

# Sistemas y Control

---

CLASE DE PRÁCTICO. HOJA 4, CLASE 1

# Hoja 4. Ejercicio 2

2) Se considera un satélite en órbita alrededor de la tierra. La posición instantánea está descrita en coordenadas polares respecto al centro de la tierra, el eje de la tierra orientado de Sur a Norte y un punto fijo del plano ecuatorial.

$x(t)$ , integrado por sus coordenadas y velocidades polares representa el estado del sistema en el instante  $t$ :  $x = (r \dot{r} \theta \dot{\theta} \varphi \dot{\varphi})^t$

Considere el vector  $u(t)$  de entradas, que representan las fuerzas (en coordenadas polares) que los motores de corrección de trayectoria ejercen en las direcciones polares:  $V = (u_r u_\theta u_\varphi)^t$

La salida del sistema es la posición instantánea:  $y = (r \theta \varphi)^t$

El sistema viene descrito por:  $\begin{cases} \dot{x} = f(x,u) \\ y = Cx \end{cases}$ , donde

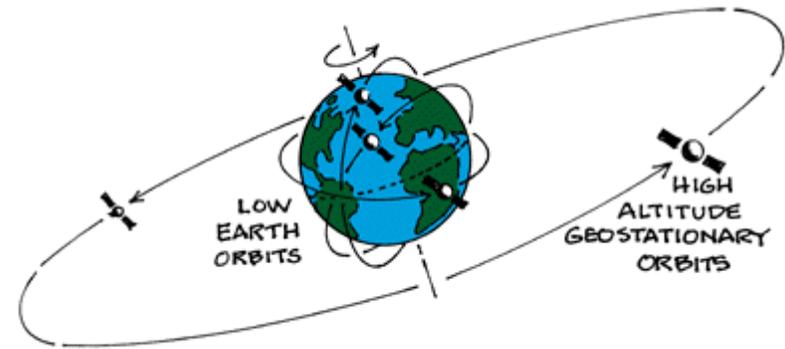
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$f(x,u) = \begin{bmatrix} \dot{r} \\ r.\dot{\theta}^2 + r.\text{sen}^2\theta.\dot{\varphi}^2 - \frac{k}{r^2} + \frac{u_R}{m} \\ \dot{\theta} \\ -\frac{2.\dot{r}.\dot{\theta}}{r} + \text{sen}\theta.\text{cos}\theta.\dot{\varphi}^2 + \frac{u_\theta}{m.r} \\ \dot{\varphi} \\ -\frac{2.\dot{r}.\dot{\varphi}}{r} - \frac{2.\text{cos}\theta}{\text{sen}\theta}.\dot{\theta}.\dot{\varphi} + \frac{u_\varphi}{m.r.\text{sen}\theta} \end{bmatrix}$$

El satélite se encuentra en órbita ecuatorial geostacionaria es decir:

$$x_0(t) = [ r_0 \ 0 \ \pi/2 \ 0 \ \omega_0 t \ \omega_0 ] , \quad u_0(t) = 0, \quad r_0^3.\omega_0^2 = k \text{ (cte.)}$$

Se consideran pequeñas perturbaciones alrededor de dicha órbita. Obtenga un modelo lineal para representar esas perturbaciones, del tipo:  $\begin{cases} \dot{x} = A.x + B.u \\ y = C.x \end{cases}$



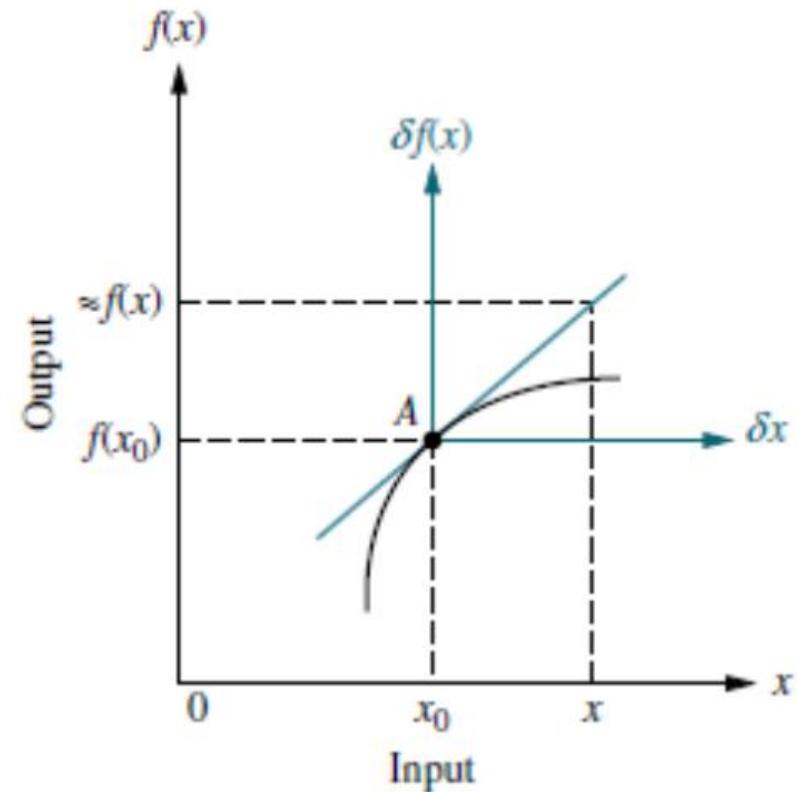
# Linealización

---

- Los sistemas reales tienen la pésima costumbre de no comportarse como nos gustaría
- La amplia mayoría de los sistemas que podemos observar tienen comportamientos no lineales.
- Por qué invertimos tanto tiempo estudiando sistemas lineales?

