

Señales y Sistemas

Sistemas Lineales e Invariantes en el Tiempo (SLITs)

Instituto de Ingeniería Eléctrica



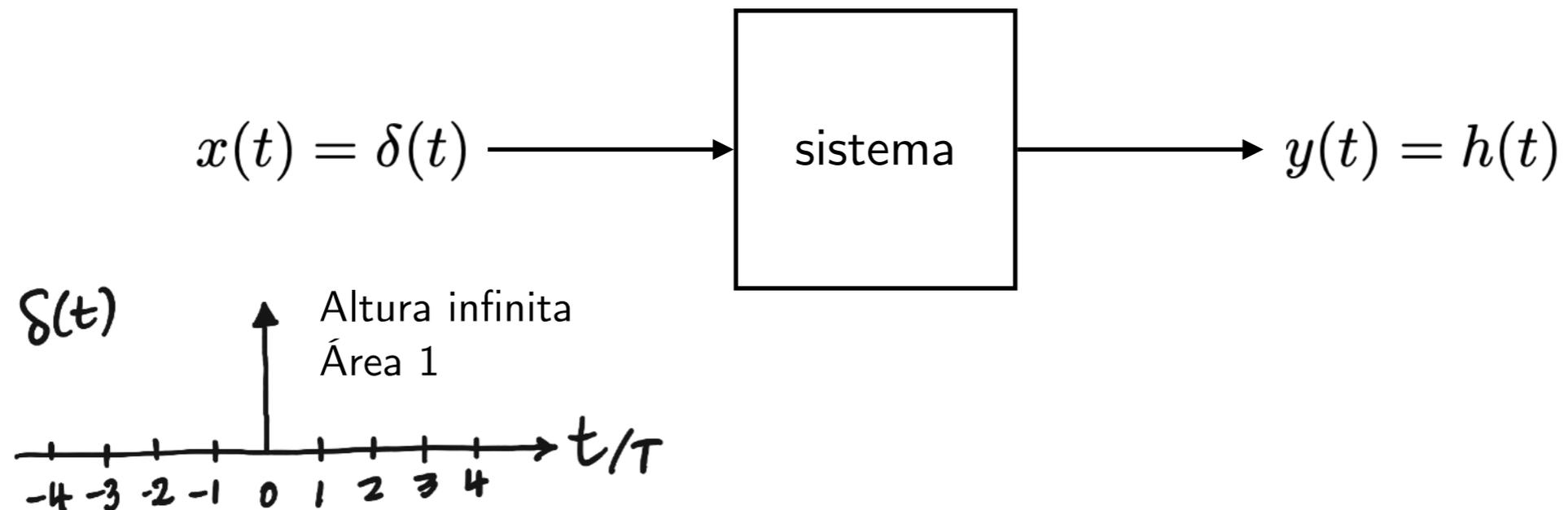
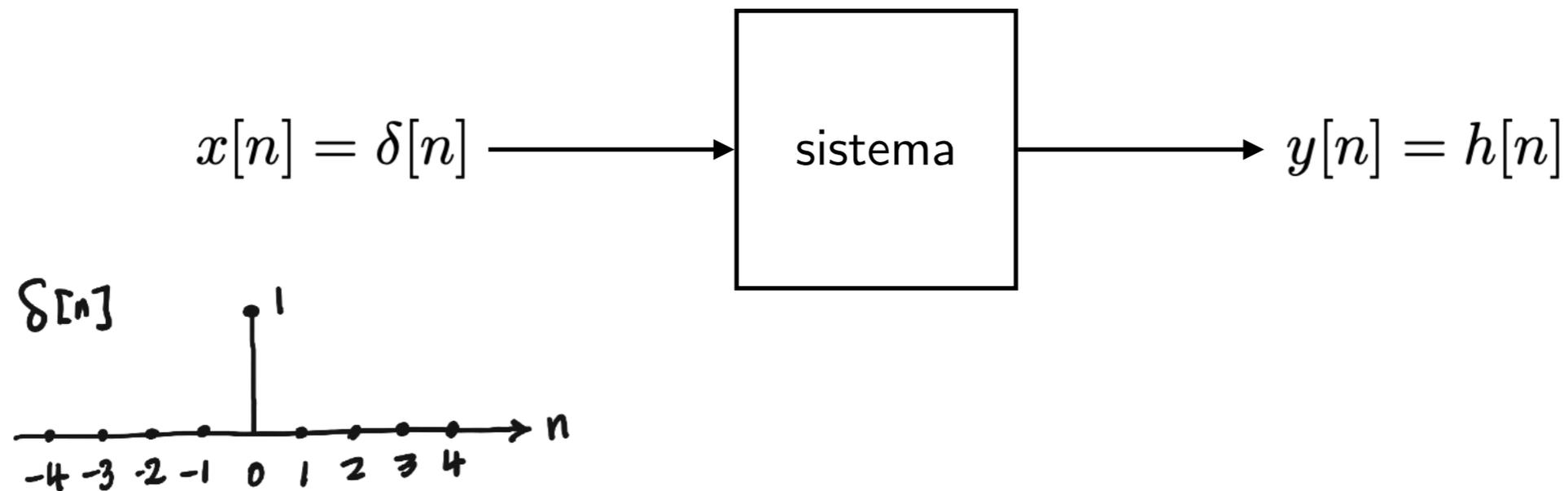
FACULTAD DE
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Respuesta al impulso

- La respuesta al impulso es la salida de un sistema cuando la entrada es un impulso (una delta).



Ejemplos

Teorema de convolución para SLITs

- Sea un sistema lineal e invariante en el tiempo (SLIT) con respuesta al impulso $h[n]$, entonces, para una entrada arbitraria $x[n]$ la salida $y[n]$ es la convolución entre la entrada y la respuesta al impulso.

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{k=+\infty} x[k] h[n - k]$$

- Pasos de la demostración.

1. Mostrar que se puede escribir: $x[n] = \sum_k x[k] \delta[n - k]$

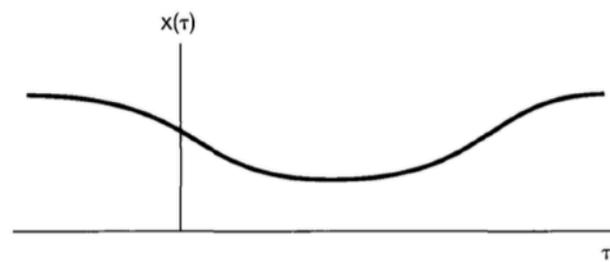
2. Usar invarianza temporal: $\delta[n - k] \longrightarrow \boxed{\text{sistema}} \longrightarrow h[n - k]$

3. Usar linealidad: $y[n] = \sum_k x[k] h[n - k]$

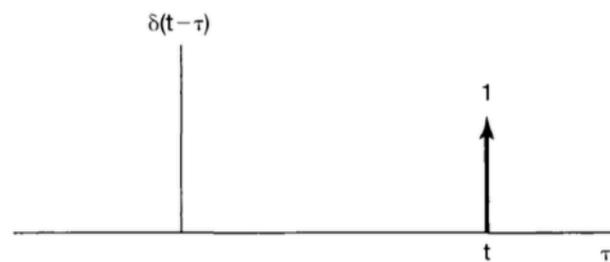
Teorema de convolución para SLITs

- Lo mismo vale para un SLIT de variable continua

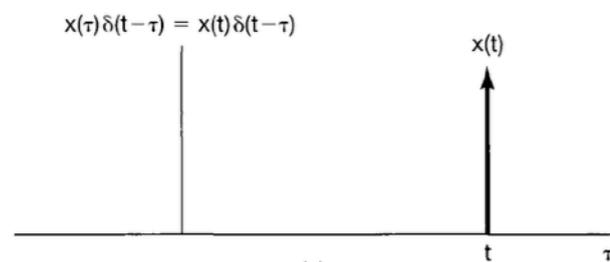
$$y(t) = (x * h)(t) = \int_{\tau=-\infty}^{\tau=+\infty} x(\tau) h(t - \tau) d\tau$$



