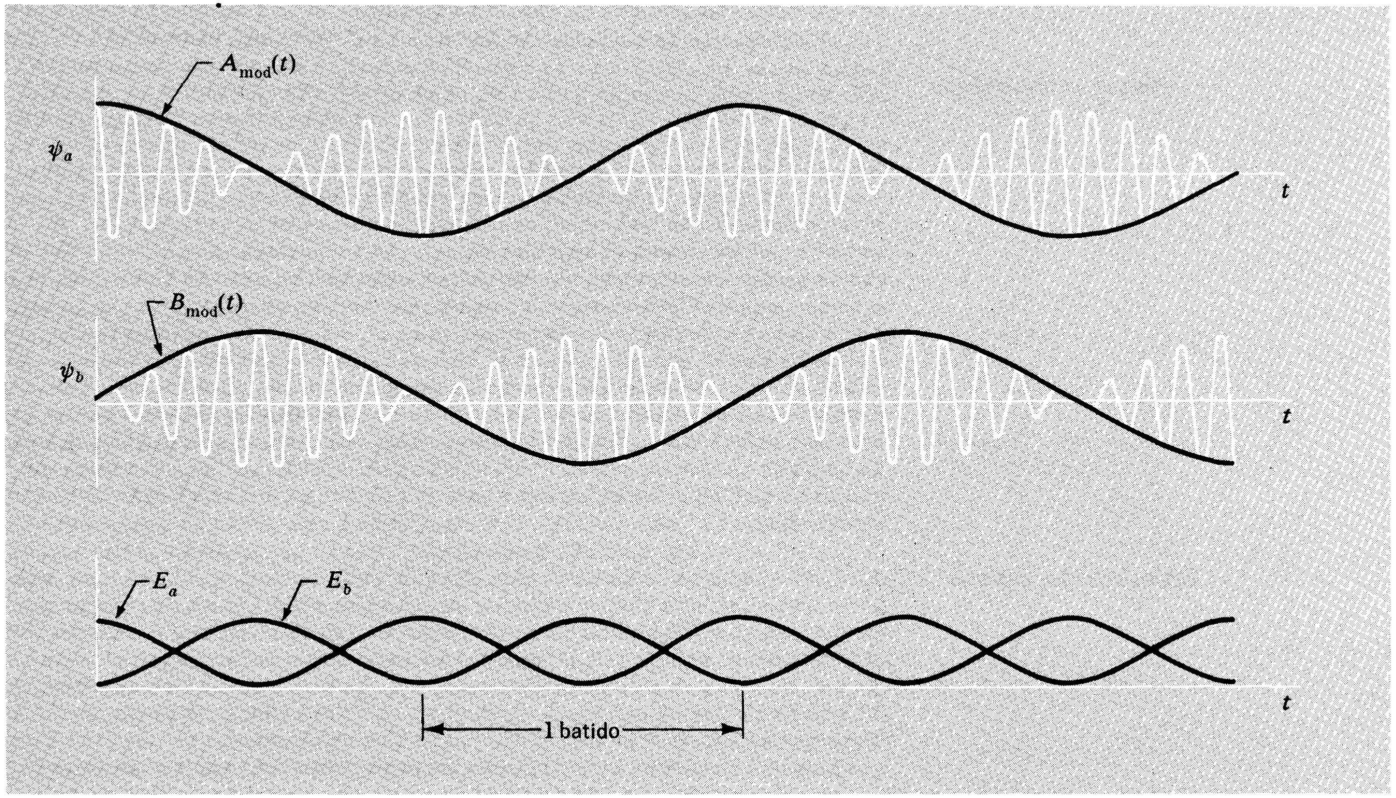
Batido



Batido

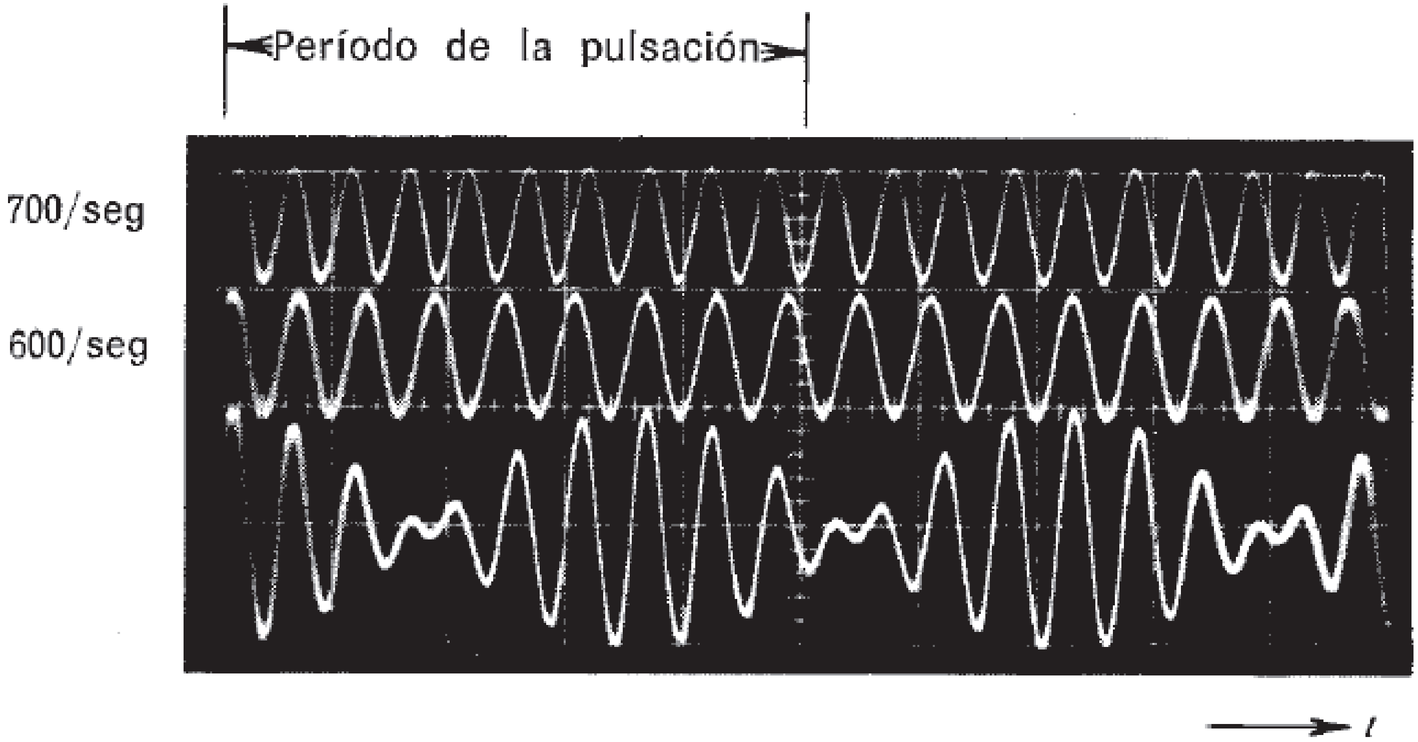
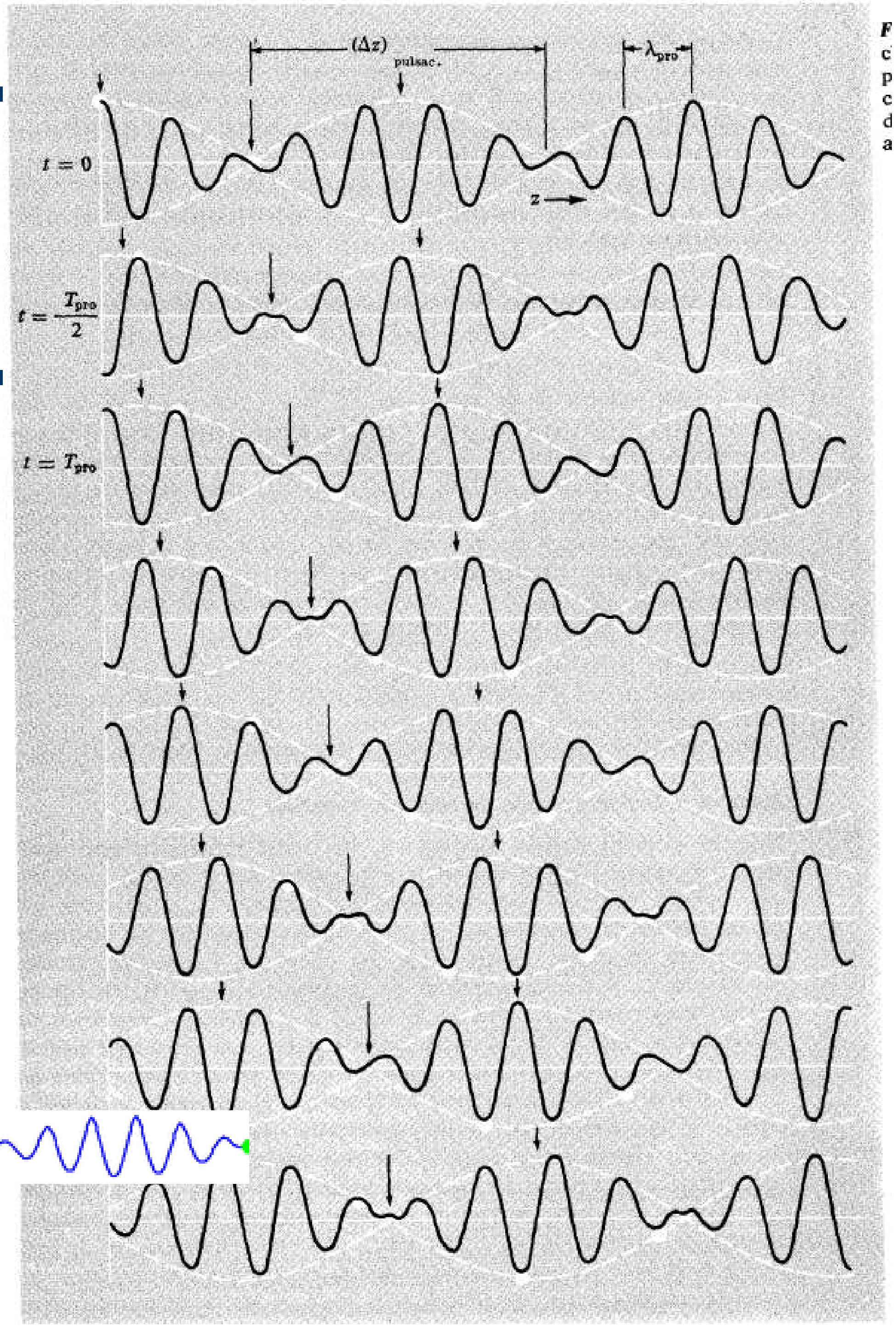
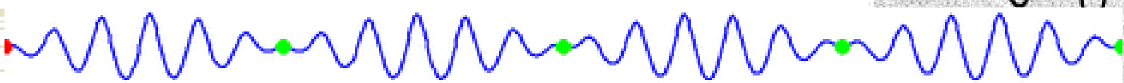