# Linealización

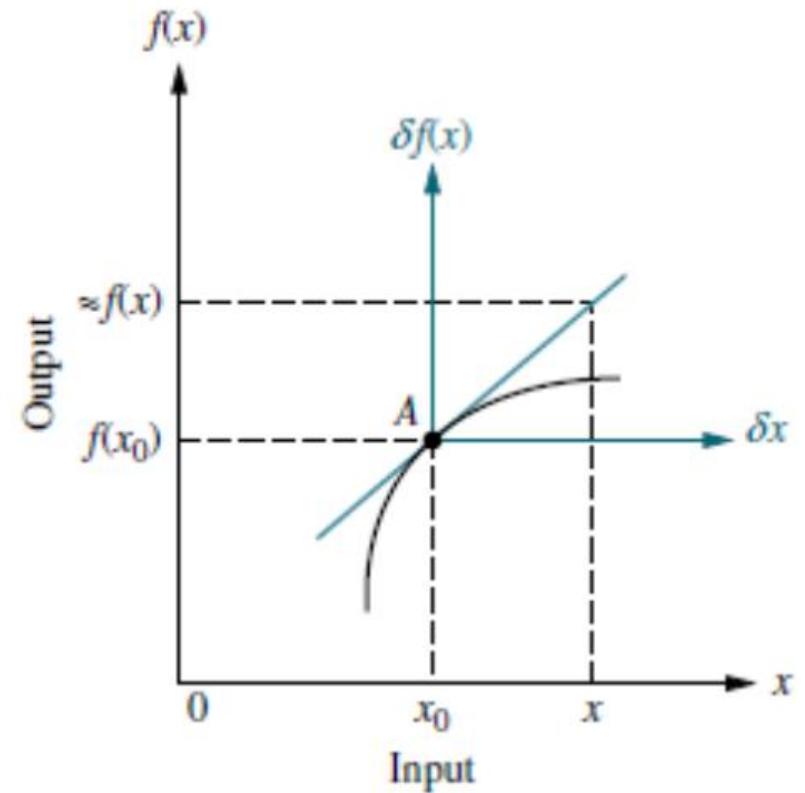
- Los sistemas reales tienen la pésima costumbre de no comportarse como nos gustaría
- La amplia mayoría de los sistemas que podemos observar tienen comportamientos no lineales.
- Por qué invertimos tanto tiempo estudiando sistemas lineales?
  - Podemos definir una representación lineal de nuestros sistemas válida para un “entorno pequeño” de un punto
  - Conforme nos alejamos del punto, es esperable que el error (diferencia entre el modelo no lineal y el lineal) aumente
    - Realidad  $\neq$  Modelo no lineal  $\neq$  Modelo lineal



# Linealización

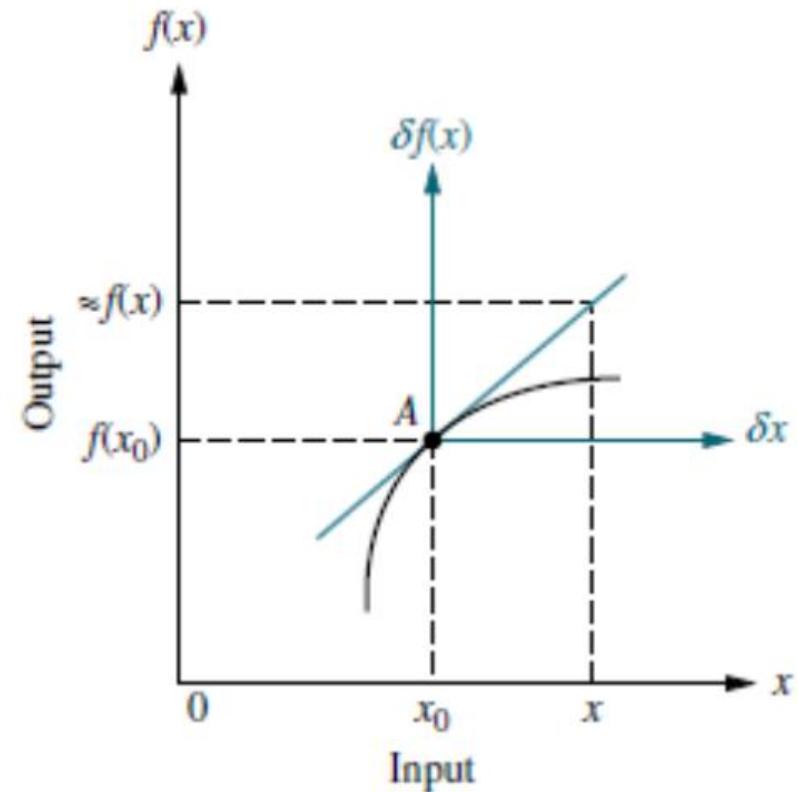
---

- El concepto detrás de la linealización nos es familiar desde hace mucho tiempo (desarrollo de Taylor de 1er orden)