(a)



(b)



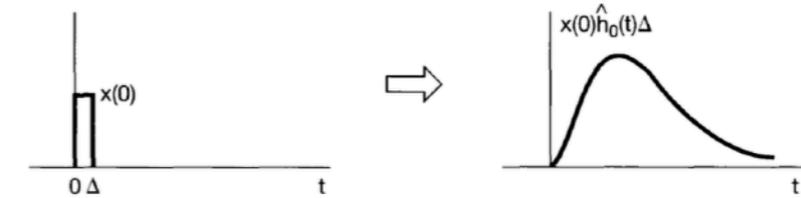
(c)

$$\int_{-\infty}^{+\infty} x(t) \delta(t - \tau) d\tau =$$

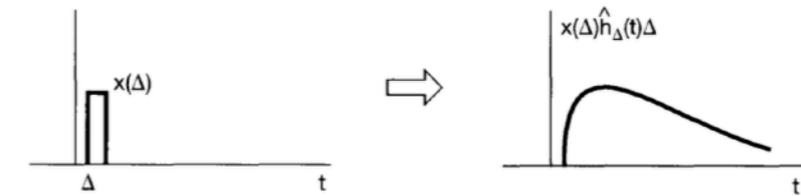
$$x(t) \int_{-\infty}^{+\infty} \delta(t - \tau) d\tau = x(t)$$

Funciones de \mathcal{T}

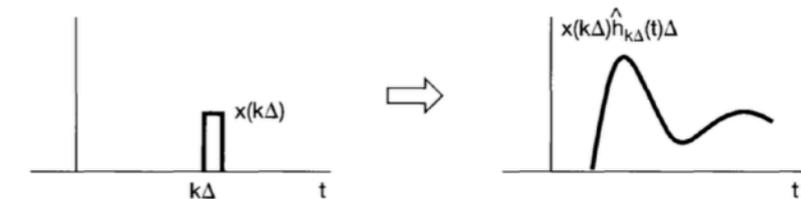
$$x(\tau) \delta(t - \tau) = x(t) \delta(t - \tau)$$



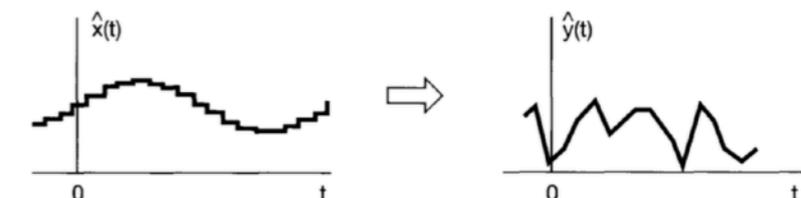
(b)



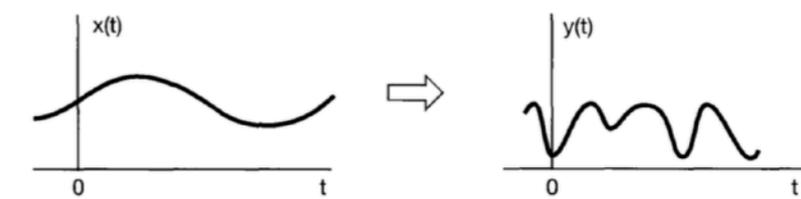
(c)



(d)

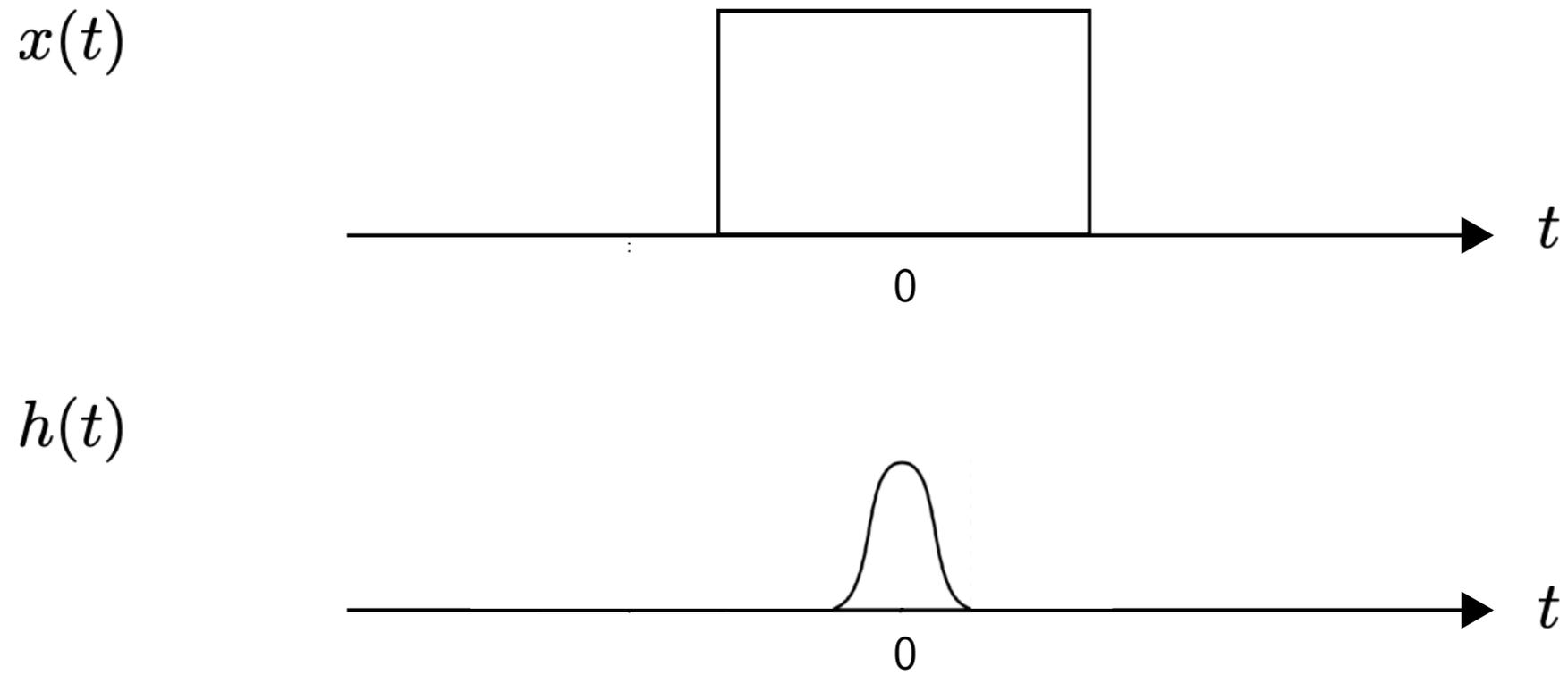


(e)



