Clase 41:

Integrales de Rieman

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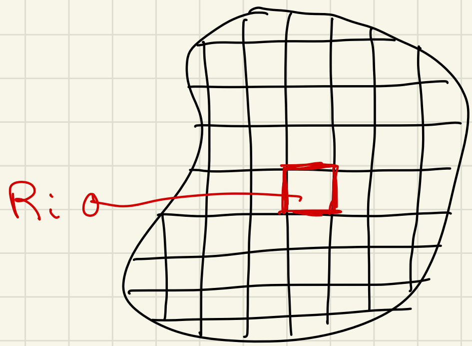
Integrales de Riemann

$D \subseteq \mathbb{R}^2$ $\qquad f : D \longrightarrow \mathbb{R}$ $\qquad \iint_D f$

Partición de D



$$\mathcal{P} = \bigcup_{i \in I} R_i$$

$$s(f, \mathcal{P}) = \sum_{i \in I} m(R_i) \inf_{x \in R_i} (f(x))$$

$$S(f, \mathcal{P}) = \sum_{i \in I} m(R_i) \sup_{x \in R_i} (f(x))$$

$$s(f) = \sup_{\mathcal{P} \text{ partición de } D} (s(f, \mathcal{P}))$$

$$S(f) = \inf_{\substack{\mathcal{P} \text{ partición} \\ \text{de } D}} \left(S(f,\mathcal{P}) \right)$$

f es integrable en D

$$\iff \quad s(f) = S(f) = \iint_D f$$

$$SR(f,\mathcal{P}) = \sum_{i \in I} m(R_i) \cdot f(x_i) \qquad x_i \in R_i$$

$$\inf_{x \in R_i} (f(x)) \leq f(x_i) \leq \sup_{x \in R_i} (f(x))$$

$\Rightarrow$

$$s(f,\mathcal{P}) \leq SR(f,\mathcal{P}) \leq S(f,\mathcal{P})$$

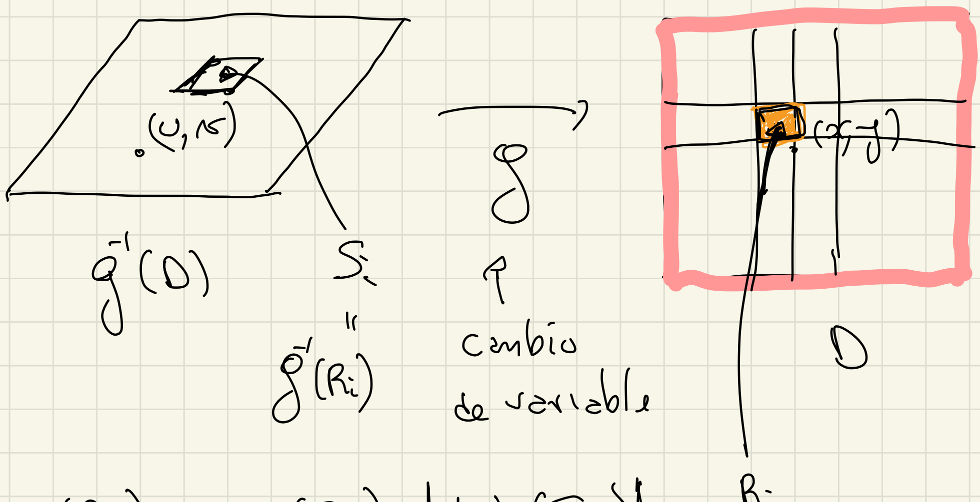
Teorema de cambio de variable

Sea $f : D \to \mathbb{R}$ continua y
$D \subseteq \mathbb{R}^2$

$g : U \to V$ un cambio de variable lineal

con $D \subseteq V$

$$\boxed{\iint_D f(x,y)\,dx\,dy = \iint_{g^{-1}(D)} f(g(u,v))\,\left|\det J_g(u,v)\right|\,du\,dv}$$