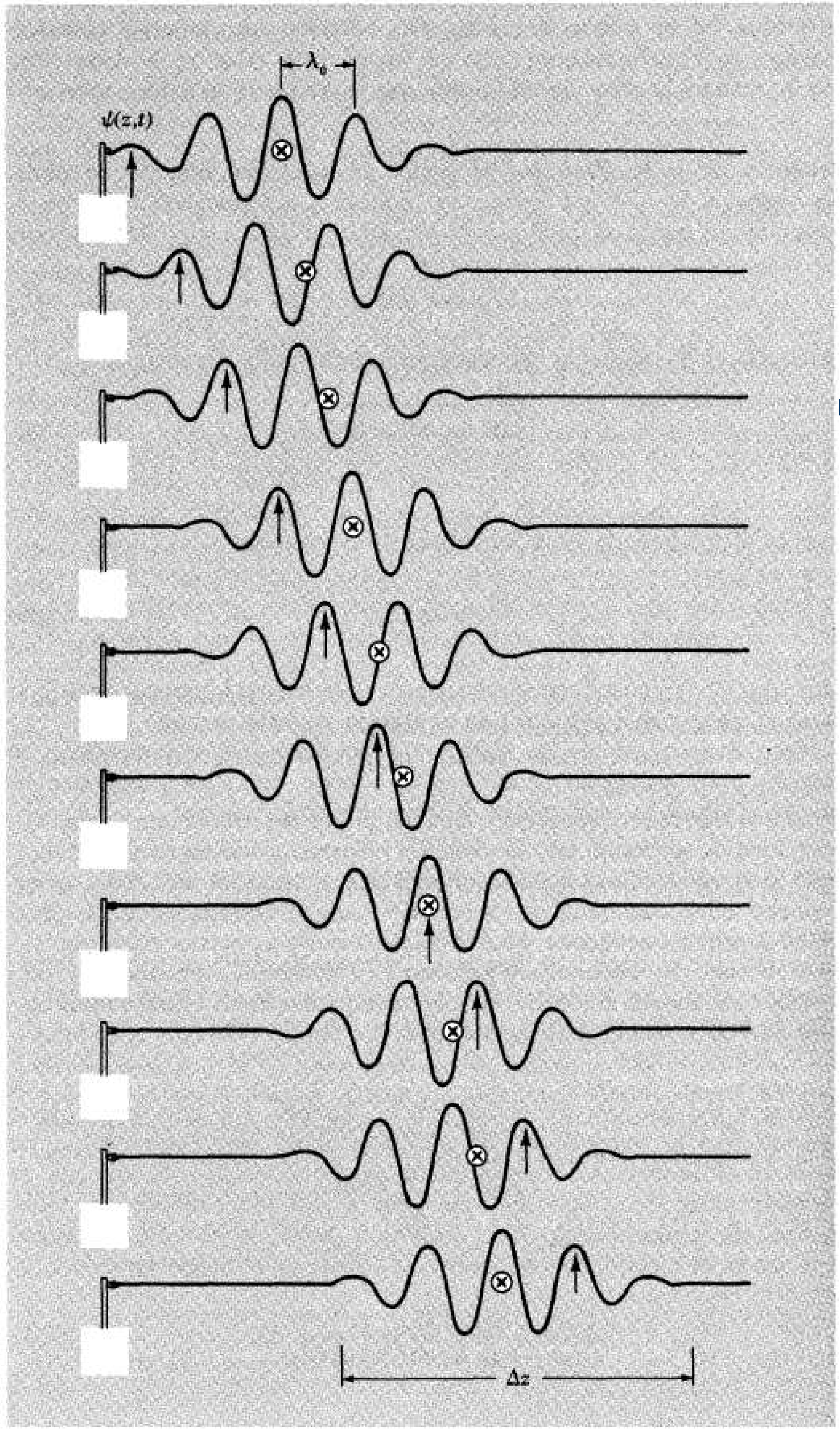
Fig. 2-6. *Superposición de sinusoides de frecuencias semejantes (600 seg^{-1} y 700 seg^{-1}) con objeto de obtener pulsaciones. (Fotografía de Jon Rosenfeld, Education Research Center, M.I.T.)*

Velocidad de Fase y Velocidad de Grupo

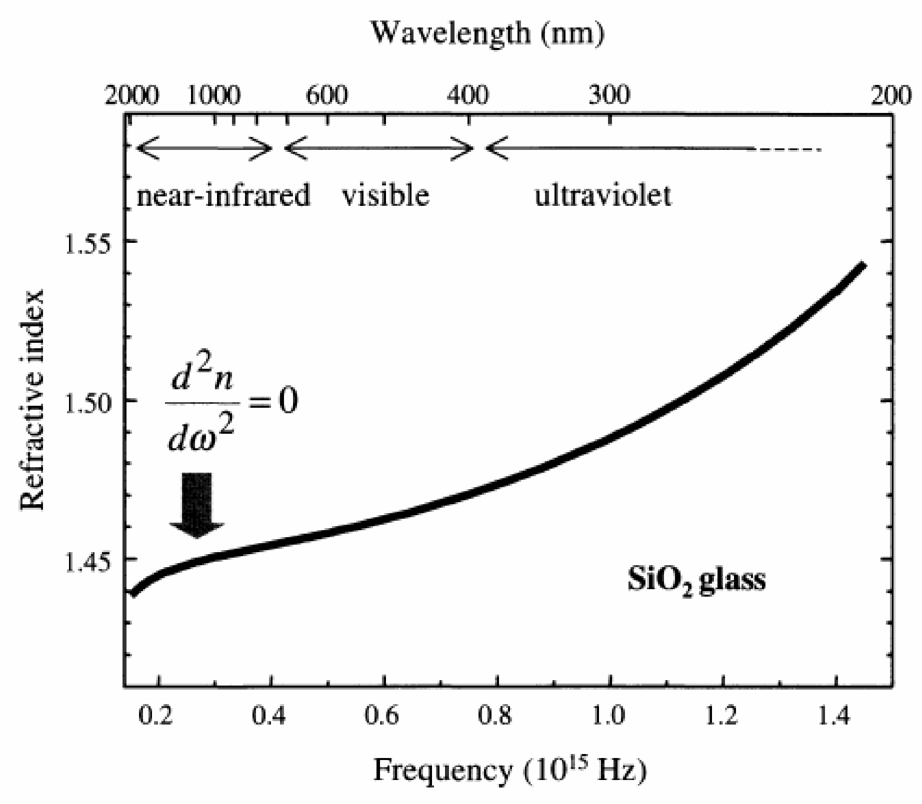




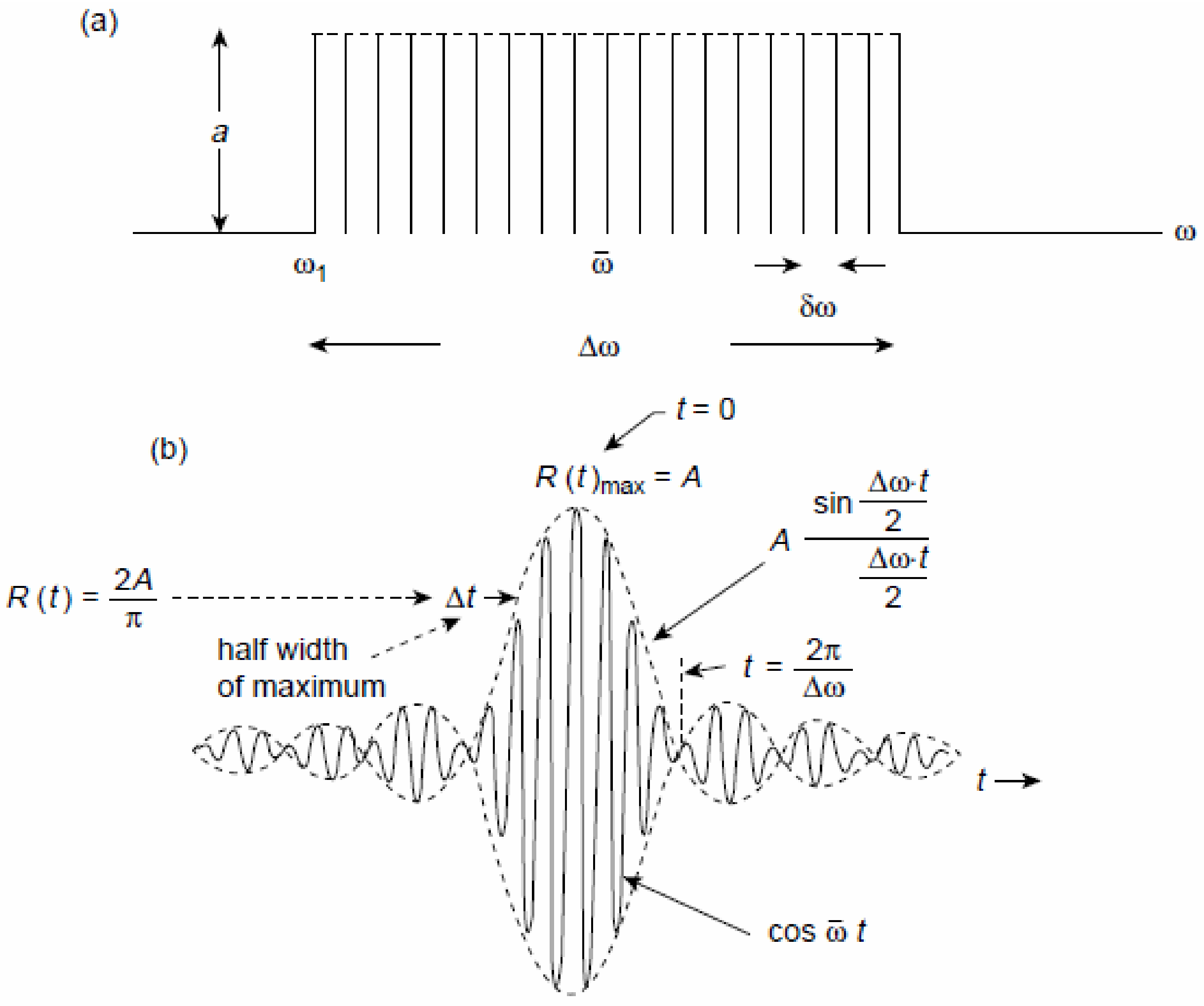
Propagación de un Pulso o Paquete de Ondas