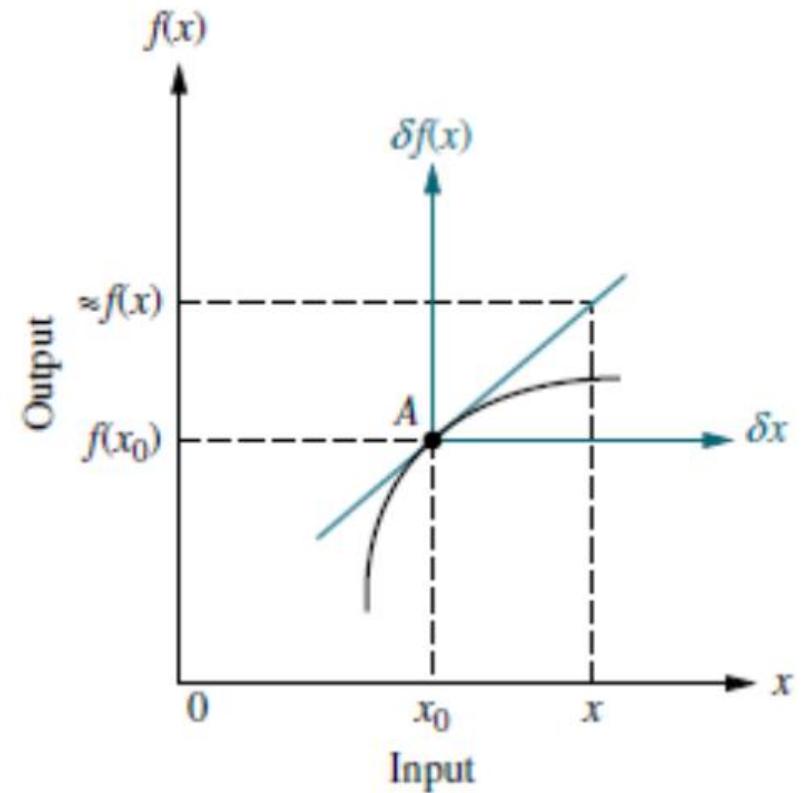
# Linealización

- El concepto detrás de la linealización nos es familiar desde hace mucho tiempo (desarrollo de Taylor de 1er orden)
- Sea  $m_a$  la pendiente de  $f(x)$  en  $x_0$
- $m_a = \left. \frac{\delta f(x)}{\delta x} \right|_{x_0}$
- $f(x) \approx f(x_0) + \left. \frac{\delta f(x)}{\delta x} \right|_{x_0} (x - x_0)$



# Linealización

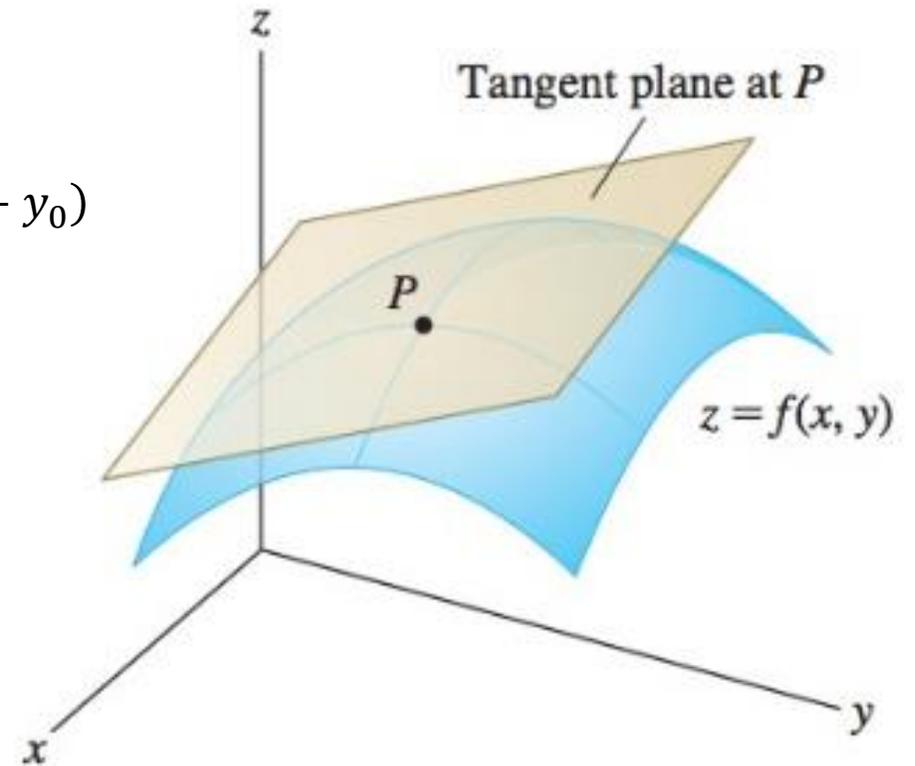
- El concepto es sencillamente extrapolable a dimensiones superiores



