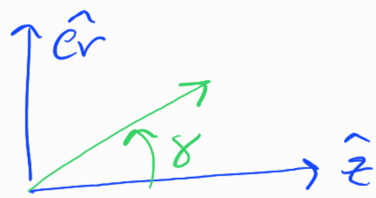
Óptica, Examen, 10 febrero de 2022

Ej 1) a) $$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right)=\nabla n$$



proyectamos la ec. según los versores (fijos) $\hat{z}, \hat{e_r}$!

$\hat{z}$) $$\underbrace{\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right)\cdot\hat{z}}=\nabla n\cdot\hat{z}=\frac{\partial n}{\partial z}=0$$

$$=\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\cdot\hat{z}\right)=\frac{d}{ds}\left(n\frac{dz}{ds}\right)=0 \;!\; n\underbrace{\frac{dz}{ds}}_{\cos\gamma}=C$$

($\hat{z}$ fijo)

$\hat{e_r}$) $$\underbrace{\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right)\cdot\hat{e_r}}=\nabla n\cdot\hat{e_r}=\frac{\partial n}{\partial r}$$

$$=\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\cdot\hat{e_r}\right)=\frac{d}{ds}\left(n\frac{dr}{ds}\right)=\frac{d}{ds}\left(\underbrace{n\frac{dr}{dz}\frac{dz}{ds}}_{C\frac{dr}{dz}}\right)=\frac{\partial n}{\partial r}$$

$\hat{e_r}$ fijo, a efectos de un rayo meridional

$$\Rightarrow \underbrace{\frac{d}{ds}\left(C\frac{dr}{dz}\right)}_{C\underbrace{\frac{d}{dz}\left(\frac{dr}{dz}\right)}_{\frac{d^2r}{dz^2}}\underbrace{\frac{dz}{ds}}_{C/n}}=\frac{\partial n}{\partial r} \;\therefore\; \frac{C^2}{n}\frac{d^2r}{dz^2}=\frac{\partial n}{\partial r} \;\therefore$$

$$\frac{d^2r}{dz^2}=\frac{1}{C^2}\underbrace{n\frac{\partial n}{\partial r}}_{\frac{1}{2}\frac{\partial(n^2)}{\partial r}}$$

$$\Rightarrow \boxed{\frac{d^2r}{dz^2}=\frac{1}{2C^2}\frac{\partial n^2}{\partial r}}$$

b) $n^2 = n_0^2(1-\alpha^2 r^2)$: $\dfrac{\partial n^2}{\partial r} = -2 n_0^2 \alpha^2 r$

$\mathbb{C} = n(r_0)\cos\gamma_0 = n(r_0)$

$\gamma_0 = 0$, todos los rayos paralelos al eje

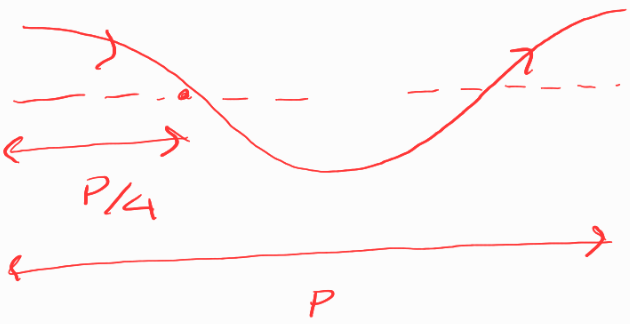
$$\Rightarrow \frac{d^2 r}{dz^2} = \frac{1}{\cancel{2}\, \underbrace{n^2(r_0)}_{\cancel{n_0^2}(1-\alpha^2 r_0^2)}} \left(-\cancel{2}\cancel{n_0^2}\alpha^2 r\right)$$

$$\frac{d^2 r}{dz^2} + \Bigg(\underbrace{\frac{\alpha^2}{1-\alpha^2 r_0^2}}_{\approx 1\ (|\alpha r_0| \ll 1,\ \text{haz fino})}\Bigg) r = 0 \ : \ \frac{d^2 r}{dz^2} + \alpha^2 r = 0$$

con $r(0) = r_0$

$\dfrac{dr}{dz}(0) = 0$

$\Rightarrow r(z) = r_0 \cos(\alpha z)$: cada rayo tiene una trayectoria de esta forma, con periodo espacial $P = 2\pi/\alpha$; el primer valor de z para el cual todos los rayos pasan por el eje es : $z = P/4 \Rightarrow \boxed{L_{min} = \dfrac{P}{4}}$