Dispersión de Velocidad de Grupo en Fibras Ópticas



Pulso Temporal de Espectro Uniforme



Pulso Temporal = Paquete de Ondas

$$\psi(x,t) = \sum_k C_k e^{j(\omega t - kx)}$$

$$\delta k = \frac{2\pi}{L} \sim \frac{\pi}{L} \text{ (para onda estacionaria)}$$

$$\text{Si } L \to \infty \Rightarrow \delta k \to 0 \text{ (cuasicontinuo)}$$

$$\psi(x,t) = \int_{-\infty}^{+\infty} dk\, p(k)\, e^{j(\omega t - kx)}$$

Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k = \sigma$ **Ancho Espectral del Pulso**

k_0 **Centro (Valor Medio) Espectral del Pulso**

$$\psi(x,t) = \int_{-\infty}^{+\infty} dk\, p(k)\, e^{j(\omega t - kx)}$$

Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k = \sigma$ **Ancho Espectral del Pulso**

k_0 **Centro (Valor Medio) Espectral del Pulso**

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

$$\gamma(k,x)$$

$$-\gamma(k,x) = \frac{(k-k_0)^2}{2\sigma^2} + jkx$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0)=\frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty}dk\,e^{\left[-\frac{(k-k_0)^2}{2\sigma^2}-jkx\right]}$$

$$\gamma(k,x)$$

$$-\gamma(k,x)=\frac{(k-k_0)^2}{2\sigma^2}+j(k-k_0)x+jk_0x$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

$$\gamma(k,x)$$

$$-\gamma(k,x) = \frac{(k-k_0)^2}{2\sigma^2} + j(k-k_0)x + jk_0x$$

$$-\gamma(k,x) = \frac{(k-k_0)^2 + 2\sigma^2 j(k-k_0)x}{2\sigma^2} + jk_0x$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

$$\gamma(k,x)$$

$$-\gamma(k,x) = \frac{\left(k - k_0 + \sigma^2 jx\right)^2}{2\sigma^2} - \frac{\left(\sigma^2 jx\right)^2}{2\sigma^2} + jk_0 x$$

$$-\gamma(k,x) = \frac{(k - k_0)^2 + 2\sigma^2 j(k - k_0)x}{2\sigma^2} + jk_0 x$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

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$$-\gamma(k,x) = \frac{\left(k - k_0 + \sigma^2 jx\right)^2}{2\sigma^2} - \frac{\left(\sigma^2 jx\right)^2}{2\sigma^2} + jk_0 x$$

$$-\gamma(k,x) = \frac{\left(k - k_0 + \sigma^2 jx\right)^2}{2\sigma^2} + \frac{\sigma^2 x^2}{2} + jk_0 x$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0)=\frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} dk\, e^{\underbrace{\left[-\frac{\left(k-k_0+\sigma^2 jx\right)^2}{2\sigma^2}-\frac{\sigma^2x^2}{2}-jk_0x\right]}_{\gamma(k,x)}}$$

$$-\gamma(k,x)=\frac{\left(k-k_0+\sigma^2 jx\right)^2}{2\sigma^2}-\frac{\left(\sigma^2 jx\right)^2}{2\sigma^2}+jk_0x$$

$$-\gamma(k,x)=\frac{\left(k-k_0+\sigma^2 jx\right)^2}{2\sigma^2}+\frac{\sigma^2x^2}{2}+jk_0x$$

Ejemplo: Pulso Gaussiano

$$\psi(x,0)=\frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty}dk\,e^{\left[-\frac{(k-k_0+\sigma^2 jx)^2}{2\sigma^2}-\frac{\sigma^2x^2}{2}-jk_0x\right]}$$

$$\psi(x,0)=e^{\left[-\frac{\sigma^2x^2}{2}-jk_0x\right]}\frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty}dk\,e^{\left[-\frac{(k-k_0+\sigma^2 jx)^2}{2\sigma^2}\right]}$$

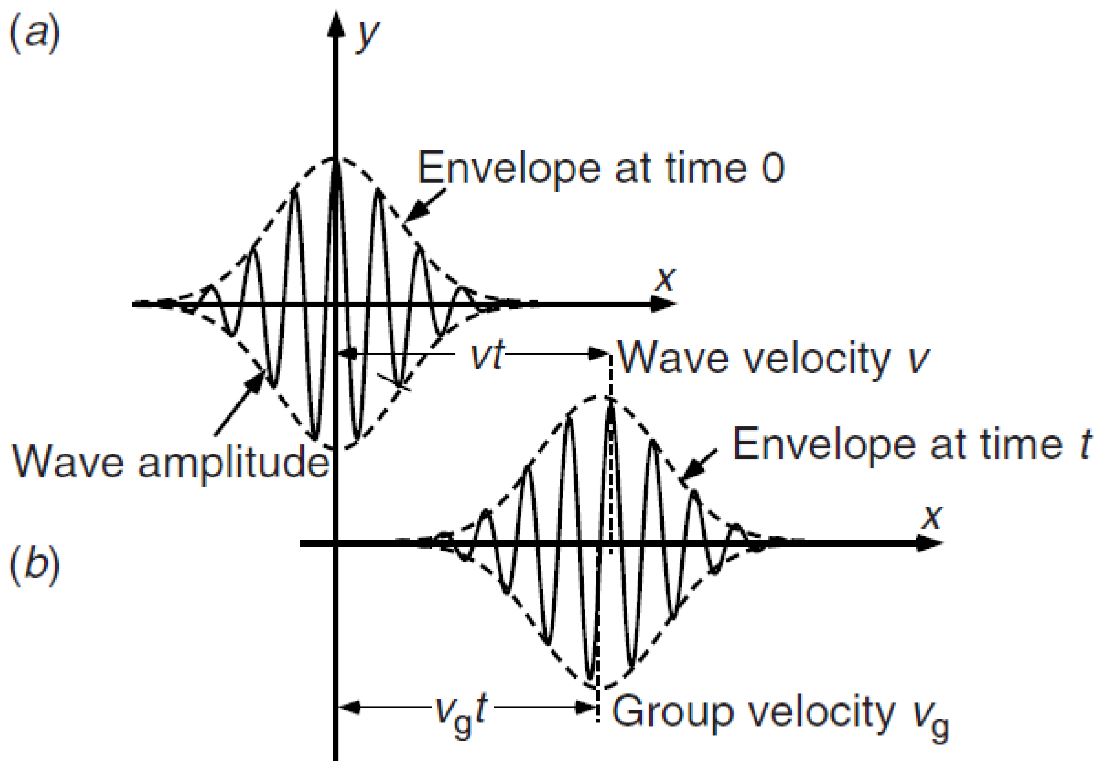
Ejemplo: Pulso Gaussiano

$$p(k)=\frac{A}{\sqrt{2\pi\sigma^2}}e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A=\int_{-\infty}^{+\infty}dk\,p(k)$$

$$\psi(x,0)=e^{\left[-\frac{\sigma^2x^2}{2}-jk_0x\right]}\frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty}dke^{\left[-\frac{(k-k_0+\sigma^2jx)^2}{2\sigma^2}\right]}$$

$$\psi(x,0)=Ae^{\left[-\frac{\sigma^2x^2}{2}-jk_0x\right]}=Ae^{-\frac{\sigma^2x^2}{2}}cos(k_0x)$$

Dispersión



(a)
y
Envelope at time 0
x
vt
Wave velocity v
Envelope at time t
Wave amplitude
x
(b)
vgt
Group velocity vg

Ejemplo: Pulso Gaussiano

$$p(k)=\frac{A}{\sqrt{2\pi\sigma^2}}e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A=\int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k=\sigma$ **Ancho Espectral del Pulso**

k_0 **Centro (Valor Medio) Espectral del Pulso**

$$\psi(x,t)=\frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2}+j(\omega t-kx)\right]}$$

Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k = \sigma$ **Ancho Espectral del Pulso**

k_0 **Centro (Valor Medio) Espectral del Pulso**

$$\psi(x,0) = A e^{\left[-\frac{\sigma^2 x^2}{2} - jk_0 x\right]} = A e^{-\frac{\sigma^2 x^2}{2}} \cos(k_0 x)$$

Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k = \sigma$ **Ancho Espectral del Pulso**

$\Delta x = \dfrac{1}{\sigma}$ **Ancho Espacial del Pulso** $(\Delta k\, \Delta x = 1)$

$$\psi(x,0) = Ae^{\left[-\frac{\sigma^2 x^2}{2} - jk_0 x\right]} = Ae^{-\frac{\sigma^2 x^2}{2}} \cos(k_0 x)$$

Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k = \sigma$ **Ancho Espectral del Pulso**

k_0 **Centro (Valor Medio) Espectral del Pulso**

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

Dispersión

Tres Casos:

1. $\omega = kv$ **Medio No Dispersivo.**
2. $\omega = kv_g + \alpha$ **Medio Dispersivo Lineal.**
3. $\omega = kv_x + \alpha + \beta\,(k\text{-}k_0)^2$ **Medio Dispersivo Cuadrático.**

(si $\alpha = 0$, $k_0 = 0$) $\beta > 0$ **Dispersión Anómala ($v_g > v_{ph}$)**

$\beta < 0$ **Dispersión Normal ($v_g < v_{ph}$)**

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

1) Medio No Dispersivo $(\omega = kv)$

(v: velocidad de fase y de grupo)

$$\omega t - kx = k(vt - x) = -k(x - vt) = -kx'(t)$$

$$x'(t) = x - vt$$

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx'(t)\right]}$$

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

1) Medio No Dispersivo ($\omega = kv$)

(v: velocidad de fase y de grupo)

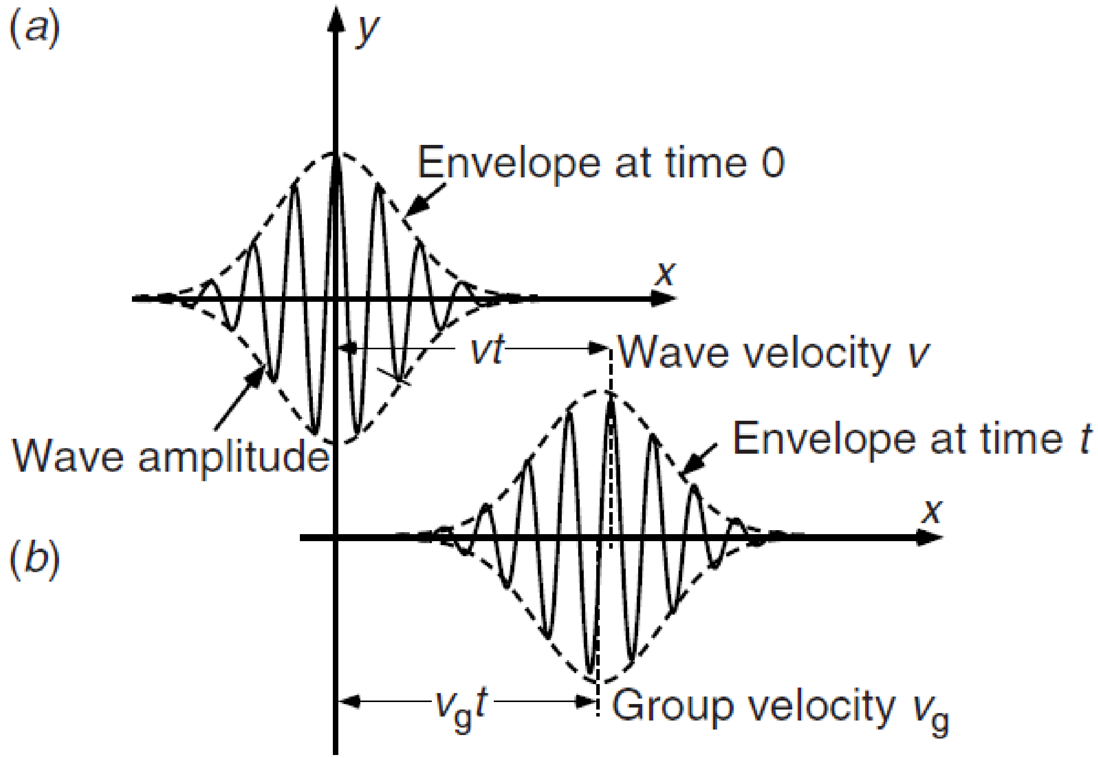
$$\psi(x,t) = \psi(x'(t),0) = \psi(x - vt, 0)$$

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx'(t)\right]}$$

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

Dispersión

(v: velocidad de fase y de grupo)



2) Medio Dispersivo Lineal ($\omega = kv_g + \alpha$)

(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

$$\omega t - kx = k\left(v_g t - x\right) + \alpha t = -kx''\left(t\right) + \alpha t$$

$$x''\left(t\right) = x - v_g t$$

$$\psi\left(x,t\right) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{\left(k-k_0\right)^2}{2\sigma^2} - jkx''\left(t\right) + j\alpha t\right]}$$

$$\psi\left(x,t\right) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{\left(k-k_0\right)^2}{2\sigma^2} + j\left(\omega t - kx\right)\right]}$$

2) Medio Dispersivo Lineal ($\omega = kv_g + \alpha$)

(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

$$\omega t - kx = k\left(v_g t - x\right) + \alpha t = -kx''\left(t\right) + \alpha t$$

$$x''\left(t\right) = x - v_g t$$

$$\psi\left(x,t\right) = \frac{e^{j\alpha t} A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{\left(k-k_0\right)^2}{2\sigma^2} - jkx''\left(t\right)\right]}$$

$$\psi\left(x,t\right) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{\left(k-k_0\right)^2}{2\sigma^2} + j\left(\omega t - kx\right)\right]}$$

2) Medio Dispersivo Lineal ($\omega = kv_g + \alpha$)

(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

$$\omega t - kx = k\left(v_g t - x\right) + \alpha t = -kx''(t) + \alpha t$$

$$x''(t) = x - v_g t$$

$$\psi(x,t) = \frac{e^{j\alpha t} A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx''(t)\right]}$$

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

2) Medio Dispersivo Lineal ($\omega = kv_g + \alpha$)

(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

$$\psi(x,t) = e^{j\alpha t}\psi(x''(t),0) = e^{j\alpha t}\psi(x - v_g t, 0)$$

$$\psi(x,t) = \frac{e^{j\alpha t} A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx''(t)\right]}$$

$$\psi(x,0) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} - jkx\right]}$$

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(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

$$\psi(x,t) = e^{j\alpha t}\psi(x''(t),0) = e^{j\alpha t}\psi(x - v_g t, 0)$$

$$\psi(x,t) = e^{j\alpha t} A e^{\left[-\frac{\sigma^2 x''^2}{2} - j(k_0 x'')\right]} =$$

$$= A e^{\left[-\frac{\sigma^2 x''^2}{2} - j(k_0 x'' - \alpha t)\right]} = A e^{-\frac{\sigma^2 x''^2}{2}} cos(k_0 x'' - \alpha t)$$

$$\psi(x,0) = A e^{\left[-\frac{\sigma^2 x^2}{2} - jk_0 x\right]} = A e^{-\frac{\sigma^2 x^2}{2}} cos(k_0 x)$$

2) Medio Dispersivo Lineal ($\omega = kv_g + \alpha$)

(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

$$\psi(x,t) = e^{j\alpha t}\psi(x''(t),0) = e^{j\alpha t}\psi(x - v_g t, 0)$$

$$\psi(x,t) = e^{j\alpha t} A e^{\left[-\frac{\sigma^2 x''^2}{2} - j(k_0 x'')\right]} =$$

$$= A e^{\left[-\frac{\sigma^2 x''^2}{2} - j(k_0 x'' - \alpha t)\right]} = A e^{-\frac{\sigma^2 x''^2}{2}} \cos(k_0 x'' - \alpha t)$$

$$\psi(x,0) = A e^{-\frac{\sigma^2 (x - v_g t)^2}{2}} \cos\left[k_0\left(x - \left(v_g + \frac{\alpha}{k_0}\right)t\right)\right]$$

2) Medio Dispersivo Lineal ($\omega = kv_g + \alpha$)

(v_g: velocidad de grupo $\neq v = v_g + \alpha/k$: velocidad de fase)

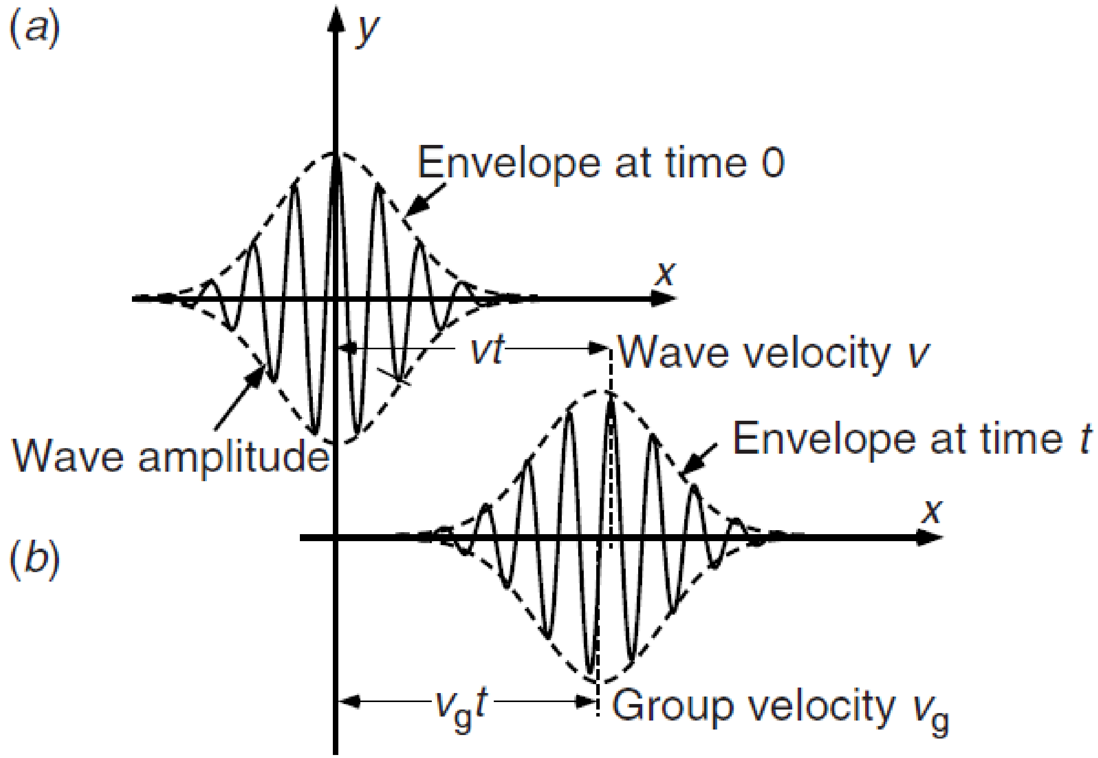
v_g: velocidad de grupo

$v = v_g + \alpha/k_0$: velocidad de fase)

$$\psi(x,0) = Ae^{-\frac{\sigma^2 (x - v_g t)^2}{2}} \cos\left[k_0\left(x - \overbrace{\left(v_g + \frac{\alpha}{k_0}\right)}t\right)\right]$$