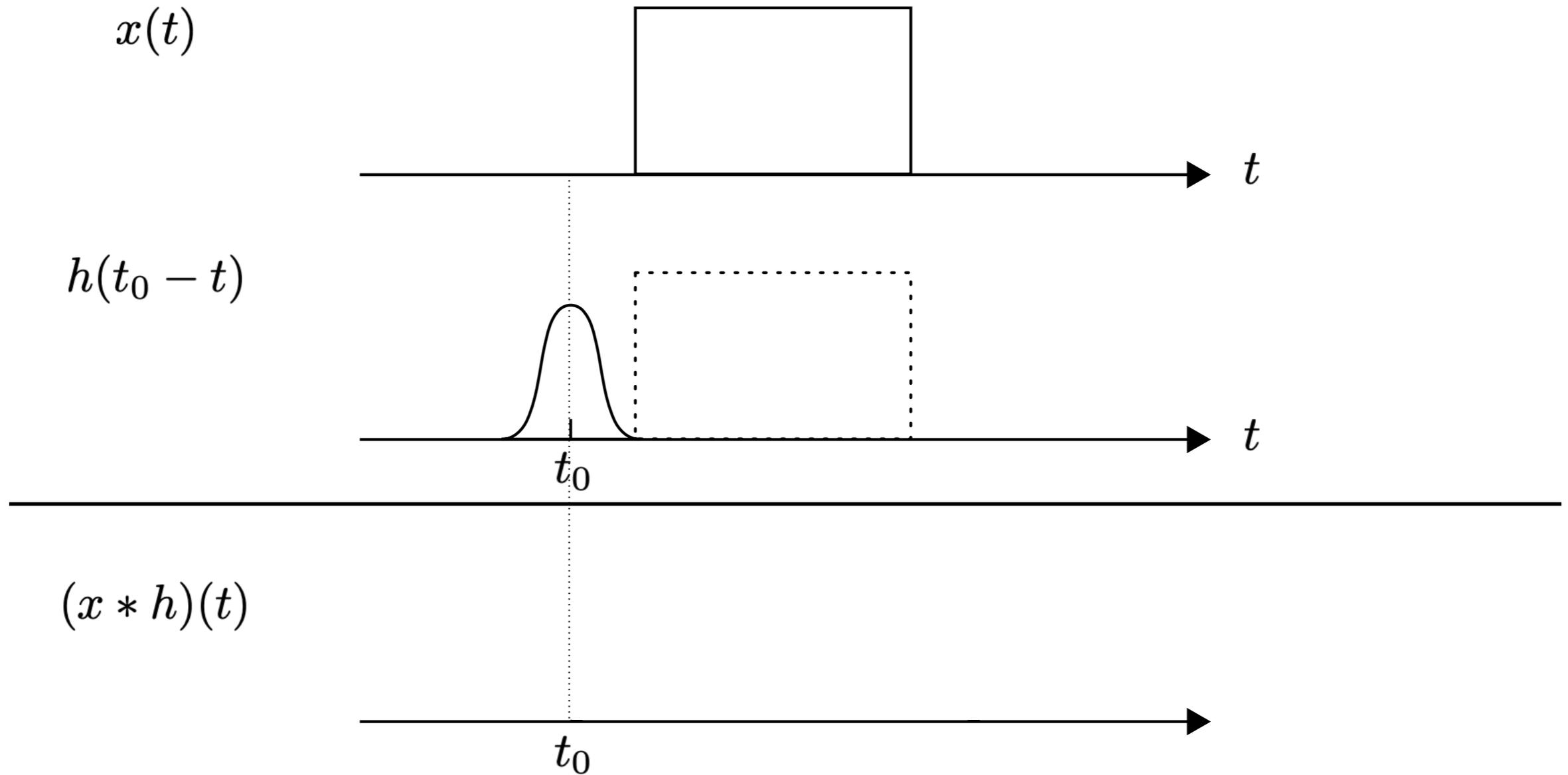
(f)

La convolución



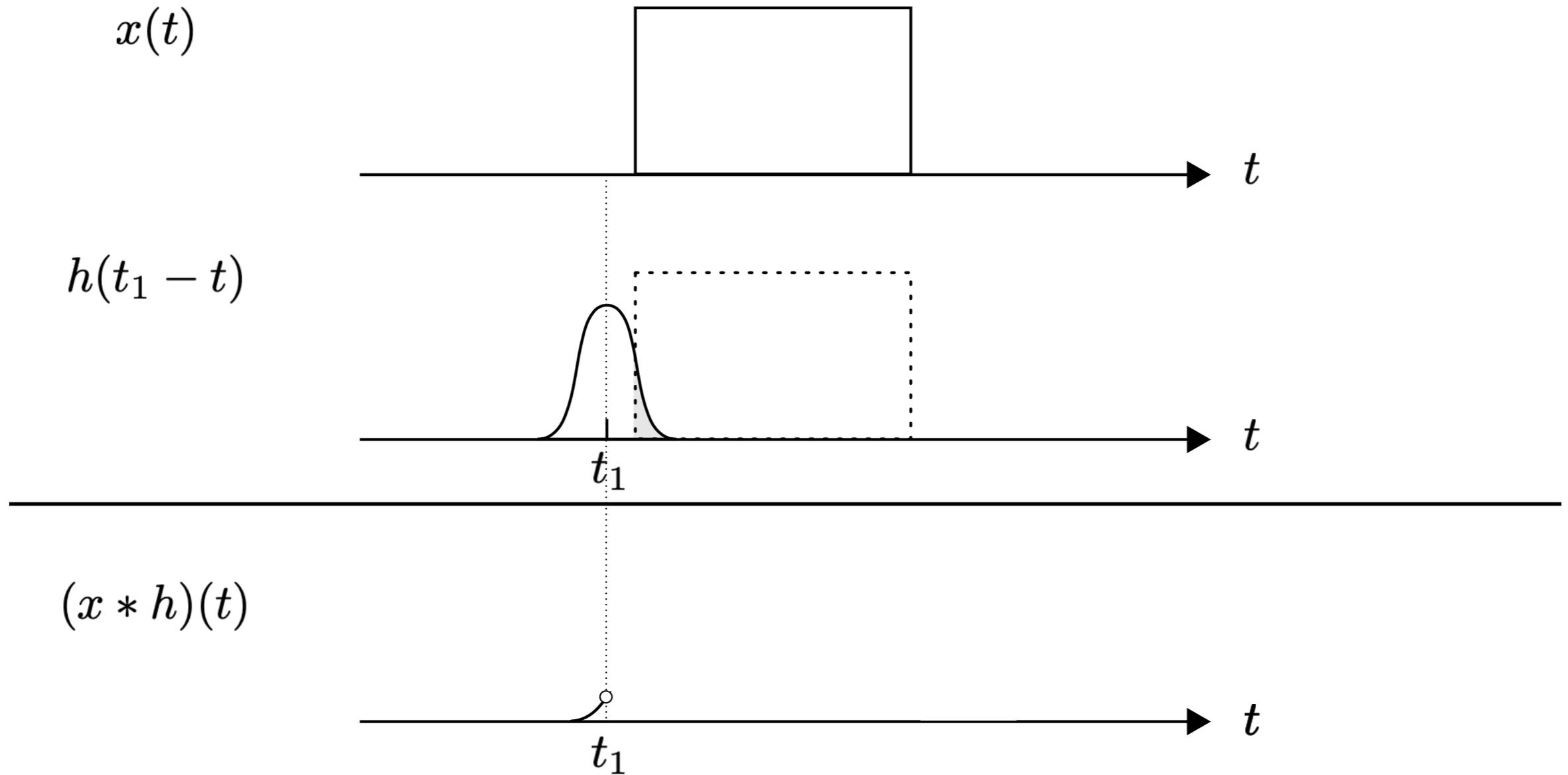
$$\text{"}x(t) * h(t)\text{"} = (x * h)(t) = \int_{\tau=-\infty}^{\tau=+\infty} x(\tau) h(t - \tau) d\tau$$