# Linealización

- El concepto es sencillamente extrapolable a dimensiones superiores.

$$f(x, y) = f(x_0, y_0) + \left. \frac{\delta f(x, y)}{\delta x} \right|_{(x_0, y_0)} (x - x_0) + \left. \frac{\delta f(x, y)}{\delta y} \right|_{(x_0, y_0)} (y - y_0)$$

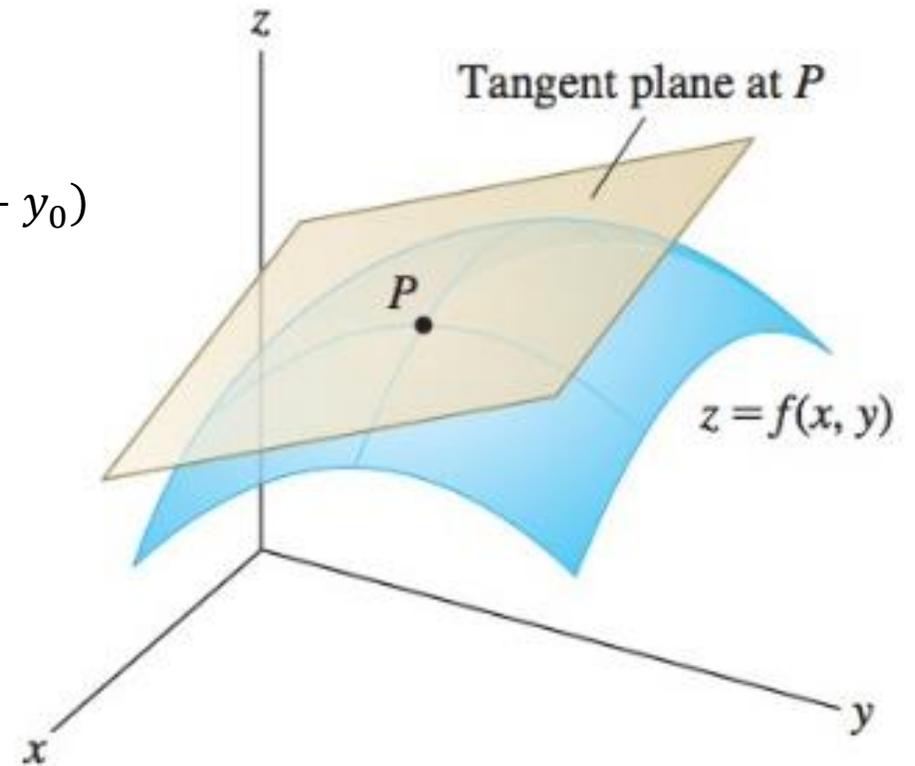


# Linealización

- El concepto es sencillamente extrapolable a dimensiones superiores.

$$f(x, y) = f(x_0, y_0) + \left. \frac{\delta f(x, y)}{\delta x} \right|_{(x_0, y_0)} (x - x_0) + \left. \frac{\delta f(x, y)}{\delta y} \right|_{(x_0, y_0)} (y - y_0)$$

En la imagen,  $P$

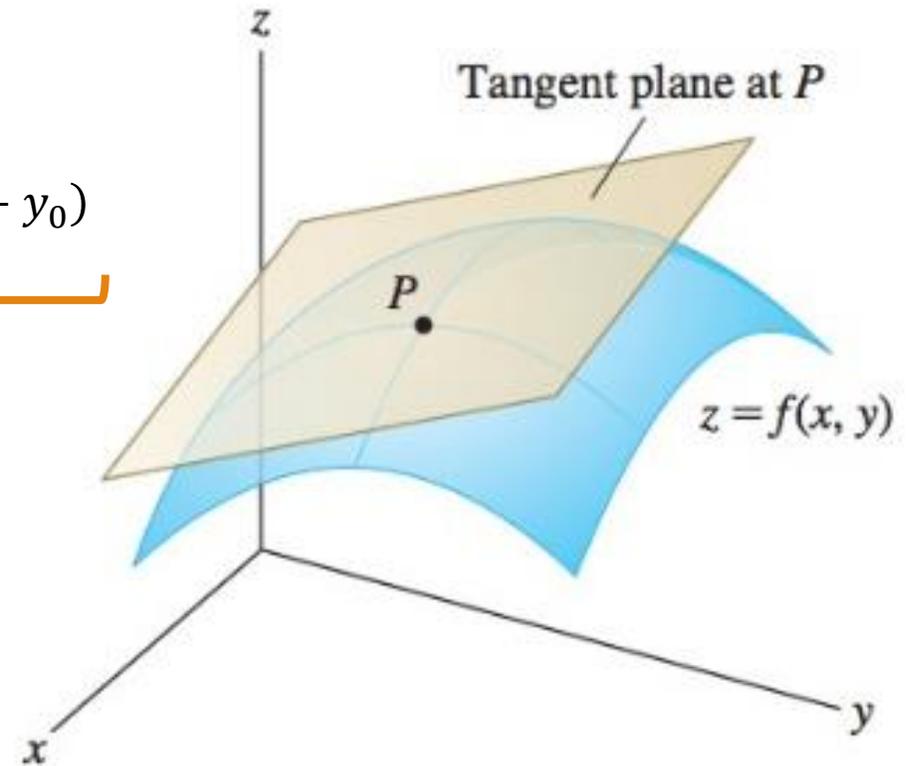


# Linealización

- El concepto es sencillamente extrapolable a dimensiones superiores.