Dispersión

(v_g: velocidad de grupo $\neq$ v: velocidad de fase)



3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k\text{-}k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$-\gamma(k,x,t) = \frac{(k-k_0)^2}{2\sigma^2} + jkx - j\left(kv_x + \alpha + \beta(k-k_0)^2\right)t$$

$$-\gamma(k,x,t) = (k-k_0)^2\left(\frac{1}{2\sigma^2} - j\beta t\right) + jk(x - v_x t) - j\alpha\, t$$

$$\gamma(k,x,t)$$

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k\text{-}k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

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$$-\gamma(k,x,t) = (k-k_0)^2\left(\frac{1}{2\sigma^2} - j\beta t\right) + jk(x - v_x t) - j\alpha\,t$$

$$-\gamma(k,x,t) = \frac{(k-k_0)^2}{2\sigma'^2} + jkx'''(t) - j\alpha\,t$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$-\gamma(k,x,t) = \frac{(k-k_0)^2}{2\sigma'^2} + j(k-k_0)x'''(t) + $$

$$+ jk_0 x'''(t) - j\alpha\, t$$

$$-\gamma(k,x,t) = \frac{(k-k_0)^2}{2\sigma'^2} + jkx'''(t) - j\alpha\, t$$

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$$+ jk_0 x'''(t) - j\alpha\, t$$

$$-\gamma(k,x,t) = \frac{(k-k_0)^2 + 2\sigma'^2\, j(k-k_0)x'''(t)}{2\sigma'^2} +$$

$$+ jk_0 x'''(t) - j\alpha\, t$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$-\gamma(k,x,t) = \frac{\left(k - k_0 + \sigma'^2\, jx'''(t)\right)^2}{2\sigma'^2} - \frac{\left(\sigma'^2\, jx'''(t)\right)^2}{2\sigma'^2} + \\ + jk_0 x'''(t) - j\alpha\, t$$

$$-\gamma(k,x,t) = \frac{\left(k - k_0\right)^2 + 2\sigma'^2\, j\left(k - k_0\right)x'''(t)}{2\sigma'^2} + \\ + jk_0 x'''(t) - j\alpha\, t$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$-\gamma(k,x,t) = \frac{\left(k-k_0+\sigma'^2\, jx'''(t)\right)^2}{2\sigma'^2} - \frac{\left(\sigma'^2\, jx'''(t)\right)^2}{2\sigma'^2} + $$
$$+ jk_0 x'''(t) - j\alpha\, t$$

$$-\gamma(k,x,t) = \frac{\left(k-k_0+\sigma'^2\, jx'''(t)\right)^2}{2\sigma'^2} + \frac{\sigma'^2\, x'''(t)^2}{2} + $$
$$+ jk_0 x'''(t) - j\alpha\, t$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k\text{-}k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\gamma(k,x,t)$$

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx)\right]}$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{\left(k-k_0+\sigma'^2 jx'''(t)\right)^2}{2\sigma'^2} - \frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]}$$

$$-\gamma(k,x,t) = \frac{\left(k-k_0+\sigma'^2 jx'''(t)\right)^2}{2\sigma'^2} + \frac{\sigma'^2 x'''(t)^2}{2} + jk_0 x'''(t) - j\alpha t$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

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$$\psi(x,t) = \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{\left(k-k_0+\sigma'^2 jx'''(t)\right)^2}{2\sigma'^2} - \frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]}$$

$$\psi(x,t) = \frac{e^{\left[-\frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]}}{\sqrt{2\pi\sigma^2}} A \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{\left(k-k_0+\sigma'^2 jx'''(t)\right)^2}{2\sigma'^2}\right]}$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$$\psi(x,0) = e^{\left[-\frac{\sigma^2 x^2}{2} - jk_0 x\right]} \frac{A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk e^{\left[-\frac{(k-k_0+\sigma^2 jx)^2}{2\sigma^2}\right]}$$

$$\psi(x,0) = Ae^{\left[-\frac{\sigma^2 x^2}{2} - jk_0 x\right]} = Ae^{-\frac{\sigma^2 x^2}{2}} \cos(k_0 x)$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = \frac{e^{\left[-\frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]} A}{\sqrt{2\pi\sigma^2}} \sqrt{2\pi\sigma'^2}$$

$$\psi(x,t) = \frac{e^{\left[-\frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]} A}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} dk\, e^{\left[-\frac{\left(k-k_0+\sigma'^2 jx'''(t)\right)^2}{2\sigma'^2}\right]}$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\,\frac{\sigma'}{\sigma}\,e^{\left[-\frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]}$$

$$\frac{1}{\sigma'^2} = \frac{1}{\sigma^2} - 2j\beta t \Rightarrow \sigma'^2 = \frac{1}{\frac{1}{\sigma^2} - 2j\beta t} = \frac{\sigma^2}{1 - 2j\beta\sigma^2 t}$$

$$\frac{1}{2\sigma'^2} = \frac{1}{2\sigma^2} - j\beta t; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k\text{-}k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\,\frac{\sigma'}{\sigma}\,e^{\left[-\frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]}$$

$$\frac{1}{\sigma'^2} = \frac{1}{\sigma^2} - 2j\beta t \Rightarrow \sigma'^2 = \frac{1}{\dfrac{1}{\sigma^2} - 2j\beta t} = \frac{\sigma^2}{1 - 2j\beta\sigma^2 t}$$

$$\sigma'^2 = \sigma^2\,\frac{1 + 2j\beta\sigma^2 t}{1 + 4\beta^2\sigma^4 t^2}\,; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma'^2 x'''(t)^2}{2} - jk_0 x'''(t) + j\alpha t\right]}$$

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4 t^2)}\right]}e^{\left[-\frac{2j\beta\sigma^4 t x'''(t)^2}{2(1+4\beta^2\sigma^4 t^2)} - jk_0 x'''(t) + j\alpha t\right]}$$

$$\sigma'^2 = \sigma^2\frac{1+2j\beta\sigma^2 t}{1+4\beta^2\sigma^4 t^2}; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4t^2)}\right]}e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{(1+4\beta^2\sigma^4t^2)}\right)x'''(t)+j\alpha t\right]}$$

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4t^2)}\right]}e^{\left[-\frac{2j\beta\sigma^4 t x'''(t)^2}{2(1+4\beta^2\sigma^4t^2)}-jk_0x'''(t)+j\alpha t\right]}$$

$$\sigma'^2 = \sigma^2\frac{1+2j\beta\sigma^2 t}{1+4\beta^2\sigma^4t^2}; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

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$$\psi(x,t) = A\,\frac{\sigma'}{\sigma}\,e^{\left[-\frac{\sigma^2 (x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}\,e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{\left(1+4\beta^2\sigma^4 t^2\right)}\right)x'''(t)+j\alpha\,t\right]}$$

$$\left|\psi(x,t)\right| = A\,\frac{\left|\sigma'\right|}{\sigma}\,e^{\left[-\frac{\sigma^2 (x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}$$

$$\sigma'^2 = \sigma^2\,\frac{1+2j\beta\sigma^2 t}{1+4\beta^2\sigma^4 t^2}\,; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

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($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\left|\sigma'^2\right| = \sigma^2 \frac{\left|1 + 2j\beta\sigma^2 t\right|}{1 + 4\beta^2\sigma^4 t^2} = \sigma^2 \frac{\sqrt{1 + 4\beta^2\sigma^4 t^2}}{1 + 4\beta^2\sigma^4 t^2} = \frac{\sigma^2}{\sqrt{1 + 4\beta^2\sigma^4 t^2}}$$

$$\left|\psi(x,t)\right| = A\,\frac{\left|\sigma'\right|}{\sigma}\, e^{\left[-\frac{\sigma^2 (x - v_x t)^2}{2\left(1 + 4\beta^2\sigma^4 t^2\right)}\right]}$$

$$\sigma'^2 = \sigma^2 \frac{1 + 2j\beta\sigma^2 t}{1 + 4\beta^2\sigma^4 t^2}\,; \qquad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\left|\sigma'^2\right| = \sigma^2 \frac{\left|1+2j\beta\sigma^2 t\right|}{1+4\beta^2\sigma^4 t^2} = \sigma^2 \frac{\sqrt{1+4\beta^2\sigma^4 t^2}}{1+4\beta^2\sigma^4 t^2} = \frac{\sigma^2}{\sqrt{1+4\beta^2\sigma^4 t^2}}$$

$$\left|\psi(x,t)\right| = A\frac{\left|\sigma'\right|}{\sigma} e^{\left[-\frac{\sigma^2 (x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}$$

$$\left|\sigma'\right| = \frac{\sigma}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}}\,; \quad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{\left(1+4\beta^2\sigma^4 t^2\right)}\right)x'''(t)+j\alpha\, t\right]}$$

$$\left|\psi(x,t)\right| = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}$$

$$\left|\sigma'\right| = \frac{\sigma}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}}\,; \qquad x'''(t) = x - v_x t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