La convolución



$$“x(t) * h(t)” = (x * h)(t) = \int_{\tau} x(\tau) h(t - \tau) d\tau$$

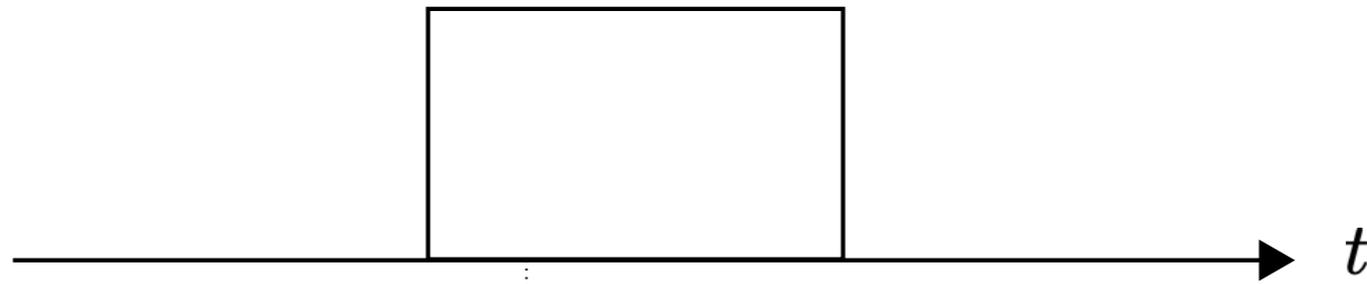
La convolución



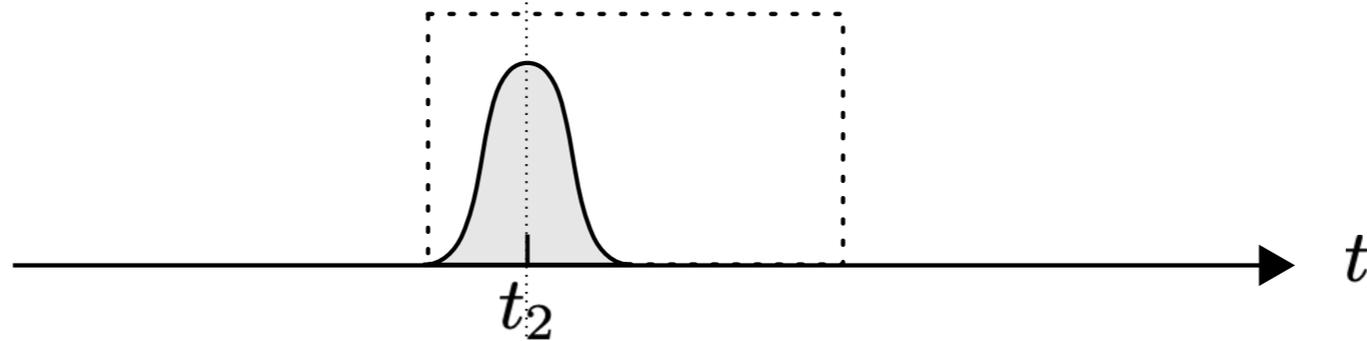
$$“x(t) * h(t)” = (x * h)(t) = \int_{\tau} x(\tau) h(t - \tau) d\tau$$