$$m(R_i) = m(S_i) \cdot \left|\det(J_g)\right|$$

$$\iint_D f(x,y)\,dx\,dy =$$

$$SR(f,\mathcal{P}) = \sum_{i\in I} m(R_i)\cdot f(x_i,y_i)$$

$$\Big\downarrow \mathcal{P}$$

$$\iint_D f$$

$$= \sum_{i\in I} m(S_i)\,|\det(J_g)|\, f(x_i,y_i)$$

$$= \sum_{i\in I} m(S_i)\,|\det(J_g)|\cdot f(g(u_i,v_i))$$

$$\Big\downarrow \mathcal{P}$$

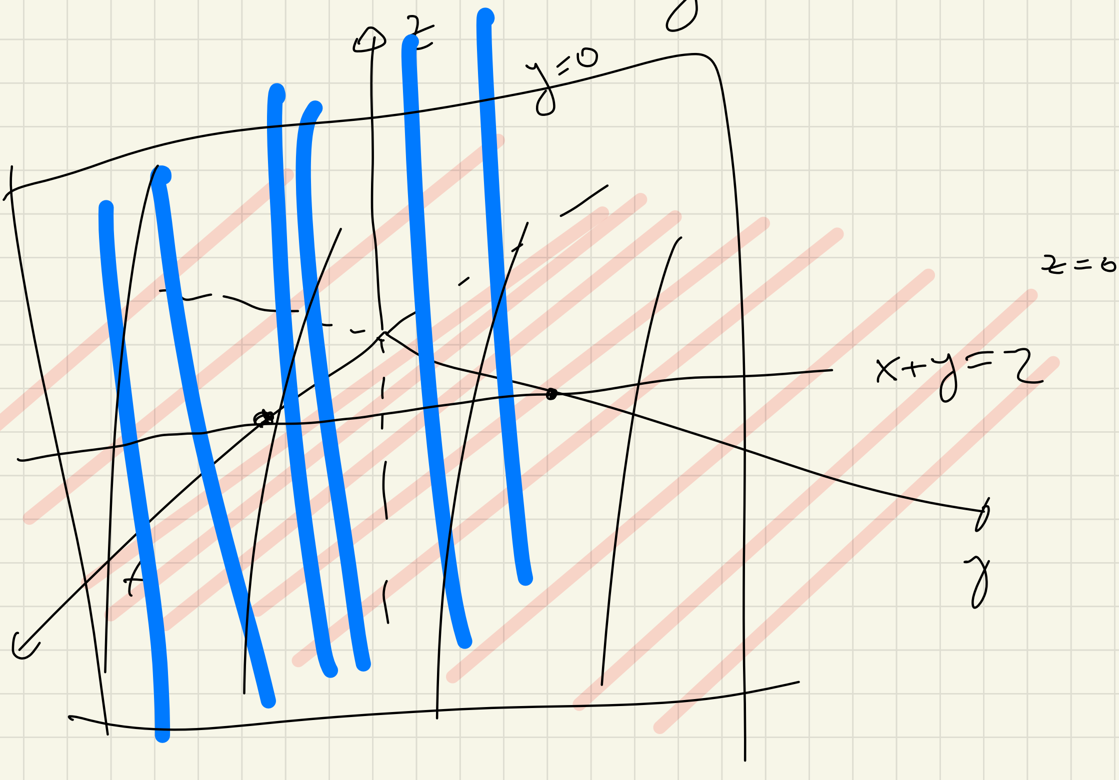
$$\iint_{g^{-1}(D)} f(g(u,v))\cdot|\det J_g(u,v)|$$

$f(x,y,z) = x$

D región limitada por

$\cap$
$\mathbb{R}^3$ $z=0, \ y=0, \ x+y=2,$

$x+y+z=6.$



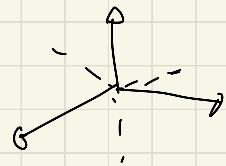
D región limitada por

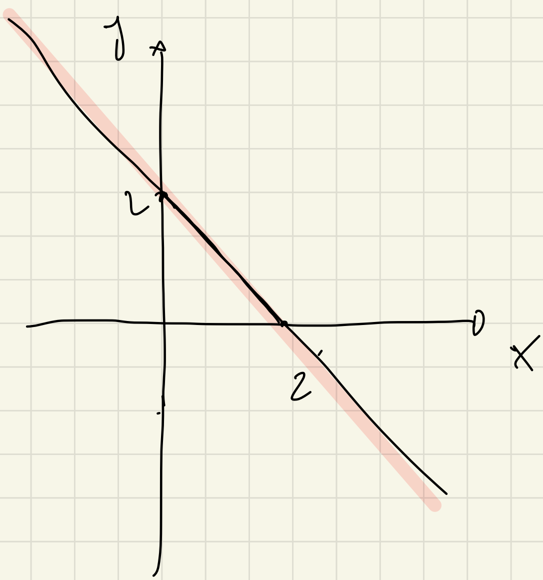
en $\mathbb{R}^3$ $x=y$ $z=0$, $y=0$, $x+y=2$, $x+y+z=6$.

