Ejercicio N°1

$$E_0 \longrightarrow P_0 = \varepsilon_0 \chi_0 E_0$$

$$E(t) = E_0\left[1-\Theta(t)\right] \longrightarrow P(t) = P_0\, e^{-\frac{t}{\tau}}$$

$$\Theta(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$

parte a: $P(t) = \varepsilon_0 \int_0^\infty d\xi\, K(\xi) E(t-\xi)$ Ec 2.5 Stenzel

$$P(t) = \varepsilon_0 \int_0^\infty d\xi\, K(\xi) E_0 \underbrace{\left[1-\Theta(t-\xi)\right]}$$

$$\begin{matrix} 1 & t-\xi<0 \sim \xi>t \\ 0 & t-\xi>0 \sim \xi<t \end{matrix}$$

$$P(t) = \varepsilon_0 E_0 \int_t^\infty d\xi\, K(\xi) = -\varepsilon_0 E_0 \int_\infty^t d\xi\, K(\xi)$$

$$\frac{dP(t)}{dt} = -\varepsilon_0 E_0 K(t)$$

$\parallel$

$0 \quad t<0 \Rightarrow K(\xi)=0$

$-\frac{1}{\tau}P_0 e^{-\frac{t}{\tau}} \quad t>0 \Rightarrow -\frac{1}{\tau}\varepsilon_0\chi_0 E_0\, e^{-\frac{t}{\tau}} = -\varepsilon_0 E_0 K(t)$

$$\Rightarrow \boxed{K(\xi) = \begin{cases} 0 & \xi<0 \\ \frac{\chi_0}{\tau} e^{-\frac{\xi}{\tau}} & \xi>0 \end{cases}}$$

parte b:

$$\hat{\chi}(\omega) = \int_0^\infty d\xi\, K(\xi)\, e^{i\omega\xi} \quad \text{Ec 2.7 Stenzel}$$

$$\hat{\chi}(\omega) = \frac{\chi_0}{\tau}\int_0^\infty d\xi\, e^{\left(i\omega-\frac{1}{\tau}\right)\xi} = \frac{\chi_0}{\tau}\left.\frac{e^{\left(i\omega-\frac{1}{\tau}\right)\xi}}{i\omega-\frac{1}{\tau}}\right|_0^\infty = -\frac{\chi_0}{\tau}\frac{1}{i\omega-\frac{1}{\tau}}$$

$$\Rightarrow \boxed{\hat{\chi}(\omega) = \frac{\chi_0}{1-i\omega\tau}}$$

parte c: $\hat{\chi}(\omega) = \chi_0 \frac{1+i\omega\tau}{1+\omega^2\tau^2} \Rightarrow \epsilon' = 1 + \frac{\chi_0}{1+\omega^2\tau^2}$

$$\epsilon'' = \frac{\omega\tau\chi_0}{1+\omega^2\tau^2}$$

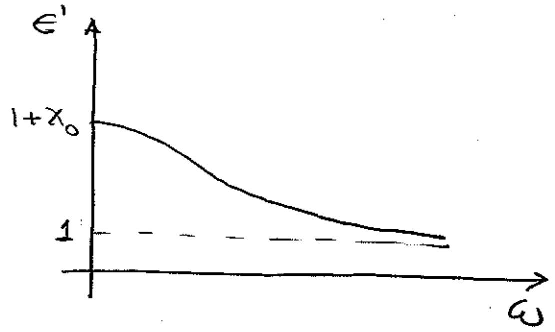
$\omega \to 0 \Rightarrow \epsilon' \to 1+\chi_0 \,,\ \epsilon'' \to 0$