La convolución

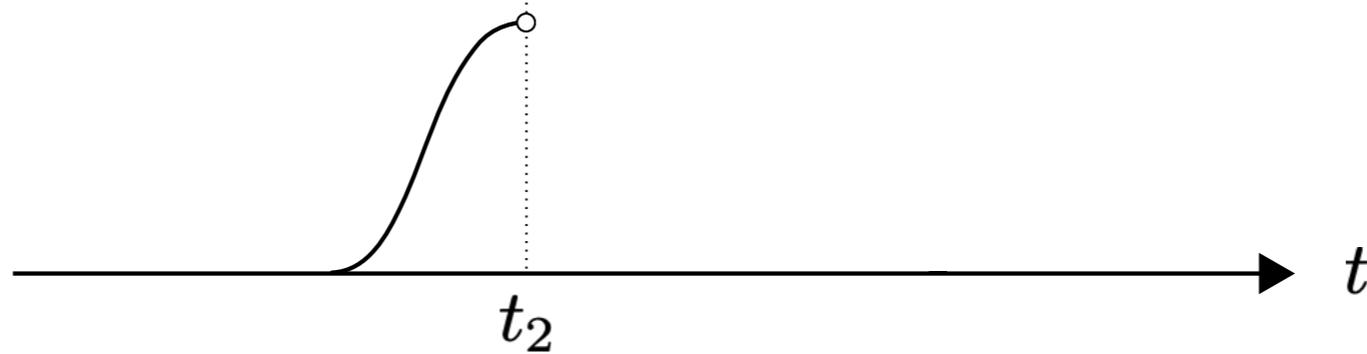
$x(t)$



$h(t_2 - t)$



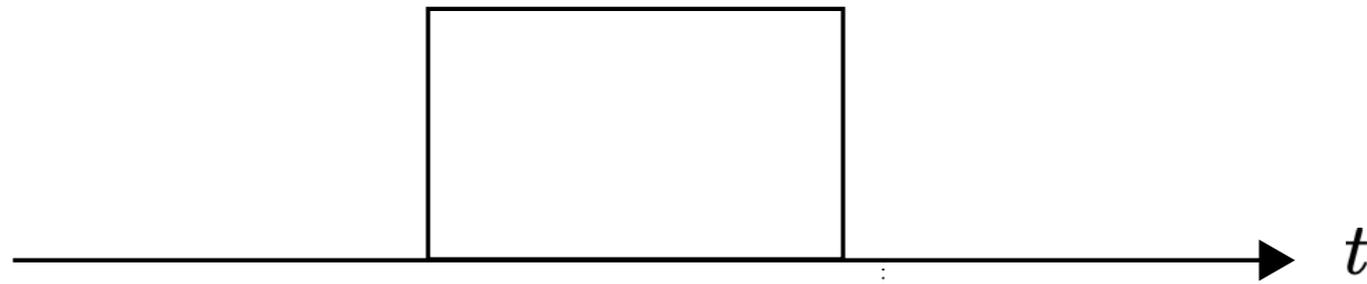
$(x * h)(t)$



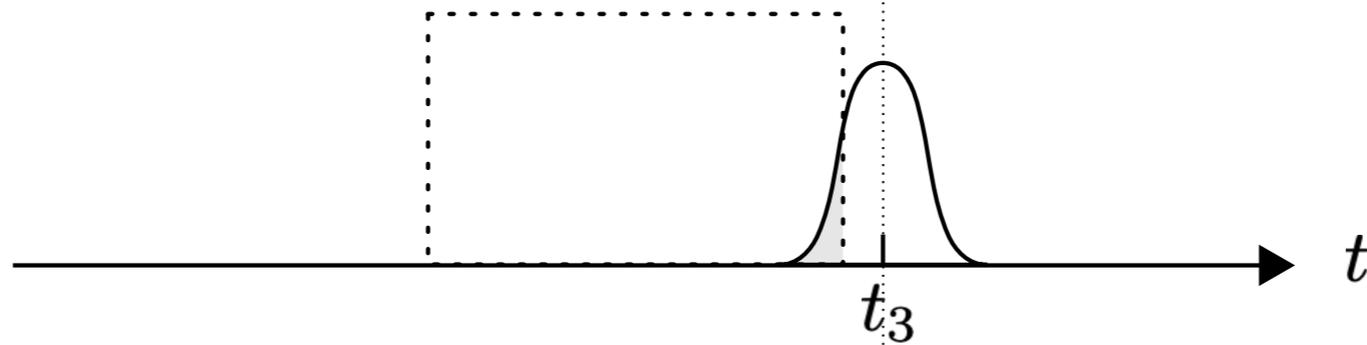
$$“x(t) * h(t)” = (x * h)(t) = \int_{\tau} x(\tau) h(t - \tau) d\tau$$

La convolución

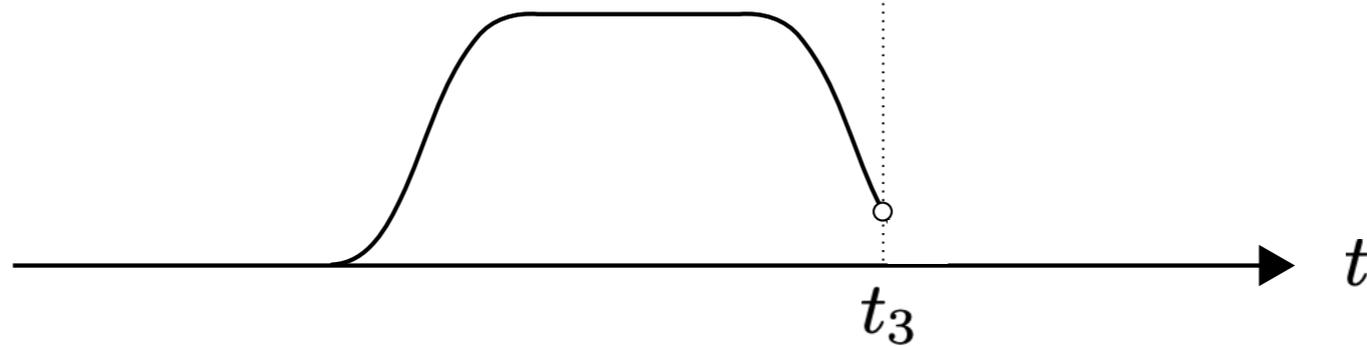
$x(t)$



$h(t_3 - t)$



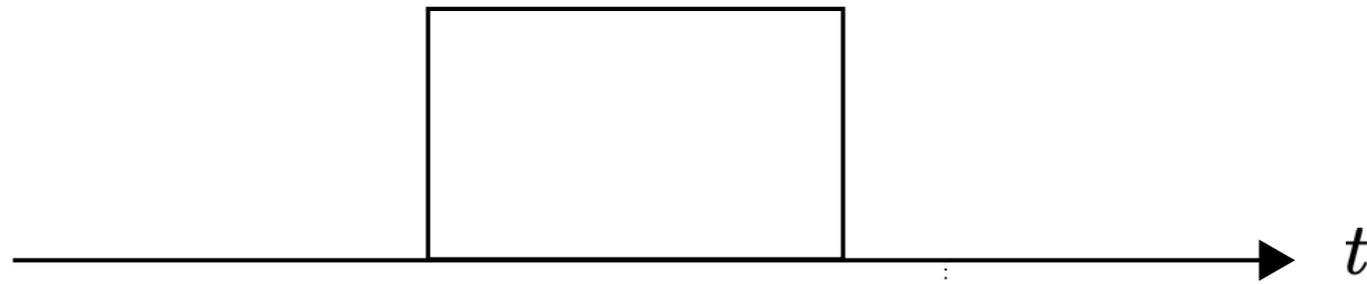
$(x * h)(t)$



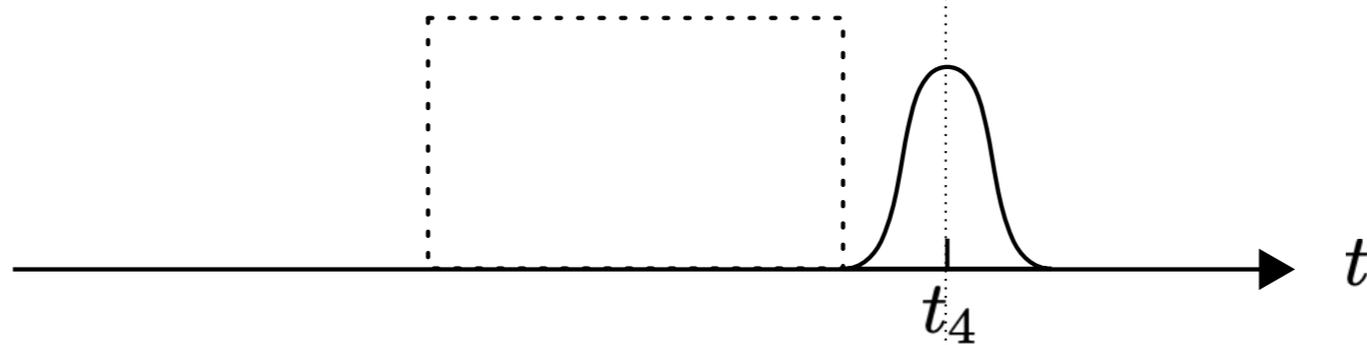
$$“x(t) * h(t)” = (x * h)(t) = \int_{\tau} x(\tau) h(t - \tau) d\tau$$

La convolución

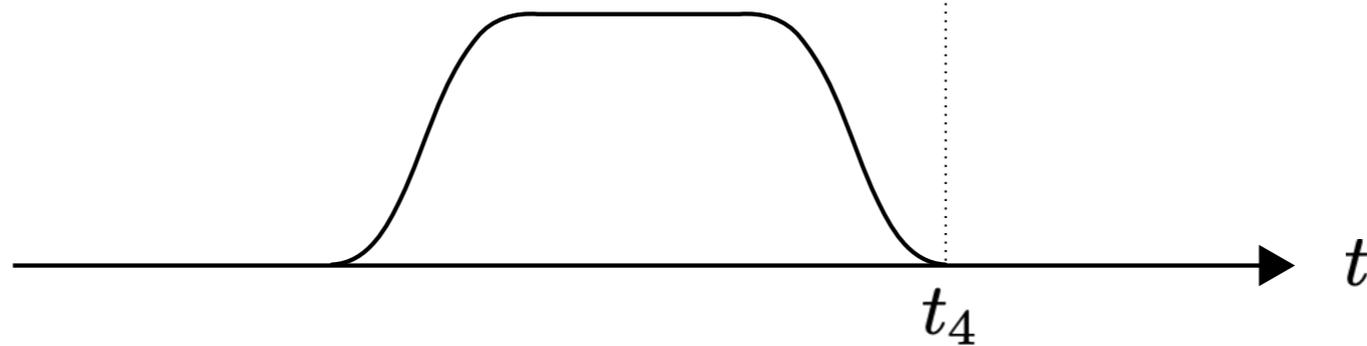
$x(t)$



$h(t_4 - t)$



$(x * h)(t)$



$$“x(t) * h(t)” = (x * h)(t) = \int_{\tau} x(\tau) h(t - \tau) d\tau$$

Ver ejemplo 2.7

Ejemplos

Propiedades de los SLITs

- Causalidad

- Un SLIT es causal sii $h[n] = 0 \quad \forall n < 0$ $h(t) = 0 \quad \forall t < 0$

- Sin memoria

- Un SLIT no tiene memoria sii $h[n] = C \delta[n]$ $h(t) = C \delta(t)$

- Estabilidad BIBO

- Un SLIT es BIBO estable sii $\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$ $\int_{\tau=-\infty}^{+\infty} |h(\tau)| d\tau < \infty$

(Estabilidad BIBO: Para **toda** señal de entrada **acotada**
la señal de salida es **acotada**.)