Ej 2) $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \underbrace{C_R \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix}}_{\text{circular dcha.}} + \underbrace{C_L \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ +j \end{pmatrix}}_{\text{circular izq}}$,

donde los coeficientes de la combinación verifican:

$$\left.\begin{aligned} E_x &= \frac{1}{\sqrt{2}}(C_R + C_L) \\ E_y &= \frac{-j}{\sqrt{2}}(C_R - C_L) \end{aligned}\right| \Rightarrow \begin{cases} C_R = \dfrac{E_x + j E_y}{\sqrt{2}} \\ C_L = \dfrac{E_x - j E_y}{\sqrt{2}} \end{cases}$$

b) $$\underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{J_D} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \left(C_R \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} + C_L \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ j \end{pmatrix} \right)$$

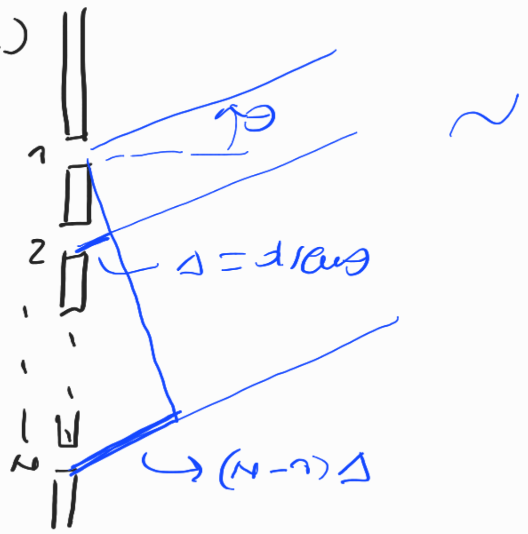
$$= t_R\, C_R \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -j \end{pmatrix} + t_L\, C_L \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ +j \end{pmatrix} =$$

$$= \frac{t_R}{2} (E_x + j E_y) \begin{pmatrix} 1 \\ -j \end{pmatrix} + \frac{t_L}{2} (E_x - j E_y) \begin{pmatrix} 1 \\ +j \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}(t_R + t_L) E_x + \frac{j}{2}(t_R - t_L) E_y \\ -\frac{j}{2}(t_R - t_L) E_x + \frac{1}{2}(t_R + t_L) E_y \end{pmatrix}$$

$$\Rightarrow \boxed{ J_D = \begin{pmatrix} \frac{1}{2}(t_R + t_L) & \frac{j}{2}(t_R - t_L) \\ -\frac{j}{2}(t_R - t_L) & \frac{1}{2}(t_R + t_L) \end{pmatrix} }$$

Ej 3) a)



• P

$$E_P = E_1 + E_2 + \cdots + E_N$$

$$E_P = \frac{A_0^{(1)}}{r_1} e^{j(\omega t - K r_1)} + \frac{A_0^{(2)}}{r_2} e^{j(\omega t - K r_2)} + \cdots + \frac{A_0^{(N)}}{r_N} e^{j(\omega t - K r_N)}$$

$$\left(K = \frac{2\pi}{\lambda}\right)$$

$$\simeq \frac{A_0^{(1)}}{r_1} e^{j(\omega t - K r_1)} \underbrace{\left[1 + e^{-jK \underbrace{(r_2 - r_1)}_{\Delta}} + \cdots e^{-jK \underbrace{(r_N - r_1)}_{(N-1)\Delta}}\right]}$$

$$\sum_{n=0}^{N-1} \left(e^{-jK\Delta}\right)^n = \frac{1 - e^{-jKN\Delta}}{1 - e^{-jK\Delta}} =$$

$$= \frac{e^{-jKN\Delta/2}}{e^{-jK\Delta/2}} \underbrace{\left(\frac{e^{jKN\Delta/2} - e^{-jKN\Delta/2}}{e^{jK\Delta/2} - e^{-jK\Delta/2}}\right)}_{\frac{\operatorname{sen}(NK\Delta/2)}{\operatorname{sen}(K\Delta/2)}}$$