$$f(x, y) = f(x_0, y_0) + \underbrace{\frac{\delta f(x, y)}{\delta x} \Big|_{(x_0, y_0)} (x - x_0) + \frac{\delta f(x, y)}{\delta y} \Big|_{(x_0, y_0)} (y - y_0)}_{\text{Plano tangente a } f(x, y) \text{ en } P}$$

En la imagen,  $P$



# Linealización

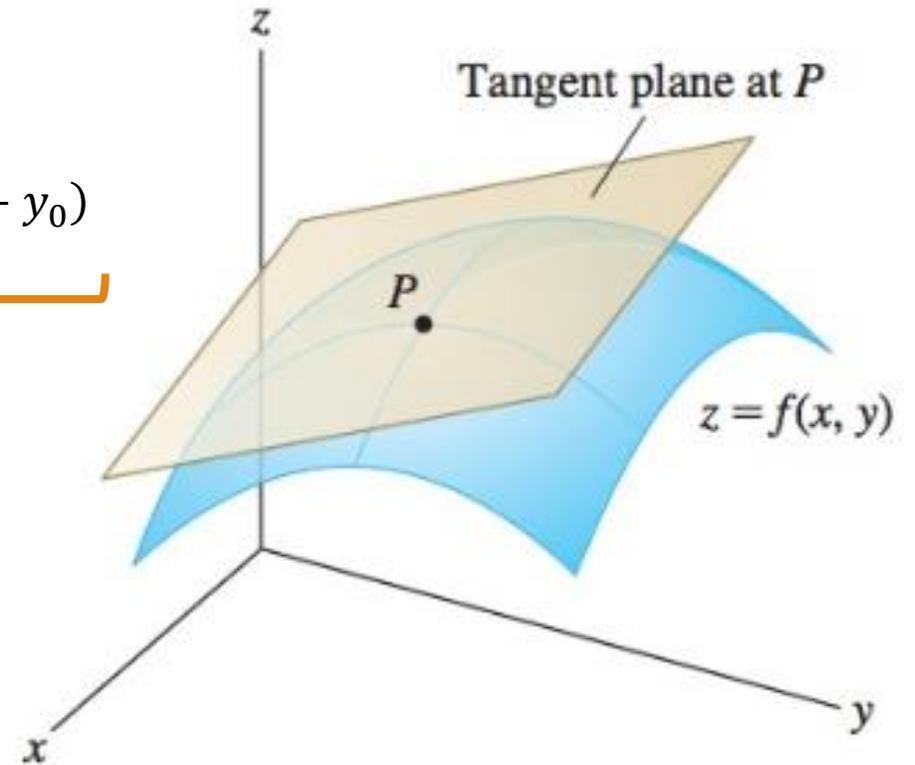
- El concepto es sencillamente extrapolable a dimensiones superiores.

$$f(x, y) = f(x_0, y_0) + \underbrace{\frac{\delta f(x, y)}{\delta x} \Big|_{(x_0, y_0)} (x - x_0) + \frac{\delta f(x, y)}{\delta y} \Big|_{(x_0, y_0)} (y - y_0)}_{\text{Plano tangente a } f(x, y) \text{ en } P}$$



En la imagen,  $P$

- $f(x_0, y_0)$  es constante, y es un componente FUNDAMENTAL de la linealización
  - Debe ser determinado EXPLÍCITAMENTE



# Linealización

- El concepto es sencillamente extrapolable a dimensiones superiores.

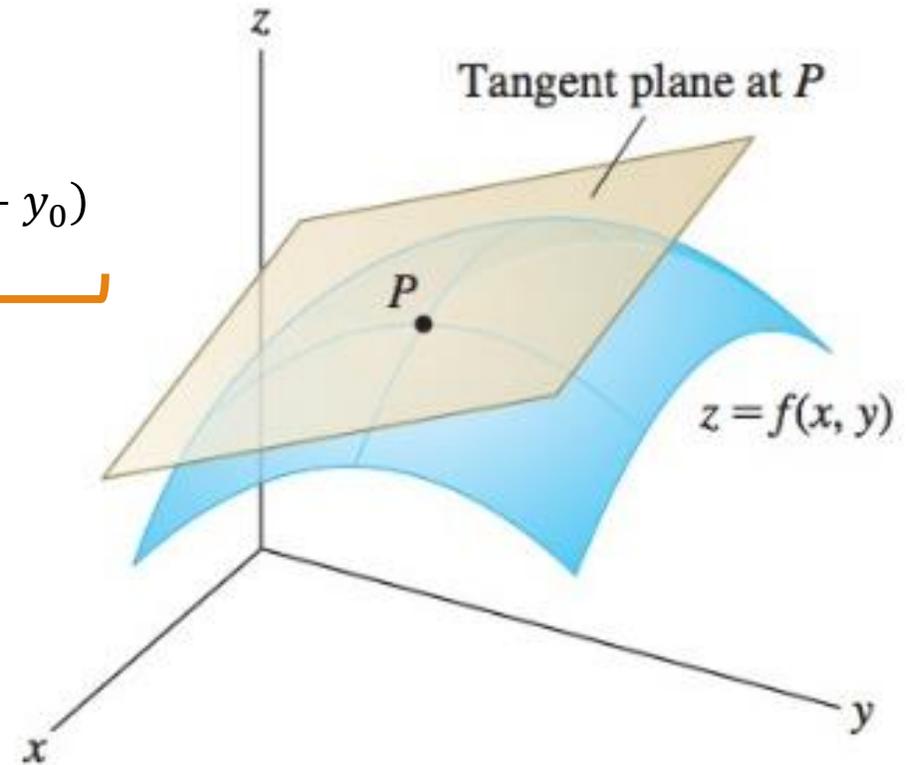
$$f(x, y) = f(x_0, y_0) + \underbrace{\frac{\delta f(x, y)}{\delta x} \Big|_{(x_0, y_0)} (x - x_0) + \frac{\delta f(x, y)}{\delta y} \Big|_{(x_0, y_0)} (y - y_0)}_{\text{Plano tangente a } f(x, y) \text{ en } P}$$