otra pared vertical

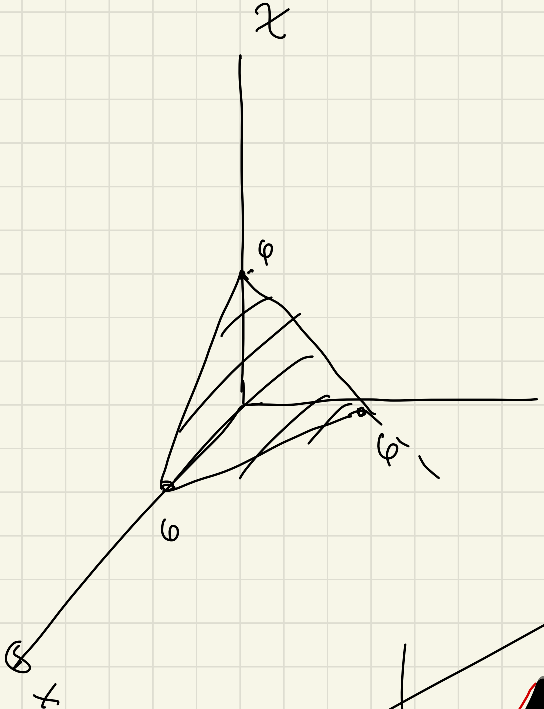
plano formado $\overrightarrow{O_{xy}}$ "piso"

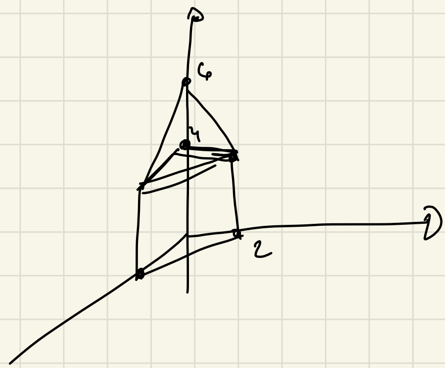
plano formado por $\overrightarrow{O_{zx}}$ "pared vertical"

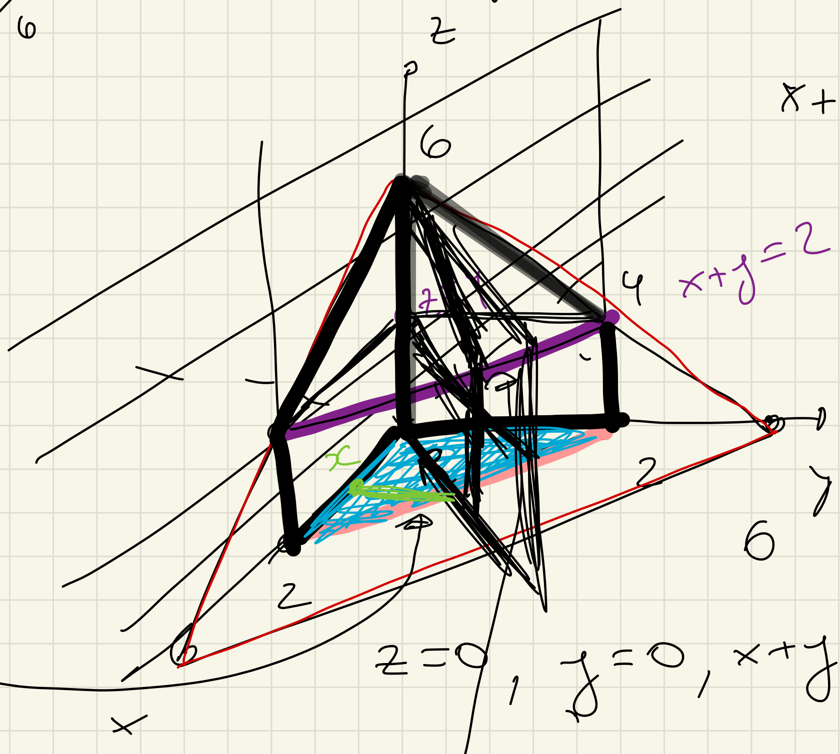




la pared $x+y=2$ vista de arriba







$x+y+z=6$

$z=6-x-y$

$z=0, \; y=0, \; x+y=2,$

$\underline{x+y+z=6}.$

$\{x+y+z=6\} \cap \{x+y=2\}$

$(z=0) \cap \{x+y=2\}$

$$\iiint_D x \, dx\,dy\,dz \overset{\text{TF}}{=} \iint_R \int_0^{6-x-y} x \, dz \; dx\,dy$$

$$= \iint_R (6-x-y)x \, dx\,dy$$

TF

$$= \int_0^2 \left(\int_0^{2-x} (6-x-y)\,x \, dy \right) dx$$