Cálculo de integrales a partir de propiedades

1. Sea $f : [2,8] \to \mathbb{R}$ integrable tal que $\int_2^8 f(x)\,dx = 20$ y $\int_8^4 f(x)\,dx = 12$.

   Calcular $\int_2^4 f(x)\,dx$

   Sabemos que:

   - $\int_2^8 f(x)\,dx \underbrace{=}_{Aditividad\ del\ Inervalo} \int_2^4 f(x)\,dx + \int_4^8 f(x)\,dx$

     $\Rightarrow \int_2^4 f(x)\,dx = \int_2^8 f(x)\,dx - \int_4^8 f(x)\,dx$

   - $\int_2^8 f(x).dx \underbrace{=}_{Letra} 20$

   - $\int_4^8 f(x)\,dx = -\int_8^4 f(x)\,dx \underbrace{=}_{Letra} -12$

   Entonces:

   $$\int_2^4 f(x)\,dx = 20 - (-12) = 32$$

2. Suponga que f y g son dos funciones integrables y que

   $$\int_1^2 f(x)\,dx = -4, \qquad \int_1^5 f(x)\,dx = 6, \qquad \int_1^5 g(x)\,dx = 8$$

   Calcular:

a) $\int_2^2 g(x)\,dx$	*b)* $\int_5^1 g(x)\,dx$	*c)* $\int_1^2 3f(x)\,dx$
d) $\int_2^5 f(x)\,dx$	*e)* $\int_1^5 (f(x) - g(x))\,dx$	*f)* $\int_1^5 (4f(x) - g(x))\,dx$

   (a) $\int_2^2 g(x)\,dx \underbrace{=}_{Area\ debajo\ de\ un\ punto} 0$

   (b) $\int_5^1 g(x)\,dx \underbrace{=}_{aditividad\ respecto\ al\ intervalo} -\int_1^5 g(x)\,dx \underbrace{=}_{Letra} -8$

   (c) $\int_2^4 3.f(x)\,dx \underbrace{=}_{Linealidad} 3.\int_2^4 f(x)\,dx \underbrace{=}_{Letra} -12$

(d) $\int_1^5 f(x)\,dx \underbrace{=}_{Aditividad\ del\ Inervalo} = \int_1^2 f(x)\,dx + \int_2^5 f(x)\,dx$

$\Rightarrow \int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx \underbrace{=}_{Letra} 6 - (-4) = 10$

(e) $\int_1^5 (f(x) - g(x))\,dx \underbrace{=}_{Linealidad} \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx \underbrace{=}_{Letra} 6 - 8 = -2$

(f) $\int_1^5 (4.f(x) - g(x))\,dx \underbrace{=}_{Linealidad} 4.\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx \underbrace{=}_{Letra} 4.6 - 8 = 16$