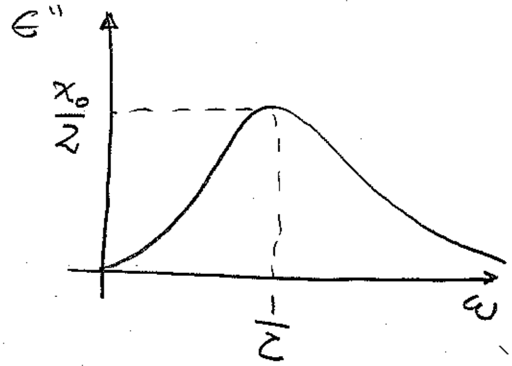
$\omega \to \infty \Rightarrow \epsilon' \to 1 \,,\ \epsilon'' \to 0$

$$\frac{d\epsilon'}{d\omega} = -\frac{2\omega\tau^2\chi_0}{(1+\omega^2\tau^2)^2} = 0 \Leftrightarrow \omega = 0$$

$$\frac{d\epsilon''}{d\omega} = \frac{\tau\chi_0}{1+\omega^2\tau^2} - \omega\tau\frac{2\omega\tau^2\chi_0}{(1+\omega^2\tau^2)^2} = \frac{\tau\chi_0}{(1+\omega^2\tau^2)^2}\left(1+\omega^2\tau^2-2\omega^2\tau^2\right)$$

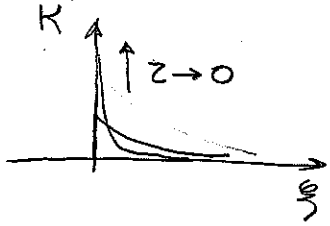
$$\frac{d\epsilon''}{d\omega} = 0 \Leftrightarrow \omega^2\tau^2 = 1 \Rightarrow \boxed{\omega_{max} = \frac{1}{\tau}}$$





parte d: $\tau \to 0 \Rightarrow K(\xi) \to \chi_0\delta(\xi)$

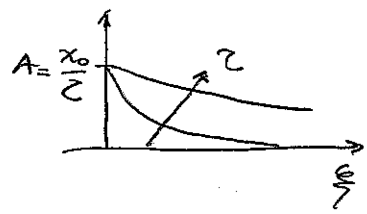
$$\frac{\chi_0}{\tau}\int_0^\infty d\xi\, e^{-\frac{\xi}{\tau}} = \frac{\chi_0}{\tau}\left.\frac{e^{-\frac{\xi}{\tau}}}{-\frac{1}{\tau}}\right|_0^\infty = \chi_0$$



$$\boxed{\epsilon' \to 1+\chi_0 \,,\ \epsilon'' \to 0}$$

Respuesta instantánea del material de susceptibilidad constante (s/pérdidas)

parte e: $\tau \to \infty,\ \chi_0 \to \infty$ c/ $\frac{\chi_0}{\tau} = A = \text{cte}$



$\Rightarrow K(\xi) \to A\,\Theta(\xi)$

$$\epsilon' \to 1 + \frac{A\tau}{\omega^2\tau^2} = 1 + \frac{A}{\omega^2\tau} \to 1$$

$$\epsilon'' \to \frac{\omega\tau A\tau}{\omega^2\tau^2} = \frac{A}{\omega}$$

$$\left.\begin{array}{l} \vec{J} = \sigma\vec{E} \\ \vec{D} = \varepsilon_0\vec{E} \qquad \frac{\partial\vec{D}}{\partial t} = -i\omega\vec{E} \end{array}\right\} \quad \vec{J} + \frac{\partial\vec{D}}{\partial t} = \sigma - i\omega\varepsilon_0 \sim -i\omega\varepsilon_0\epsilon_{eff}$$

$$\epsilon_{eff} = 1 - \frac{\sigma}{i\omega\varepsilon_0} = 1 + \frac{i\sigma}{\omega\varepsilon_0}$$