En la imagen,  $P$

- $f(x_0, y_0)$  es constante, y es un componente FUNDAMENTAL de la linealización
  - Debe ser determinado **EXPLÍCITAMENTE**

PARCIAL / EXAMEN



# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ \quad x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases} \quad \begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = f(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} + \frac{\delta f}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot [x(t) - x^*(t)] + \frac{\delta f}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot [u(t) - u^*(t)] \\ \dot{y}(t) = g(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} + \frac{\delta g}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot [x(t) - x^*(t)] + \frac{\delta g}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot [u(t) - u^*(t)] \end{cases}$$

# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases} \quad \begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = f(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} + \frac{\delta f}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \frac{\delta f}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \\ \dot{y}(t) = g(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} + \frac{\delta g}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \frac{\delta g}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \end{cases}$$

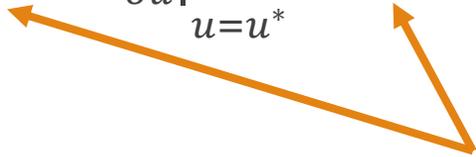
# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases} \quad \begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = f(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} + \frac{\delta f}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \frac{\delta f}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \\ \dot{y}(t) = g(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} + \frac{\delta g}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \frac{\delta g}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \end{cases}$$



Cambio de variable: pequeños apartamientos de la pos. de eq.

# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases} \quad \begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

$$\begin{cases} \dot{x}(t) - f(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} = \frac{\delta f}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \frac{\delta f}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \\ \dot{y}(t) - g(x(t), y(t), t) \Big|_{\substack{x=x^* \\ u=u^*}} = \frac{\delta g}{\delta x} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \frac{\delta g}{\delta u} \Big|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \end{cases}$$

# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

$$\begin{cases} \dot{\tilde{x}}(t) = \left. \frac{\delta f}{\delta x} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \left. \frac{\delta f}{\delta u} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \\ \tilde{y}(t) = \left. \frac{\delta g}{\delta x} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \left. \frac{\delta g}{\delta u} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \end{cases}$$

# Linealización

---

Llevando el concepto al espacio que contiene todas las entradas y todos los estados

$$\begin{cases} u(t) = u^*(t) + \tilde{u}(t) \\ x(t) = x^*(t) + \tilde{x}(t) \\ x(t_0) = x_0^* + \tilde{x}_0 \\ y(t) = y^*(t) + \tilde{y}(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = f(x(t), y(t), t) \\ y(t) = g(x(t), y(t), t) \end{cases}$$

$$\begin{cases} \dot{\tilde{x}}(t) = \left. \frac{\delta f}{\delta x} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \left. \frac{\delta f}{\delta u} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \\ \tilde{y}(t) = \left. \frac{\delta g}{\delta x} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{x}(t) + \left. \frac{\delta g}{\delta u} \right|_{\substack{x=x^* \\ u=u^*}} \cdot \tilde{u}(t) \end{cases}$$



La expresión a la que arribamos es lineal  
(derivada de edos. y salida son CL de  
edos. y entradas)

# Hoja 4. Ejercicio 2

---

$$\frac{d}{dt} \begin{bmatrix} r \\ \dot{r} \\ \theta \\ \dot{\theta} \\ \varphi \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ r\dot{\theta}^2 + r \cdot \text{sen}^2\theta \dot{\varphi}^2 - \frac{k}{r^2} + \frac{u_R}{m} \\ \dot{\theta} \\ \frac{-2\dot{r}\dot{\theta}}{r} + \text{sen}\theta \cos\theta \dot{\varphi}^2 + \frac{u_\theta}{mr} \\ \dot{\varphi} \\ \frac{-2\dot{r}\dot{\theta}}{r} + \frac{2\cos\theta}{\text{sen}\theta} \dot{\varphi}^2 + \frac{u_\varphi}{mr \text{sen}\theta} \end{bmatrix}$$

$$x_0(t) = \left[ r_0 \quad 0 \quad \frac{\pi}{2} \quad 0 \quad \omega_0 t \quad \omega_0 \right]$$