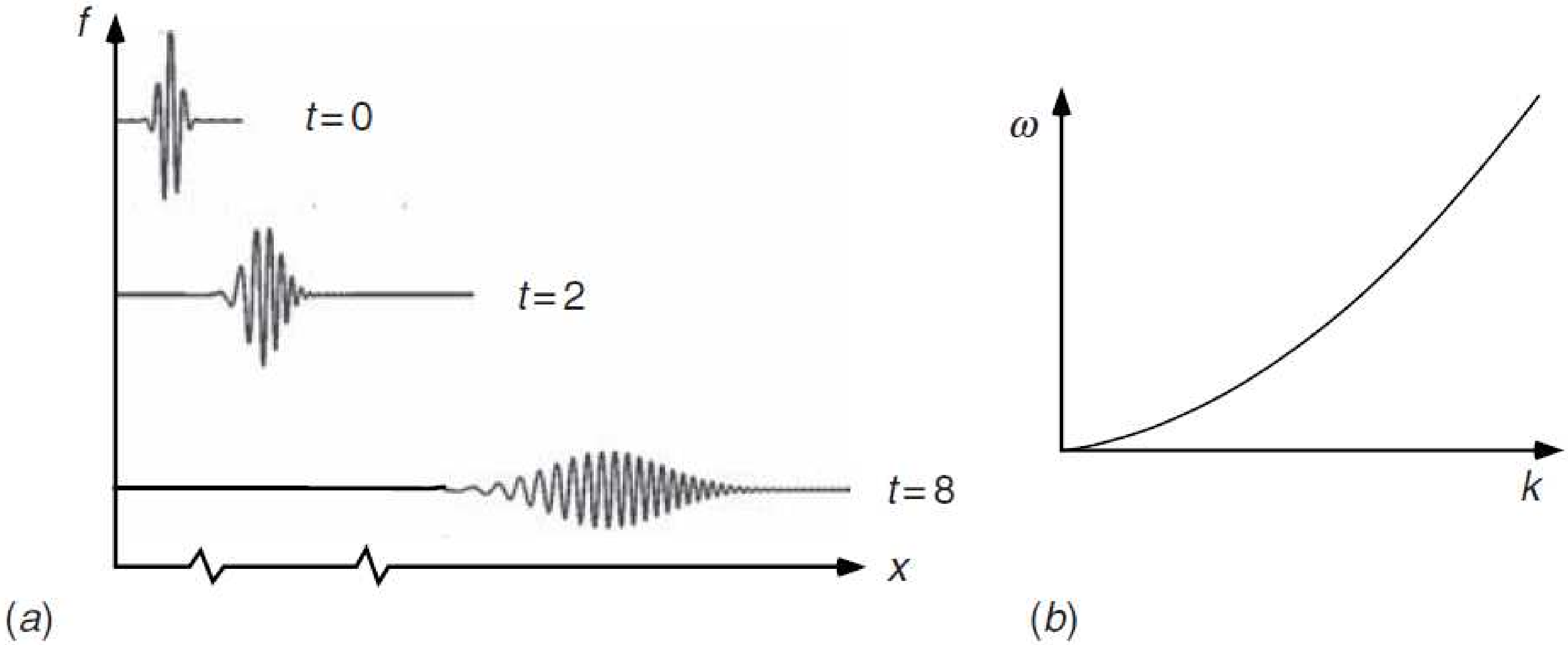
$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4t^2)}\right]}e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{(1+4\beta^2\sigma^4t^2)}\right)x'''(t)+j\alpha t\right]}$$

$$|\psi(x,t)| = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4t^2}}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4t^2)}\right]}$$

$v_x = v_g(k_0)$: velocidad de grupo

$$|\sigma'| = \frac{\sigma}{\sqrt[4]{1+4\beta^2\sigma^4t^2}}; \quad x'''(t) = x - v_x t$$

Distorsión



3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

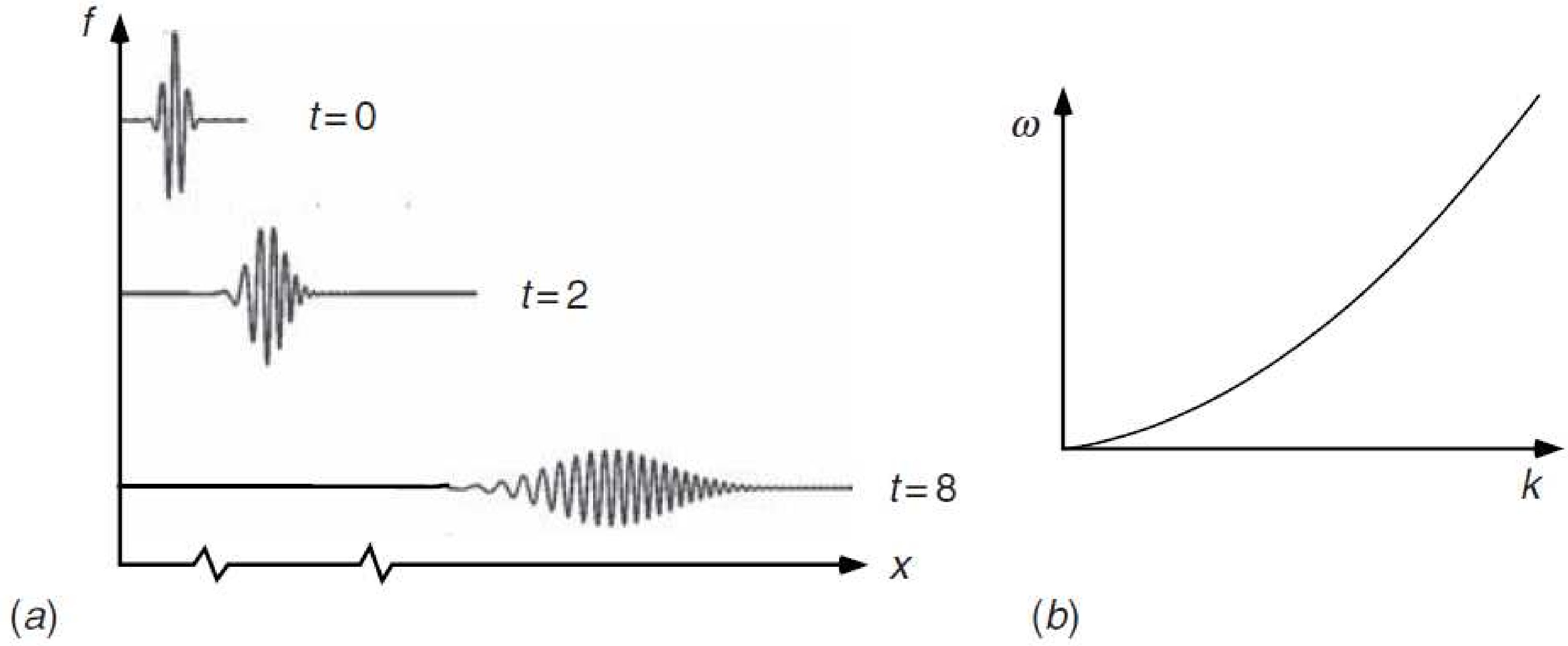
($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{\left(1+4\beta^2\sigma^4 t^2\right)}\right)x'''(t)+j\alpha t\right]}$$

$$\left|\psi(x,t)\right| = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}$$

$$A_{max}(t) = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}} \Rightarrow A_{max}(t)\downarrow si \quad t\uparrow$$

Distorsión



Ejemplo: Pulso Gaussiano

$$p(k) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-k_0)^2}{2\sigma^2}} \qquad A = \int_{-\infty}^{+\infty} dk\, p(k)$$

$\Delta k = \sigma$ **Ancho Espectral del Pulso**

$\Delta x = \frac{1}{\sigma}$ **Ancho Espacial del Pulso** $(\Delta k\, \Delta x = 1)$

$$\psi(x,0) = A e^{\left[-\frac{\sigma^2 x^2}{2} - jk_0 x\right]} = A e^{-\frac{\sigma^2 x^2}{2}} \cos(k_0 x)$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

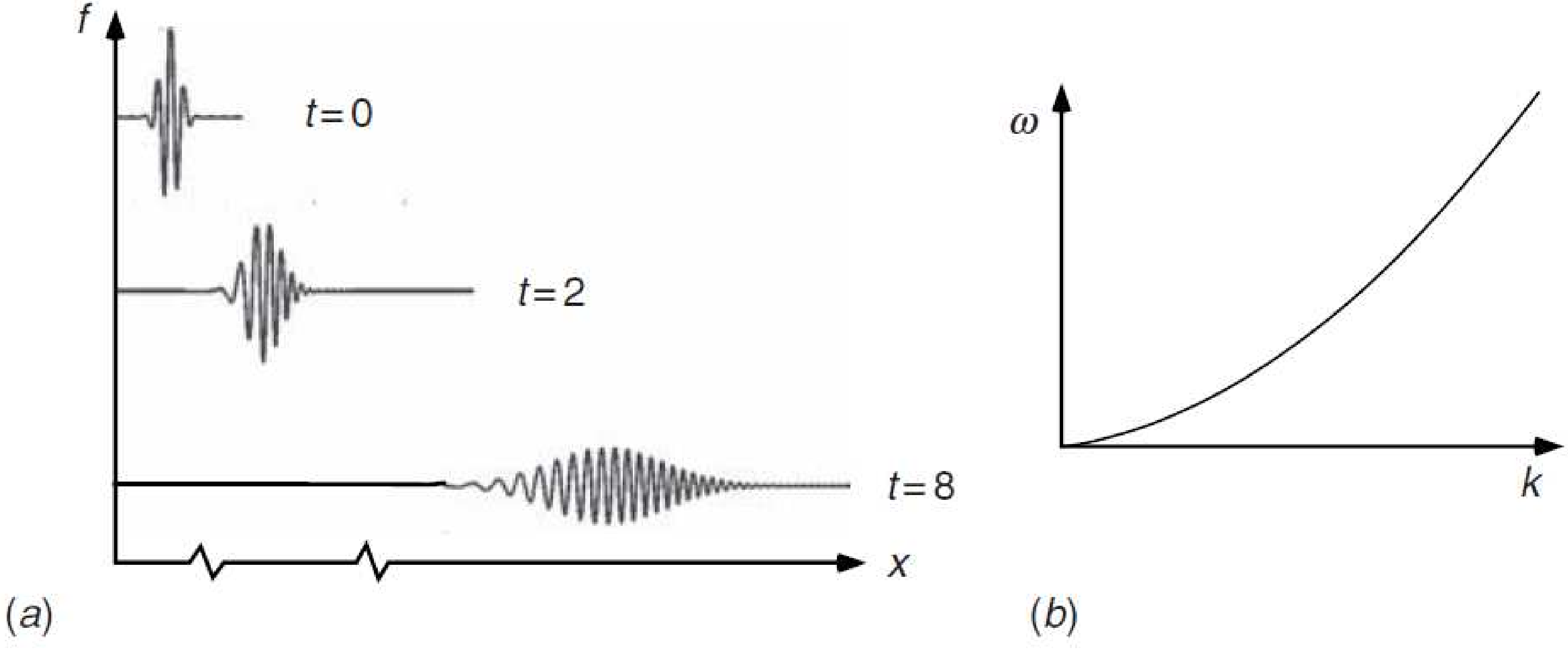
($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\,\frac{\sigma'}{\sigma}\,e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4 t^2)}\right]}\,e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{(1+4\beta^2\sigma^4 t^2)}\right)x'''(t)+j\alpha t\right]}$$