$\Rightarrow \epsilon' = 1$

$$\epsilon'' = \frac{\sigma}{\omega\varepsilon_0} \approx \frac{A}{\omega} \Rightarrow \boxed{A = \frac{\sigma}{\varepsilon_0}}$$

parte f: $LF = -\,\mathrm{Im}\,\dfrac{1}{\hat{\epsilon}(\omega)} = -\,\mathrm{Im}\,\dfrac{1}{\epsilon' + i\epsilon''} = -\,\mathrm{Im}\,\dfrac{\epsilon' - i\epsilon''}{\epsilon'^2 + \epsilon''^2}$

$$LF = \frac{\epsilon''}{\epsilon'^2 + \epsilon''^2}$$

$$LF = \frac{\chi_0\,\omega\tau}{1+\omega^2\tau^2}\;\frac{1}{\left(1 + \dfrac{\chi_0}{1+\omega^2\tau^2}\right)^2 + \dfrac{\omega^2\tau^2\chi_0^2}{(1+\omega^2\tau^2)^2}} =$$

$$= \frac{\chi_0\,\omega\tau\,(1+\omega^2\tau^2)}{(1+\omega^2\tau^2+\chi_0)^2 + \omega^2\tau^2\chi_0^2}$$

$\chi_0 \ll 1 \Rightarrow LF = \dfrac{\chi_0\,\omega\tau\,(1+\omega^2\tau^2)}{(1+\omega^2\tau^2)^2} = \boxed{\dfrac{\chi_0\,\omega\tau}{1+\omega^2\tau^2} = \epsilon'' = LF}$

$\Rightarrow \omega_{max} = \dfrac{1}{\tau} \Rightarrow \boxed{LF_{max} = \dfrac{\chi_0}{2}}$

Ejercicio N° 2

$$m\left(\frac{dv}{dt}+\frac{v}{\tau}\right)=-eE$$

parte a: $J=-n^*e\vec{v}=\sigma\vec{E}$

$$\vec{E}=E_0e^{-i\omega t}\Rightarrow\vec{v}=\vec{v}_0e^{-i\omega t}\Rightarrow\frac{d\vec{v}}{dt}=-i\omega\vec{v}$$

$$\left(-i\omega+\frac{1}{\tau}\right)\vec{v}=-\frac{e\vec{E}}{m}\Rightarrow\vec{v}=-\frac{e\vec{E}}{m}\frac{1}{\frac{1}{\tau}-i\omega}=-\frac{e\tau}{m}\frac{\vec{E}}{1-i\omega\tau}$$

$$\vec{J}=\frac{e^2n^*\tau}{m}\frac{\vec{E}}{1-i\omega\tau}\Rightarrow\sigma=\underbrace{\frac{e^2n\tau}{m}}_{\sigma(0)}\frac{1}{1-i\omega\tau}=\boxed{\sigma(0)\frac{1+i\omega\tau}{1+\omega^2\tau^2}=\sigma(\omega)}$$

parte b. $\chi'(0)=\frac{2}{\pi}\int_0^\infty d\Omega\frac{\chi''(\Omega)}{\Omega}$

$$\sigma'(0)=\frac{2}{\pi}\int_0^\infty d\Omega\frac{\sigma''(\Omega)}{\Omega}=\frac{2}{\pi}\int_0^\infty\frac{d\Omega}{\Omega}\frac{\sigma(0)\Omega\tau}{1+\Omega^2\tau^2}=\frac{2\sigma(0)\tau}{\pi}\int_0^\infty\frac{d\Omega}{1+\Omega^2\tau^2}=$$

$$=\frac{2\sigma(0)}{\tau\pi}\frac{1}{\frac{1}{\tau}}\operatorname{arct}\frac{\Omega}{\frac{1}{\tau}}\Bigg|_0^\infty=\qquad\qquad\frac{1}{\tau^2}\ \ {}^{\prime\prime}\ \ \frac{1}{\frac{1}{\tau^2}+\Omega^2}$$

$$=\frac{2\sigma(0)}{\pi}\left(\frac{\pi}{2}-0\right)=\sigma(0)\ ✓$$

parte c: $\epsilon_{eff}=1-\frac{\sigma}{i\omega\varepsilon_0}$ (ver parte e Ej. 1)

$$\epsilon=1-\frac{\sigma(0)}{i\omega\varepsilon_0}\frac{1}{1-i\omega\tau}=1-\frac{\sigma(0)}{\varepsilon_0}\frac{1}{\omega(\omega\tau+i)}=1-\frac{\sigma(0)}{\varepsilon_0}\frac{\omega\tau-i}{\omega(\omega^2\tau^2+1)}$$