$$u_0(t) = 0$$

$$r_0^3 \omega_0^2 = k$$

# Hoja 4. Ejercicio 2

---

$$\tilde{r} = \sum_{x_i} \left. \frac{\delta \dot{r}(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \dot{r}(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\tilde{r} = \left. \frac{\delta \dot{r}(x, u)}{\delta r} \right|_{(x_0, u_0)} (\tilde{r}) + \left. \frac{\delta \dot{r}(x, u)}{\delta \dot{r}} \right|_{(x_0, u_0)} (\dot{\tilde{r}}) + \left. \frac{\delta \dot{r}(x, u)}{\delta \theta} \right|_{(x_0, u_0)} (\tilde{\theta}) + \dots + \left. \frac{\delta \dot{r}(x, u)}{\delta \dot{\theta}} \right|_{(x_0, u_0)} (\dot{\tilde{\theta}}) + \dots$$

# Hoja 4. Ejercicio 2

---

$$\tilde{r} = \sum_{x_i} \left. \frac{\delta \dot{r}(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \dot{r}(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\tilde{r} = \left. \frac{\delta \dot{r}(x, u)}{\delta r} \right|_{(x_0, u_0)} (\tilde{r}) + \left. \frac{\delta \dot{r}(x, u)}{\delta \dot{r}} \right|_{(x_0, u_0)} (\dot{\tilde{r}}) + \left. \frac{\delta \dot{r}(x, u)}{\delta \theta} \right|_{(x_0, u_0)} (\tilde{\theta}) + \dots + \left. \frac{\delta \dot{r}(x, u)}{\delta \dot{\theta}} \right|_{(x_0, u_0)} (\dot{\tilde{\theta}}) + \dots$$

$$\tilde{r} = 1 \cdot \tilde{r}$$

# Hoja 4. Ejercicio 2

---

$$\tilde{r} = \sum_{x_i} \left. \frac{\delta \ddot{r}(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \ddot{r}(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\ddot{r} = r\dot{\theta}^2 + r \cdot \text{sen}^2 \theta \dot{\phi}^2 - \frac{k}{r^2} + \frac{u_R}{m}$$

$$\frac{\delta \ddot{r}(x, u)}{\delta r} = \dot{\theta}^2 + \text{sen}^2 \theta \dot{\phi}^2 + \frac{2K}{r^3}$$

$$\frac{\delta \ddot{r}(x, u)}{\delta \dot{r}} = 0$$

$$\frac{\delta \ddot{r}(x, u)}{\delta \theta} = 2r \text{sen} \theta \cos \theta \dot{\phi}^2$$

$$\frac{\delta \ddot{r}(x, u)}{\delta \dot{\theta}} = 2r\dot{\theta}$$

$$\frac{\delta \ddot{r}(x, u)}{\delta \phi} = 0$$

$$\frac{\delta \ddot{r}(x, u)}{\delta \dot{\phi}} = 2r \text{sen}^2 \theta$$

$$\frac{\delta \ddot{r}(x, u)}{\delta u_r} = \frac{1}{m}$$

$$\frac{\delta \ddot{r}(x, u)}{\delta u_\theta} = 0$$

$$\frac{\delta \ddot{r}(x, u)}{\delta u_\phi} = 0$$

# Hoja 4. Ejercicio 2

---

$$\tilde{\theta} = \sum_{x_i} \left. \frac{\delta \dot{\theta}(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \dot{\theta}(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\dot{\theta} = \dot{\theta}$$

$$\frac{\delta \dot{\theta}(x, u)}{\delta r} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta \dot{r}} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta \theta} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta \dot{\theta}} = 1$$

$$\frac{\delta \dot{\theta}(x, u)}{\delta \varphi} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta \dot{\varphi}} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta u_r} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta u_\theta} = 0$$
$$\frac{\delta \dot{\theta}(x, u)}{\delta u_\varphi} = 0$$

# Hoja 4. Ejercicio 2

---

$$\tilde{\theta} = \sum_{x_i} \left. \frac{\delta \ddot{\theta}(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \ddot{\theta}(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\ddot{\theta} = \frac{-2\dot{r}\dot{\theta}}{r} + \text{sen}\theta\text{cos}\theta\dot{\phi}^2 + \frac{u_\theta}{mr}$$

$$\begin{aligned} \frac{\delta \ddot{\theta}(x, u)}{\delta r} &= \frac{2\dot{r}\dot{\theta}}{r^2} - \frac{u_\theta}{mr^2} \\ \frac{\delta \ddot{\theta}(x, u)}{\delta \dot{\theta}} &= \frac{-2\dot{\theta}}{r} \\ \frac{\delta \ddot{\theta}(x, u)}{\delta \dot{r}} &= \frac{2\dot{\theta}}{r} \\ \frac{\delta \ddot{\theta}(x, u)}{\delta \theta} &= (\cos^2 \theta - \text{sen}^2 \theta) \dot{\phi}^2 \\ \frac{\delta \ddot{\theta}(x, u)}{\delta \dot{\theta}} &= \frac{-2\dot{r}}{r} \end{aligned}$$

$$\begin{aligned} \frac{\delta \ddot{\theta}(x, u)}{\delta \phi} &= 0 \\ \frac{\delta \ddot{\theta}(x, u)}{\delta \dot{\phi}} &= 2\text{sen}\theta\text{cos}\theta\dot{\phi} \\ \frac{\delta \ddot{\theta}(x, u)}{\delta \dot{\phi}} &= 0 \\ \frac{\delta \ddot{\theta}(x, u)}{\delta u_r} &= 0 \\ \frac{\delta \ddot{\theta}(x, u)}{\delta u_\theta} &= \frac{1}{mr} \\ \frac{\delta \ddot{\theta}(x, u)}{\delta u_\phi} &= 0 \end{aligned}$$