$$|\psi(x,t)| = A\,\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}}\,e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4 t^2)}\right]}$$

$$(\Delta x'''(t))^2 = \frac{1+4\beta^2\sigma^4 t^2}{\sigma^2} \Rightarrow \Delta x'''(t)\uparrow si \quad t\uparrow$$

Distorsión



3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4 t^2)}\right]}e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{(1+4\beta^2\sigma^4 t^2)}\right)x'''(t)+j\alpha t\right]}$$

$$\left|\psi(x,t)\right| = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4 t^2)}\right]}$$

$$\left(\Delta x'''(t)\right)^2 = \frac{1+4\beta^2\sigma^4 t^2}{\sigma^2} \Rightarrow \Delta x'''(t)\uparrow si \quad t\uparrow$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\frac{\sigma'}{\sigma}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4t^2)}\right]}e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{(1+4\beta^2\sigma^4t^2)}\right)x'''(t)+j\alpha\, t\right]}$$

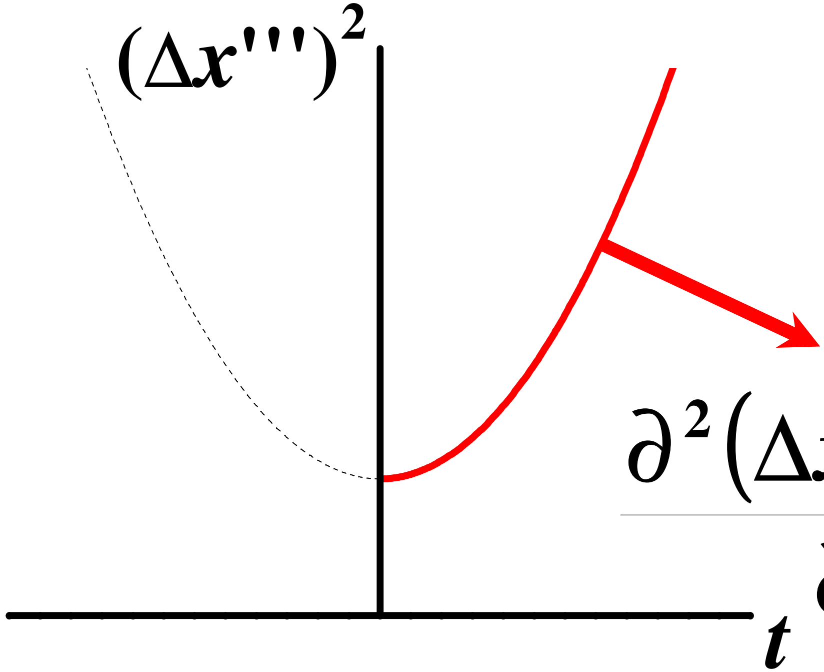
$$|\psi(x,t)| = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4t^2}}e^{\left[-\frac{\sigma^2(x-v_x t)^2}{2(1+4\beta^2\sigma^4t^2)}\right]}$$

$$(\Delta x'''(t))^2 = (\Delta x)^2 + \frac{4\beta^2t^2}{(\Delta x)^2}$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

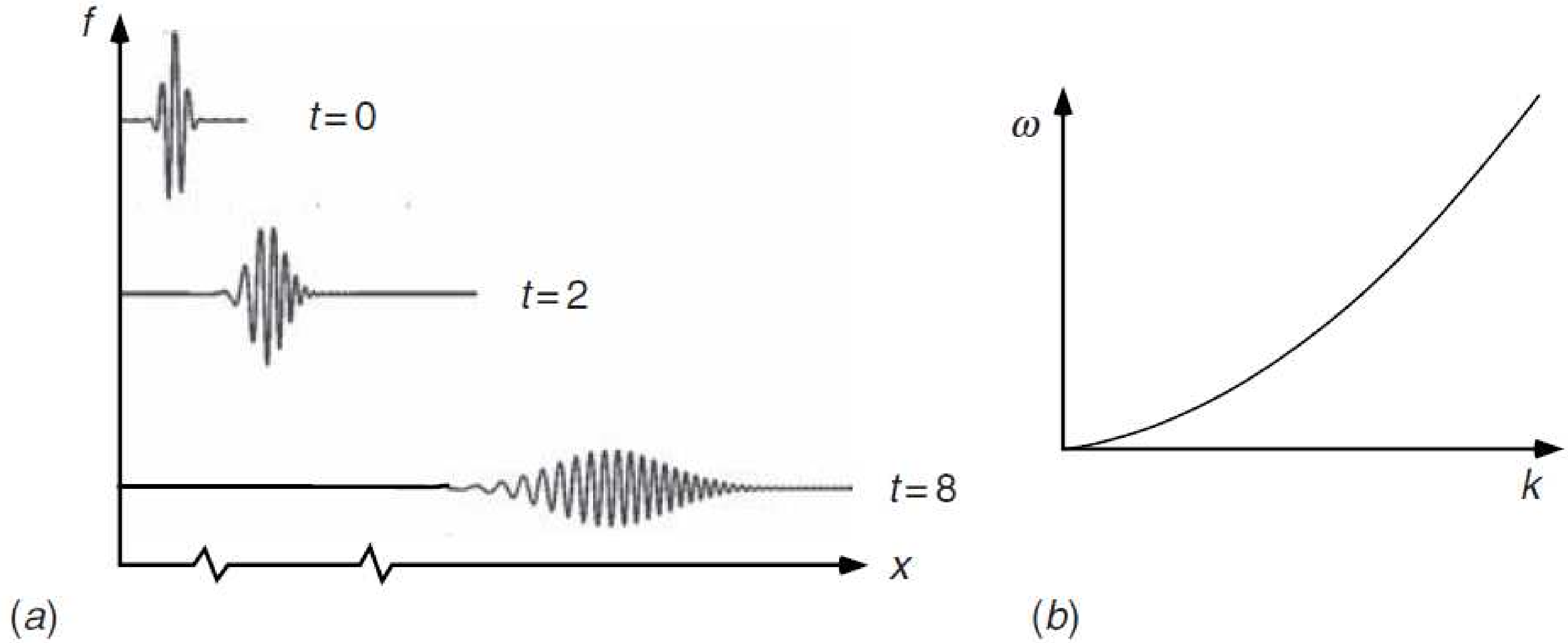
($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)



$$\frac{\partial^2 (\Delta x'''(t))^2}{\partial t^2} = \frac{8\beta^2}{(\Delta x)^2}$$

$$(\Delta x'''(t))^2 = (\Delta x)^2 + \frac{4\beta^2 t^2}{(\Delta x)^2}$$

Distorsión



3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$): velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\psi(x,t) = A\,\frac{\sigma'}{\sigma}\,e^{\left[-\frac{\sigma^2 (x-v_x t)^2}{2\left(1+4\beta^2\sigma^4 t^2\right)}\right]}\,e^{\left[-j\left(k_0+\frac{\beta\sigma^4 t x'''(t)}{\left(1+4\beta^2\sigma^4 t^2\right)}\right)x'''(t)+j\alpha\,t\right]}$$

$$\beta = \frac{1}{2}\,\frac{d^2\omega}{dk^2}$$

$$\textbf{Chirping :}\quad k_{eq} = k_0 + \frac{\beta\sigma^4 t x'''(t)}{\left(1+4\beta^2\sigma^4 t^2\right)}$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k\text{-}k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\omega = a + bk + ck^2$$

$$\gamma(k,x,t) = -\frac{(k-k_0)^2}{2\sigma^2} + j(\omega t - kx) = -\frac{(k-k_0)^2}{2\sigma^2} + j\left[\left(a+bk+ck^2\right)t - kx\right] =$$

$$= -\frac{(k-k_0)^2}{2\sigma^2} + jck^2t - jk(x-bt) + jat =$$

$$= -\frac{(k-k_0)^2}{2\sigma^2} + jc(k-k_0)^2 t + 2jckk_0t - jck_0^{\,2}t - jk(x-bt) + jat \Rightarrow$$

$$\gamma(k,x,t) = -(k-k_0)^2\left(\frac{1}{2\sigma^2} - jct\right) - jk\left[x-(b+2ck_0)\,t\right] + j\left(a-ck_0^{\,2}\right)t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)
($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$$\omega = a + bk + ck^2$$

$$-\gamma(k,x,t) = (k-k_0)^2\left(\frac{1}{2\sigma^2} - j\beta t\right) + jk(x - v_x t) - j\alpha\, t$$

$$\beta \to c = \frac{1}{2}\frac{d^2\omega}{dk^2} \qquad v_x \to b + 2ck_0 = \left.\frac{d\omega}{dk}\right|_{k=k_0}$$

$$\alpha \to a - ck_0^{\,2} \sim a \to \alpha + \beta k_0^{\,2} = \omega(0)$$

$$\gamma(k,x,t) = -(k-k_0)^2\left(\frac{1}{2\sigma^2} - jct\right) - jk\left[x - (b+2ck_0)\,t\right] + j\left(a - ck_0^{\,2}\right)t$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$\beta > 0$ Dispersión Anómala