Propiedades de los SLITs

- Conmutativo

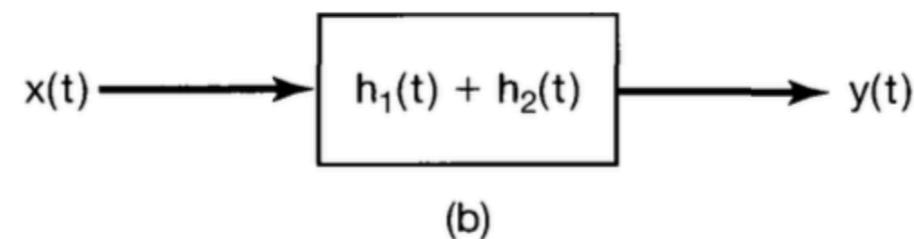
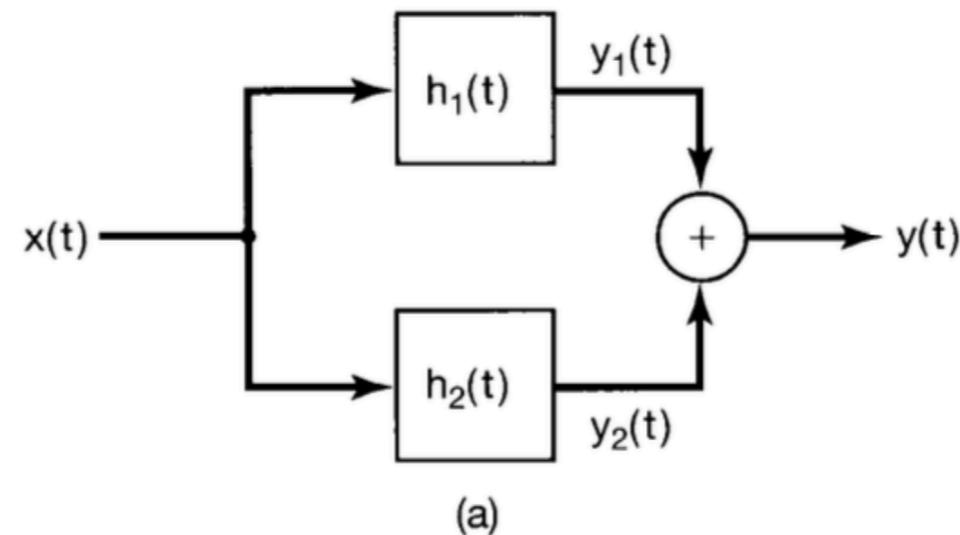
$$x[n] * h[n] = h[n] * x[n]$$

$$x(t) * h(t) = h(t) * x(t)$$

- Distributiva

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

$$x(t) * (h_1(t) + h_2(t)) = x(t) * h_1(t) + x(t) * h_2(t)$$

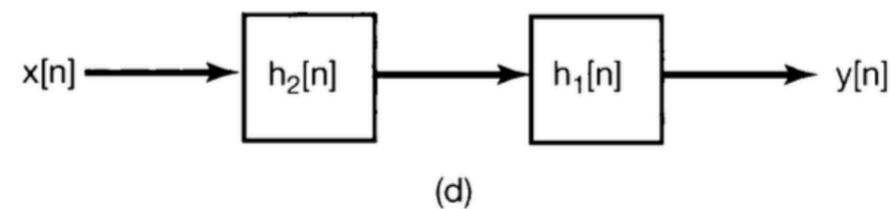
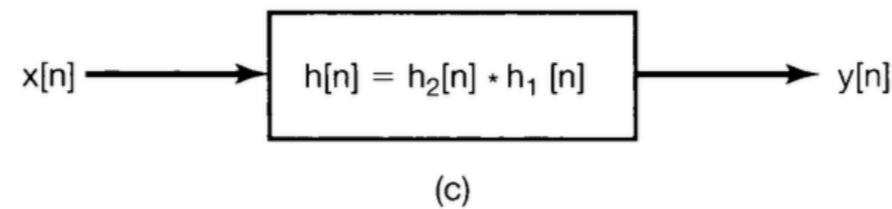
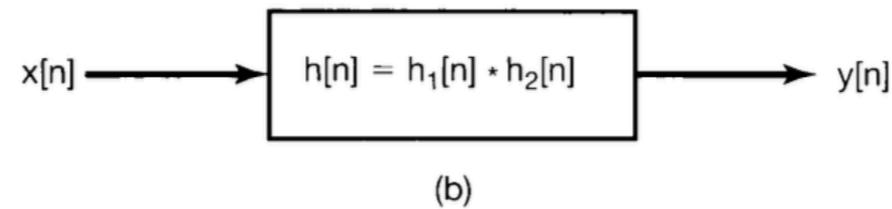
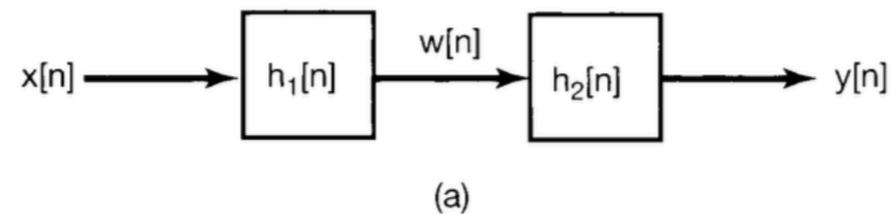


Propiedades de los SLITs

- Asociativa

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n] \quad x[n], h_1[n], h_2[n] = 0 \quad \forall n < 0$$

$$x(t) * (h_1(t) * h_2(t)) = (x(t) * h_1(t)) * h_2(t) \quad x(t), h_1(t), h_2(t) = 0 \quad \forall t < 0$$

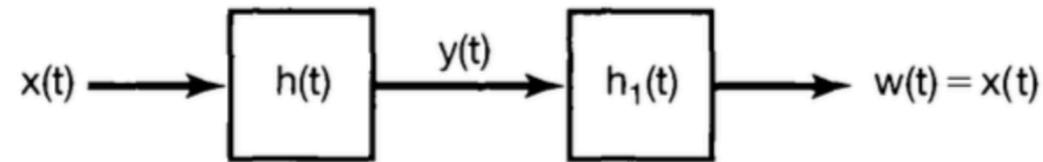


Propiedades de los SLITs

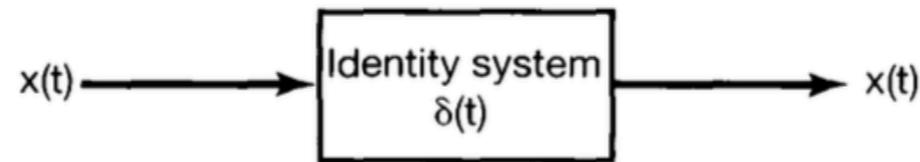
- Invertibilidad

$$h[n] * h_1[n] = \delta[n]$$

$$h(t) * h_1(t) = \delta(t)$$



(a)