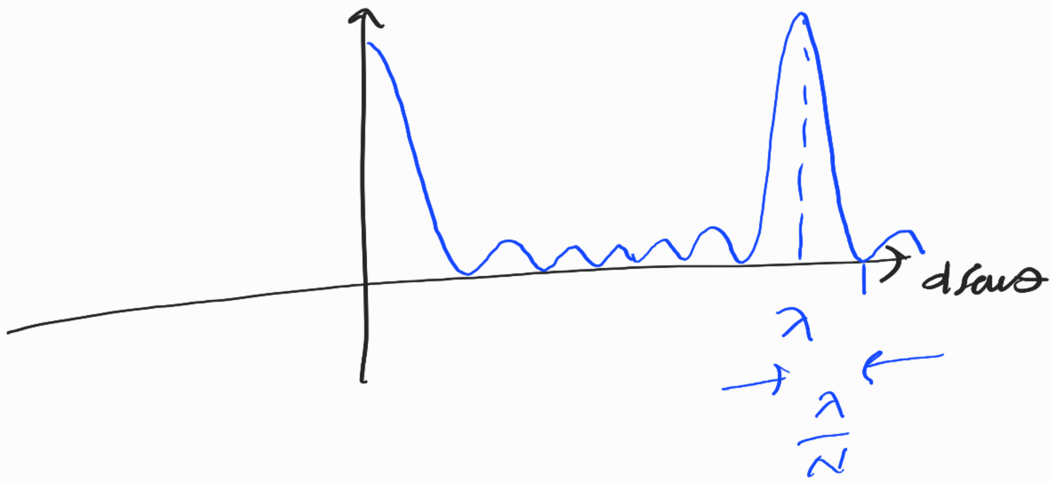
$$\Rightarrow I_P \propto E_P E_P^* = I_0 \frac{\operatorname{sen}^2(NK\Delta/2)}{\operatorname{sen}^2(K\Delta/2)}$$

(intensidad de una fuente individual)

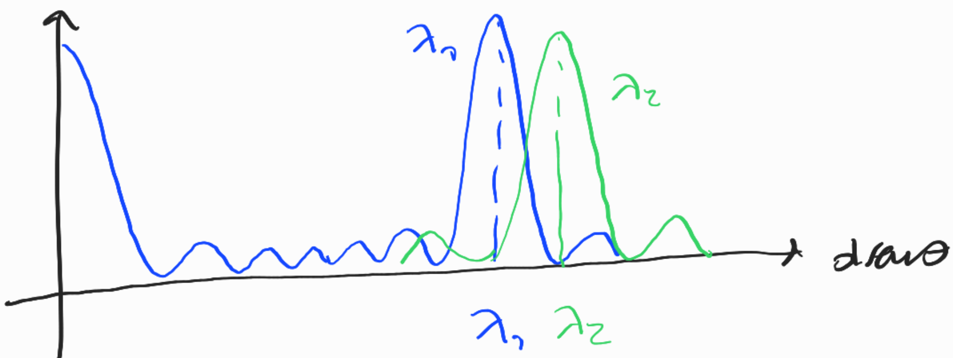
b) $I_P(\theta) = I_0 \left(\dfrac{\sin(N\pi d \sin\theta/\lambda)}{\sin(\pi d \sin\theta/\lambda)}\right)^2$

máximos principales: $d\sin\theta/\lambda = m$ (entero)

mínimos : $d\sin\theta/\lambda = \dfrac{n}{N}$, $n \neq mN$



• Con dos longitudes de onda tenemos:



Para poder separarlas necesito que : $\lambda_2 - \lambda_1 \geq \dfrac{\lambda_1}{N}$

(es decir, que los máximos estén más separados que el semi-ancho característico de λ_1)

$\Longleftrightarrow \quad N \geq \dfrac{\lambda_1}{\lambda_2 - \lambda_1} \sim \boxed{289}$