# Hoja 4. Ejercicio 2

---

$$\tilde{\phi} = \sum_{x_i} \left. \frac{\delta \phi(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \delta \left. \frac{\delta \phi(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\dot{\phi} = \phi$$

$$\frac{\delta \phi(x, u)}{\delta r} = 0$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta r} = 0$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta \dot{r}} = 0$$
$$\frac{\delta \phi(x, u)}{\delta \theta} = 0$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta \dot{\theta}} = 0$$

$$\frac{\delta \phi(x, u)}{\delta \varphi} = 0$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta \varphi} = 1$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta u_r} = 0$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta u_\theta} = 0$$
$$\frac{\delta \dot{\phi}(x, u)}{\delta u_\varphi} = 0$$

# Hoja 4. Ejercicio 2

---

$$\tilde{\varphi} = \sum_{x_i} \left. \frac{\delta \varphi(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \varphi(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\ddot{\varphi} = \frac{-2\dot{r}\dot{\theta}}{r} + \frac{2\cos\theta}{\sin\theta} \dot{\varphi}^2 + \frac{u_\varphi}{mr\sin\theta}$$

$$\frac{\delta \varphi(x, u)}{\delta r} = \frac{2\dot{r}\dot{\theta}}{r} - \frac{u_\varphi}{mr^2\sin\theta}$$

$$\frac{\delta \varphi(x, u)}{\delta \dot{r}} = \frac{-2\dot{r}}{r}$$

$$\frac{\delta \varphi(x, u)}{\delta \dot{\theta}} = \frac{2\dot{\varphi}^2(-\sin^2\theta - \cos^2\theta)}{\sin^2\theta} - \frac{u_\varphi \cos\theta}{mr\sin^2\theta}$$

$$\frac{\delta \varphi(x, u)}{\delta \dot{\theta}} = \frac{-2\dot{r}}{r}$$

$$\frac{\delta \varphi(x, u)}{\delta \varphi} = 0$$

$$\frac{\delta \varphi(x, u)}{\delta \dot{\varphi}} = \frac{4\cos\theta}{\sin\theta} \dot{\varphi}$$

$$\frac{\delta \varphi(x, u)}{\delta u_r} = 0$$

$$\frac{\delta \varphi(x, u)}{\delta u_\theta} = 0$$

$$\frac{\delta \varphi(x, u)}{\delta u_\varphi} = \frac{1}{mr\sin\theta}$$

# Hoja 4. Ejercicio 2

---

Ya tenemos todas las derivadas!

Ahora debemos evaluarlas en  $(x_0, u_0)$

# Hoja 4. Ejercicio 2

---

$$\tilde{r} = \sum_{x_i} \left. \frac{\delta \ddot{r}(x, u)}{\delta x_i} \right|_{(x_0, u_0)} (\tilde{x}_i) + \sum_{u_i} \left. \frac{\delta \ddot{r}(x, u)}{\delta u_i} \right|_{(x_0, u_0)} (\tilde{u}_i)$$

$$\ddot{r} = r\dot{\theta}^2 + r \cdot \text{sen}^2\theta \dot{\phi}^2 - \frac{k}{r^2} + \frac{u_R}{m}$$

$$\begin{aligned} \left. \frac{\delta \ddot{r}(x, u)}{\delta r} \right|_{(x_0, u_0)} &= \dot{\theta}^2 + \text{sen}^2\theta \dot{\phi}^2 + \frac{2K}{r^3} = 2\omega_0^2 \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta \dot{r}} \right|_{(x_0, u_0)} &= 0 \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta \theta} \right|_{(x_0, u_0)} &= 2r \text{sen}\theta \cos\theta \dot{\phi}^2 = 0 \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta \dot{\theta}} \right|_{(x_0, u_0)} &= 2r\dot{\theta} = 0 \end{aligned}$$

$$\begin{aligned} \left. \frac{\delta \ddot{r}(x, u)}{\delta \phi} \right|_{(x_0, u_0)} &= 0 \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta \dot{\phi}} \right|_{(x_0, u_0)} &= 2r \text{sen}^2\theta = 0 \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta u_r} \right|_{(x_0, u_0)} &= \frac{1}{m} \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta u_\theta} \right|_{(x_0, u_0)} &= 0 \\ \left. \frac{\delta \ddot{r}(x, u)}{\delta u_\phi} \right|_{(x_0, u_0)} &= 0 \end{aligned}$$

# Hoja 4. Ejercicio 2

---

Y así se sigue evaluando...