$\beta < 0$ Dispersión Normal

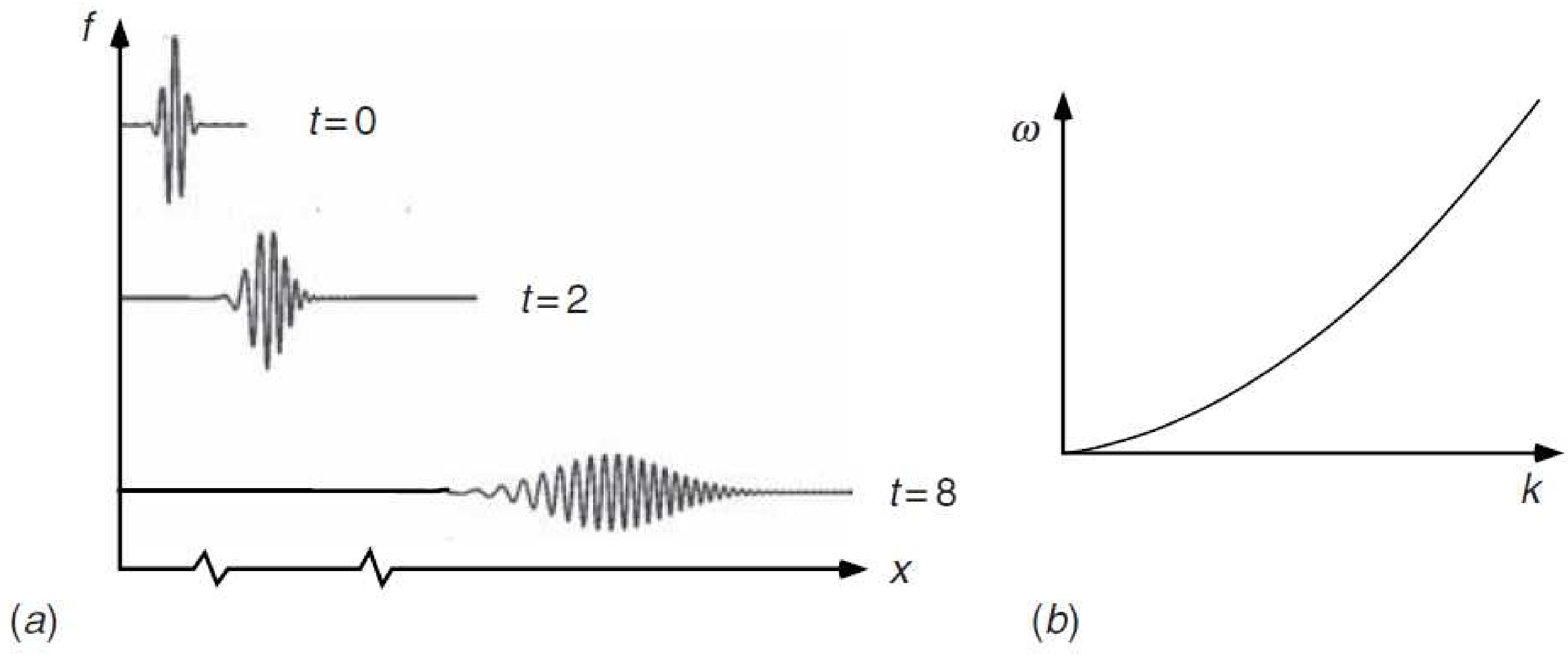
$$k_{eq} = \frac{2\pi}{\lambda_{eq}}; \quad k_0 = \frac{2\pi}{\lambda_0}$$

$$\beta > 0: x''' > 0, t > 0 \Rightarrow k_{eq} > k_0 \sim \lambda_{eq} < \lambda_0$$

$$x''' < 0, t > 0 \Rightarrow k_{eq} < k_0 \sim \lambda_{eq} > \lambda_0$$

$$\text{Chirping}: \quad k_{eq} = k_0 + \frac{\beta\sigma^4 t x'''(t)}{\left(1 + 4\beta^2\sigma^4 t^2\right)}$$

Distorsión



3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

$\beta > 0$ **Dispersión Anómala**

$\beta < 0$ **Dispersión Normal**

$$k_{eq} = \frac{2\pi}{\lambda_{eq}}; \quad k_0 = \frac{2\pi}{\lambda_0}$$

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$$\textbf{Chirping :} \quad k_{eq} = k_0 + \frac{\beta\sigma^4 t x'''(t)}{\left(1 + 4\beta^2\sigma^4 t^2\right)}$$

3) Medio Dispersivo Cuadrático

($\omega = kv_x + \alpha + \beta\,(k-k_0)^2$)

($v_x = v_g(k_0)$: velocidad de grupo en $k_0 \neq v = \omega/k$: velocidad de fase)

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$$k_{eq} = \frac{2\pi}{\lambda_{eq}}; \quad k_0 = \frac{2\pi}{\lambda_0}$$

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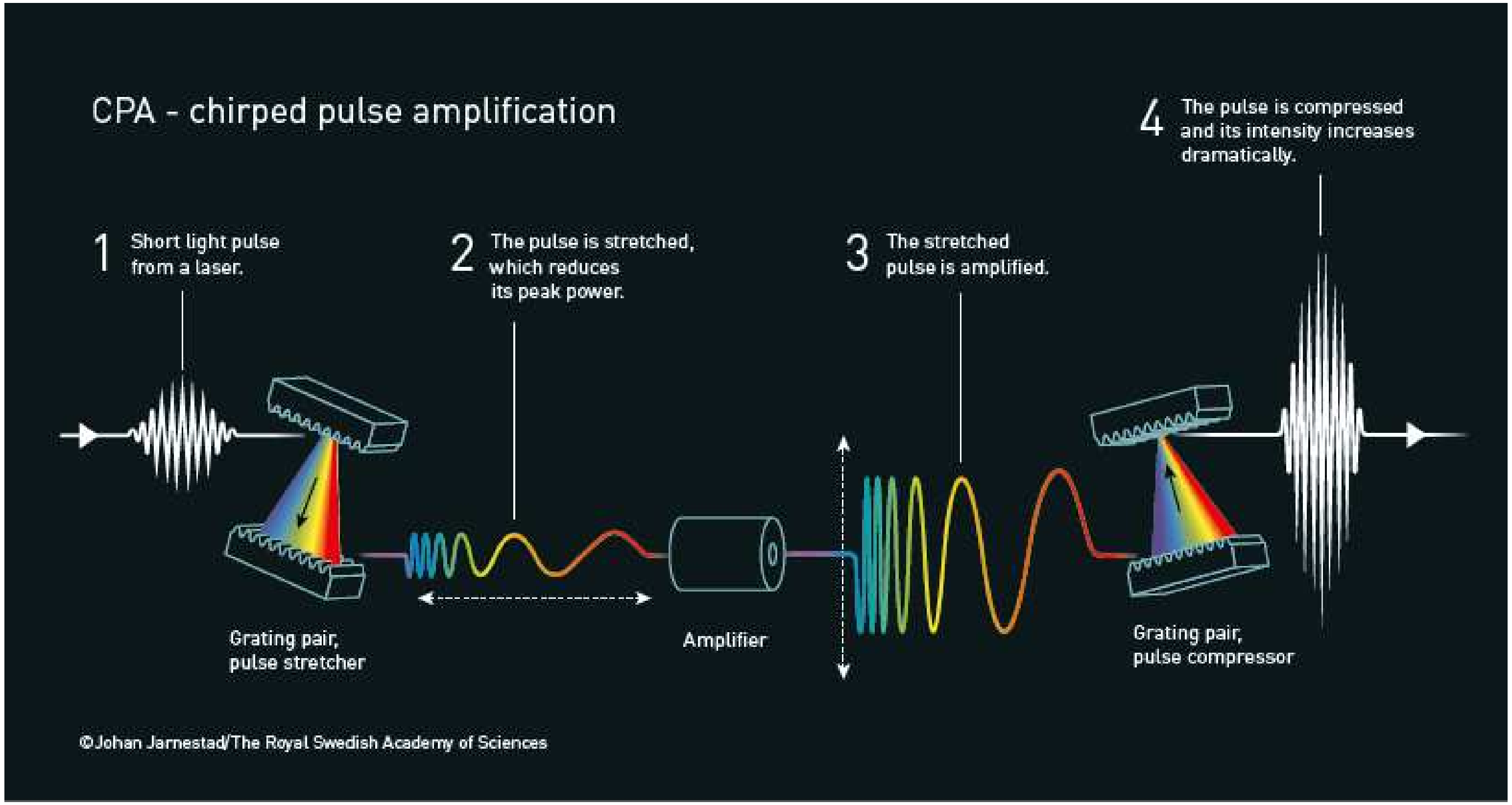
$$\textbf{Chirping}: \quad k_{eq} = k_0 + \frac{\beta\sigma^4 t x'''(t)}{\left(1 + 4\beta^2\sigma^4 t^2\right)}$$

Nobel Física 2018: Chirped Pulse Amplification



Nobel Física 2018: Chirped Pulse Amplification

$$A_{max}(t) = A\frac{1}{\sqrt[4]{1+4\beta^2\sigma^4 t^2}} \Rightarrow A_{max}(t)^2 = \frac{A^2}{\sqrt{1+4\beta^2\sigma^4 t^2}}$$

$$P_{max}(t) = \frac{P_{max}(0)}{\sigma\Delta x'''(t)} \Rightarrow P_{max}(t) = \frac{\Delta x}{\Delta x'''(t)} P_{max}(0)$$

$$\left(\Delta x'''(t)\right)^2 = \frac{1+4\beta^2\sigma^4 t^2}{\sigma^2}$$