(b)

- Respuesta al escalón

$$s[n] = u[n] * h[n] = \sum_{k=-\infty}^n h[k]$$

$$s(t) = u(t) * h(t) = \int_{\tau=-\infty}^t h(\tau) d\tau$$

Propiedades de los SLITs

- Neutro

$$x[n] * \delta[n] = x[n]$$

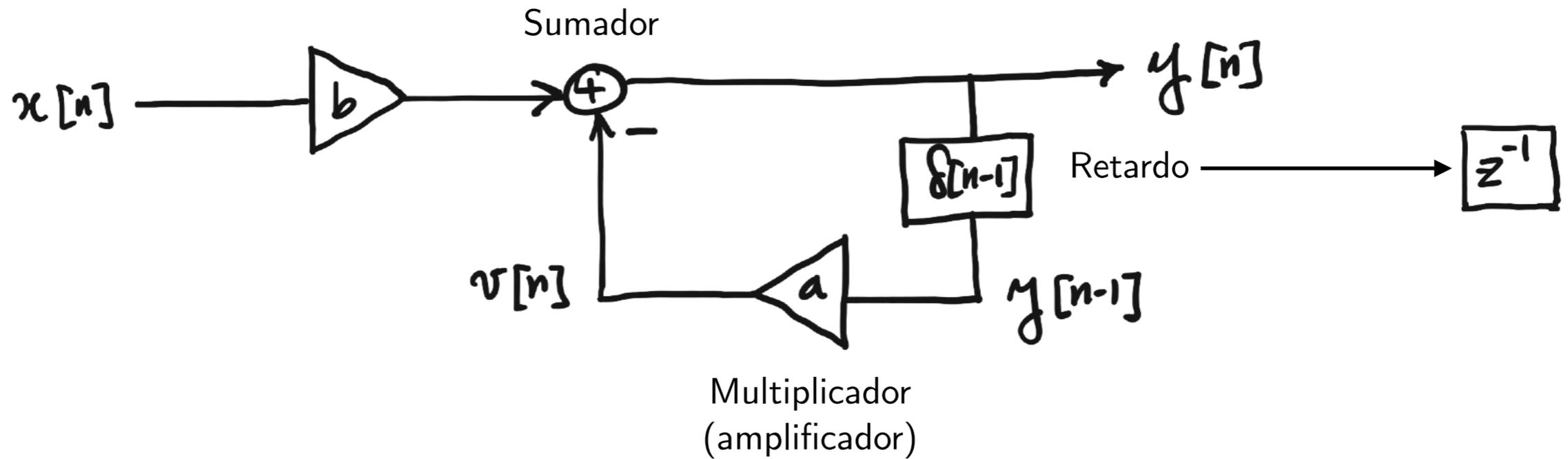
$$x(t) * \delta(t) = x(t)$$

$$x[n] + 0[n] = x[n]$$

$$x(t) + 0(t) = x(t)$$

Representación de sistemas en diagramas de bloques

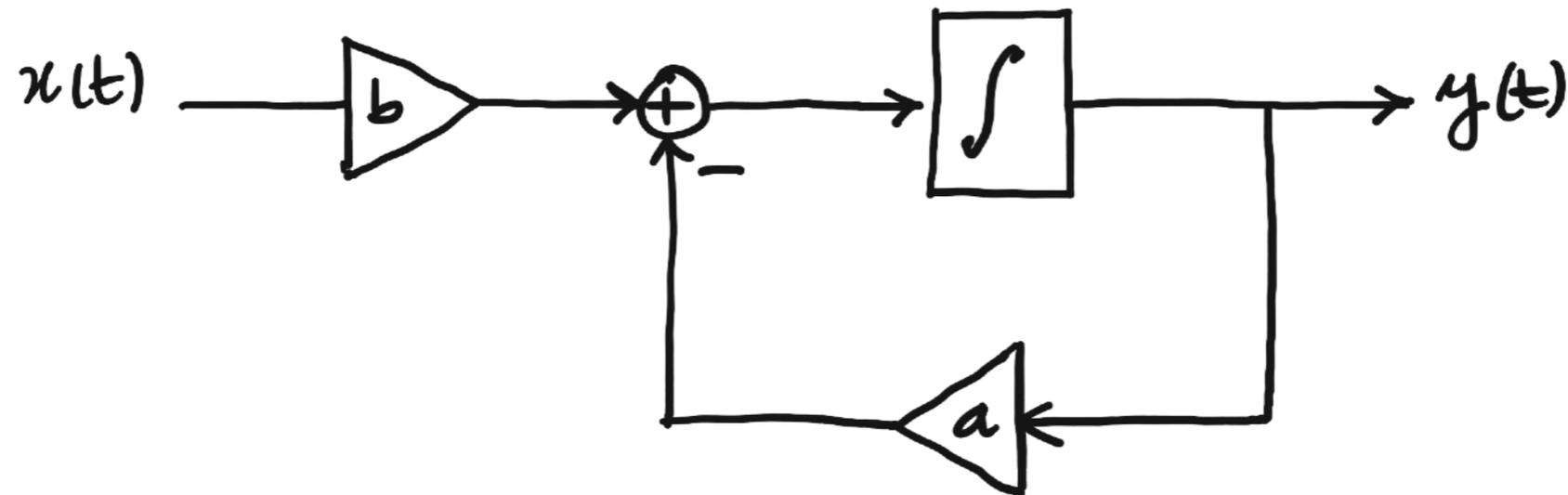
$$y[n] = -a y[n-1] + b x[n]$$



$$\left. \begin{aligned} y[n] &= b x[n] - v[n] \\ v[n] &= a y[n-1] \end{aligned} \right\} y[n] = b x[n] - a y[n-1]$$

