (5-x_0)\,a \to \boxed{a = \frac{2}{5-x_0}}$$

$$\to y_0 = b = -\frac{2x_0}{5-x_0} \qquad \text{y } \boxed{b = -\frac{2x_0}{5-x_0}}$$

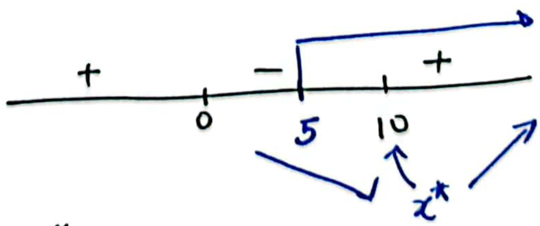
$$\underset{\text{del triángulo}}{\text{Área}} = \frac{x_0 \times y_0}{2} = \frac{x_0 \times -\frac{\cancel{2}x_0}{5-x_0}}{\cancel{2}} = \frac{x_0^2}{x_0-5} \quad \text{con } x_0 \geq 5.$$

Ahora optimizamos en función de $x_0 \geq 5$.

Sea $f(x) = \dfrac{x^2}{x-5} \Longrightarrow$

$$f'(x) = \frac{2x(x-5) - x^2}{(x-5)^2} = \frac{2x^2 - 10x - x^2}{(x-5)^2} = \frac{x^2-10x}{(x-5)^2} = \frac{x(x-10)}{\underbrace{(x-5)^2}_{\geq 0}}$$

$\operatorname{sgn}(f') = \operatorname{sgn}(x(x-10))$



El mínimo se da en $x^* = 10$ y vale:

$$f(10) = \frac{10^2}{10-5} = \frac{100}{5} = \boxed{20}$$

Ej 6:

$$\lim_{x \to 0} \frac{\int_0^x \operatorname{sen}^2(t)\,dt}{\operatorname{sen}(x) - x} \underset{\substack{\uparrow \\ \text{L'Hopital}}}{\overset{0/0}{=}} \lim_{x \to 0} \frac{\operatorname{sen}^2 x}{\cos x - 1} \underset{\substack{\uparrow \\ \text{L'Hopital}}}{\overset{0/0}{=}} \lim_{x \to 0} \frac{2\,\cancel{\operatorname{sen} x} \cdot \cos x}{-\cancel{\operatorname{sen} x}} = -2\cos(0)$$

$$= \boxed{-2}.$$

Ej 7:

$$2 \cdot \int_0^1 e^{3\sqrt{x}+1} \cdot \frac{1}{2\sqrt{x}}\,dx = \begin{bmatrix} \text{C.V.} \\ u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \end{bmatrix} = 2\int_0^1 e^{3u+1}\,du = 2 \cdot \frac{e^{3u+1}}{3} \Big|_0^1$$

$$= \frac{2}{3}\left(e^{3\cdot 1+1} - e^{3\cdot 0+1}\right) = \boxed{\frac{2}{3}\left(e^4 - e\right)}$$

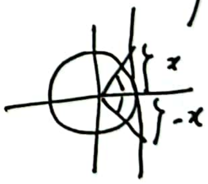
Ej 8:

$$\text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} (f(x) - g(x))\,dx = \int_{-\sqrt{3}}^{\sqrt{3}} f(x)\,dx - \int_{-\sqrt{3}}^{\sqrt{3}} g(x)\,dx = \text{I} - \text{II}$$

$$\text{I} = \int_{-\sqrt{3}}^{\sqrt{3}} \frac{18}{9+x^2}\,dx = \frac{18}{3} \cdot \operatorname{Arctg}\left(\frac{x}{3}\right)\Big|_{-\sqrt{3}}^{\sqrt{3}} = 6\left[\operatorname{Arctg}\left(\frac{\sqrt{3}}{3}\right) - \operatorname{Arctg}\left(-\frac{\sqrt{3}}{3}\right)\right] =$$

$$= 6\left[\operatorname{Arctg}\left(\frac{1}{\sqrt{3}}\right) + \operatorname{Arctg}\left(\frac{1}{\sqrt{3}}\right)\right]$$

$$= 12\,\operatorname{Arctg}\left(\frac{1}{\sqrt{3}}\right).$$



$\operatorname{Arctg}(-x) = -\operatorname{Arctg}(x)$

$$\text{II} = \int_{-\sqrt{3}}^{\sqrt{3}} \frac{9}{9-x^2}\,dx = \int_{-\sqrt{3}}^{\sqrt{3}} \frac{9}{(3-x)(3+x)}\,dx$$

$$\frac{9}{(3-x)(3+x)} = \frac{A}{3-x} + \frac{B}{3+x} = \frac{A(3+x) + B(3-x)}{(3-x)(3+x)} = \frac{\overbrace{(A-B)}^{=0}x + \overbrace{(3A+3B)}^{=9}}{(3-x)(3+x)}$$

$$\Longleftrightarrow \begin{cases} A-B=0 & \Longleftrightarrow \quad B=A \\ 3A+3B=9 & \Longleftrightarrow \quad 6A=9 \Longleftrightarrow A=\frac{9}{6}=\frac{3}{2} \text{ y } B=\frac{3}{2} \end{cases}$$

$$\rightarrow \text{II} = \int_{-\sqrt{3}}^{\sqrt{3}} \frac{3/2}{3-x} + \frac{3/2}{3+x}\, dx = \frac{3}{2}\left[-\log(3-x)+\log(3+x)\right]\Big|_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \frac{3}{2}\log\left(\frac{3+x}{3-x}\right)\Big|_{-\sqrt{3}}^{\sqrt{3}} = \frac{3}{2}\left[\log\left(\frac{3+\sqrt{3}}{3-\sqrt{3}}\right)-\log\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right)\right]$$

$$= \frac{3}{2}\log\left[\frac{3+\sqrt{3}}{3-\sqrt{3}} \div \frac{3-\sqrt{3}}{3+\sqrt{3}}\right] = \frac{3}{\cancel{2}}\log\left[\frac{3+\sqrt{3}}{3-\sqrt{3}}\right]^{\cancel{2}} = 3\log\left(\frac{3+\sqrt{3}}{3-\sqrt{3}}\right)$$

Finalmente,

$$\text{I}-\text{II} = 12\, \text{Arctg}\left(\frac{1}{\sqrt{3}}\right) - 3\log\left(\frac{3+\sqrt{3}}{3-\sqrt{3}}\right).$$

Ej 9: $$P_2(x) = \underbrace{f(1)}_{=0} + \underbrace{f'(1)}_{=\frac{2}{e}}(x-1) + \frac{\overbrace{f''(1)}^{=-\frac{6}{e}}}{2}(x-1)^2$$

- $f(1)=0$
- $f'(x) = e^{-x^4}\cdot 2x \Rightarrow f'(1) = e^{-1}\cdot 2 = \frac{2}{e}$
- $f''(x) = e^{-x^4}\underbrace{(-x^4)'}_{-4x^3}\cdot 2x + e^{-x^4}\underbrace{(2x)'}_{2} = e^{-x^4}(-8x^4+2) \Rightarrow f''(1) = e^{-1}(-8+2$

$$= -\frac{6}{e}$$

$$\Rightarrow P_2(x) = \frac{2}{e}(x-1) - \frac{3}{e}\overbrace{(x-1)^2}^{x^2-2x+1} = \frac{1}{e}\left[2x-2-3x^2+6x-3\right] = \frac{1}{e}\left[-3x^2+8x-5\right]$$

Escaneado con CamScanner