$$\boxed{\epsilon'=1-\frac{\sigma(0)}{\varepsilon_0}\frac{\tau}{\omega^2\tau^2+1}}$$

$$\boxed{\epsilon''=\frac{\sigma(0)}{\varepsilon_0}\frac{1}{\omega(\omega^2\tau^2+1)}}$$

parte d: i) $\omega\tau\gg1$ $\boxed{\epsilon'\cong1-\frac{\sigma(0)}{\varepsilon_0}\frac{\tau}{\omega^2\tau^2}=1-\frac{\omega_p^2}{\omega^2}}$

$$\omega_p^2=\frac{n^*e}{\varepsilon_0m}=\frac{\sigma(0)}{\varepsilon_0\tau}\qquad\boxed{\epsilon''\cong\frac{\sigma(0)}{\varepsilon_0}\frac{1}{\omega^3\tau^2}=\frac{\omega_p^2}{\omega^3\tau}}$$

ii) $\omega\tau\ll1\Rightarrow\boxed{\epsilon'\cong1-\frac{\sigma(0)\tau}{\varepsilon_0}=1-\omega_p^2\tau^2}$

$$\boxed{\epsilon''\cong\frac{\sigma(\omega)}{\varepsilon_0\omega}=\frac{\omega_p^2\tau}{\omega}}$$

parte e: i) $\epsilon'' \ll \epsilon'$

$$\alpha = \frac{2\omega K}{c} \cong \frac{2\omega \epsilon''}{c\, 2\sqrt{\epsilon'}} \cong \frac{\omega}{c}\frac{\omega_p^2}{\omega^3 \tau} = \boxed{\frac{\omega_p^2}{c\tau\omega^2} = \alpha(\omega)}$$

Ec 2.20a Stenzel

$K \cong \frac{\epsilon''}{2\sqrt{\epsilon'}}$ Ec. 1.27 y 1.28 Fox

ii) $\epsilon'' \gg \epsilon' \Rightarrow \hat{\epsilon} \simeq i\epsilon'' \Rightarrow \hat{n} = \sqrt{\epsilon''}\,\frac{i+1}{\sqrt{2}} \Rightarrow K = \sqrt{\frac{\epsilon''}{2}}$

$$\boxed{\alpha = \frac{2\omega}{c}\sqrt{\frac{\omega_p^2 \tau}{2\omega}} = \sqrt{\frac{2\omega_p^2 \tau \omega}{c^2}}}$$

parte f: i) $\hat{n} \cong \sqrt{\epsilon'} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \simeq 1 - \frac{\omega_p^2}{2\omega^2}$

$\epsilon'' \ll \epsilon'$

$$\boxed{R = \left| \frac{\frac{\omega_p^2}{2\omega^2}}{1 - \frac{\omega_p^2}{2\omega^2} + 1} \right|^2 \simeq \left| \frac{\omega_p^2}{4\omega^2} \right|^2 \simeq \frac{\omega_p^4}{16\omega^4}}$$

ii) $n = K = \sqrt{\frac{\omega_p^2 \tau}{2\omega}} \gg 1$

$$R = \left| \frac{n + iK - 1}{n + iK + 1} \right| \cong \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2} = \frac{\left(1 - \frac{1}{n}\right)^2 + 1}{\left(1 + \frac{1}{n}\right)^2 + 1} \cong$$

$\frac{1}{n} \ll 1$

$$\cong \frac{1 - \frac{2}{n} + 1}{1 + \frac{2}{n} + 1} = \frac{1 - 1/n}{1 + 1/n} \cong \left(1 - \frac{1}{n}\right)^2 = 1 - \frac{2}{n} = 1 - 2\sqrt{\frac{2\omega}{\omega_p^2 \tau}}$$

$$\boxed{R \simeq 1 - \sqrt{\frac{8\omega}{\omega_p^2 \tau}}}$$