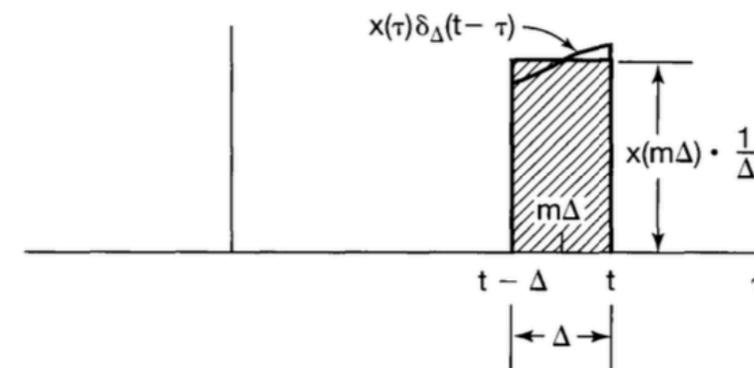
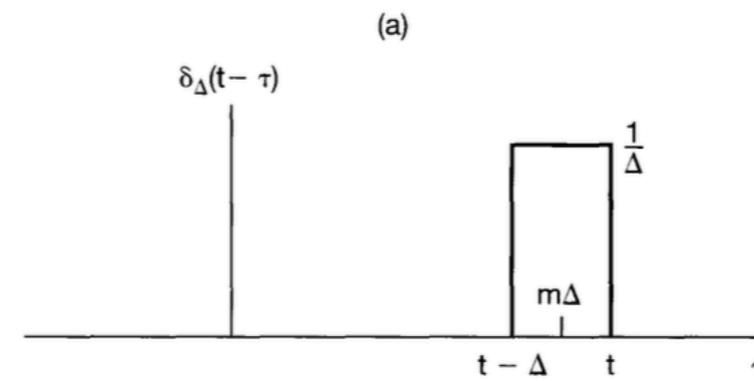
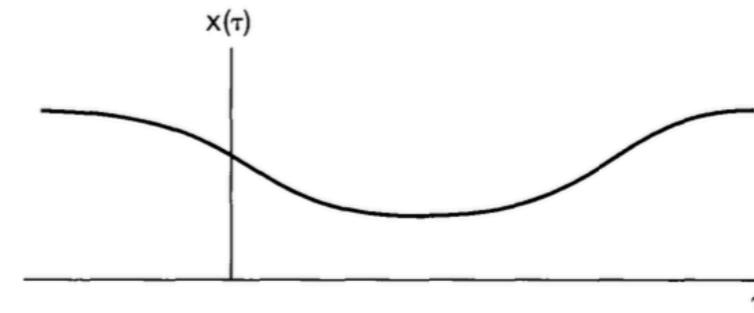
Representación de sistemas en diagramas de bloques

$$\frac{dy(t)}{dt} = -a y(t) + b x(t)$$



Funciones singulares

- $$\delta_{\Delta}(t) = \begin{cases} \frac{1}{\Delta}, & 0 \leq t < \Delta \\ 0, & \text{en otro caso} \end{cases}$$

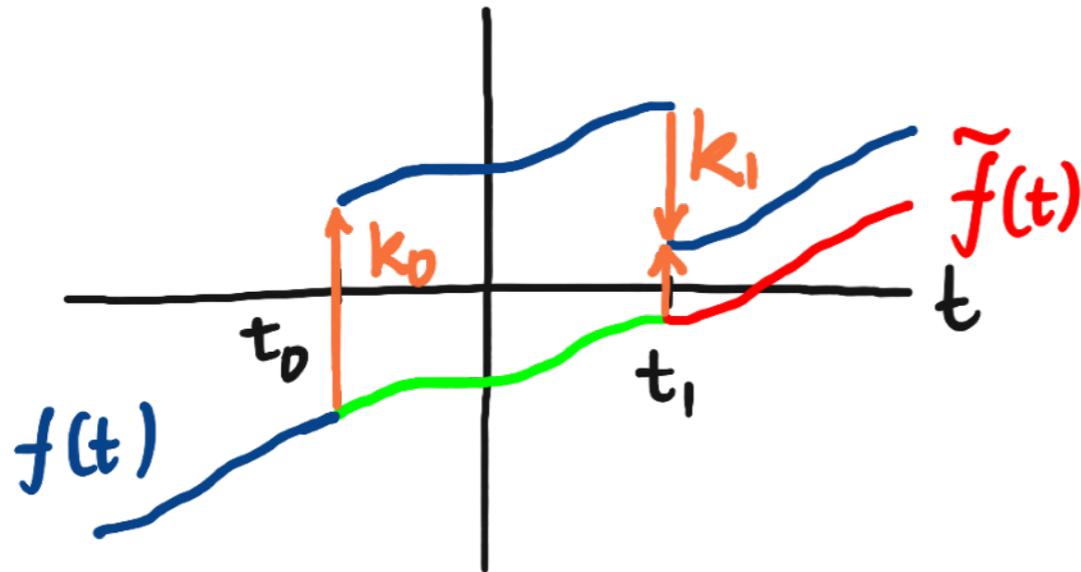


- $$x(t) * \delta(t) = x(t)$$

- $$\int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau =$$

$$\int_{-\infty}^{+\infty} x(t)\delta(t-\tau)d\tau = x(t) \int_{-\infty}^{+\infty} \delta(t-\tau)d\tau = x(t)$$

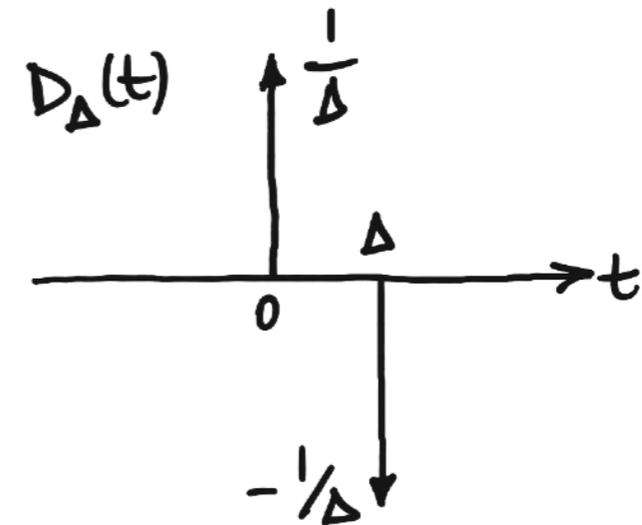
Funciones singulares



La delta nos permite modelar discontinuidades

$$f(t) = \tilde{f}(t) + K_0 u(t - t_0) + K_1 u(t - t_1)$$

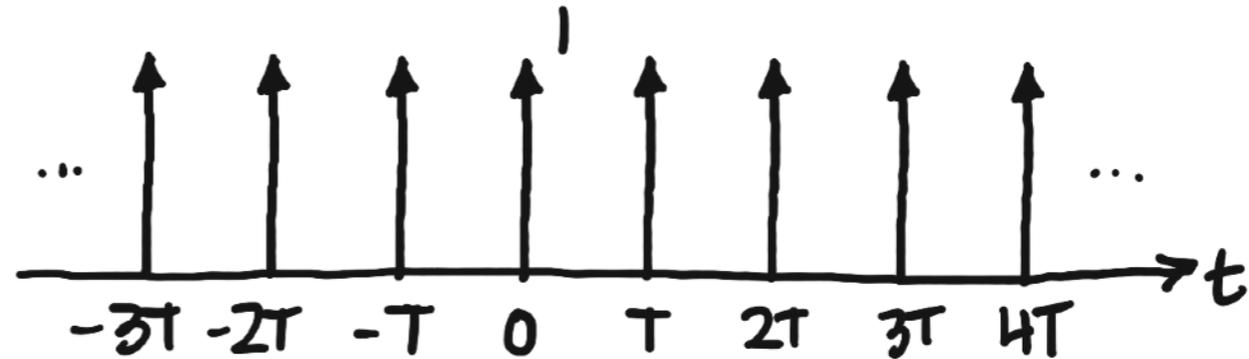
$$\delta(t) * \delta'(t) = \delta'(t) = \frac{d\delta(t)}{dt} = u_1(t) \approx D_\Delta(t)$$



Funciones singulares

- Peine de Dirac

$$p(t) = \sum_k \delta(t - kT)$$



$$\delta(t + 3T)$$

⋮

⋮

⋮

+

⋮



$$\delta(t)$$

⋮

⋮

+

⋮



$$\delta(t - 2T)$$

⋮