SyC - Hoja 5 - Ej. 1 (solamente parte a)

Valores propios de la matriz A

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix},$$

SyC - Hoja 5 - Ej. 1 (solamente parte a)

Valores propios de la matriz A

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix},$$

$$s\mathrm{I} - A = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{bmatrix}$$

SyC - Hoja 5 - Ej. 1 (solamente parte a)

Valores propios de la matriz A

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix},$$

$$sI - A = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{bmatrix}$$

$$p_A(s) := \det(sI - A) = s^3 + 6s^2 + 11s + 6 = (s+1)(s+2)(s+3)$$

$$\lambda_1 := -1 \qquad \lambda_3 := -2 \qquad \lambda_3 := -3$$

SyC - Hoja 5 - Ej. 1

Vectores propios de la matriz A

$$(\lambda_1 \mathrm{I} - A)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \lambda_1 & -1 & 0 \\ 0 & \lambda_1 & -1 \\ 6 & 11 & \lambda_1 + 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

SyC - Hoja 5 - Ej. 1

Vectores propios de la matriz A

$$(\lambda_1 \mathrm{I} - A)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \lambda_1 & -1 & 0 \\ 0 & \lambda_1 & -1 \\ 6 & 11 & \lambda_1 + 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x - y = 0 \\ -y - z = 0 \\ 6x + 11y + 5z = 0 \end{cases} \qquad \begin{cases} y = -x \\ z = -y = x \\ 6x + 11(-x) + 5x = 0 \end{cases}$$

SyC - Hoja 5 - Ej. 1

Vectores propios de la matriz A

$$(\lambda_1 \mathrm{I} - A) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \lambda_1 & -1 & 0 \\ 0 & \lambda_1 & -1 \\ 6 & 11 & \lambda_1 + 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$v_1 := \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

SyC - Hoja 5 - Ej. 1

Vectores propios de la matriz A

$$(\lambda_1 \mathrm{I} - A) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \lambda_1 & -1 & 0 \\ 0 & \lambda_1 & -1 \\ 6 & 11 & \lambda_1 + 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x - y = 0 \\ -y - z = 0 \\ 6x + 11y + 5z = 0 \end{cases} \qquad \begin{cases} y = -x \\ z = -y = x \\ 6x + 11(-x) + 5x = 0 \end{cases}$$

$$v_1 := \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

De forma análoga, se obtienen:

$$v_2 := \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}, \qquad v_3 := \begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$$

Diagonalizacíón

Sean

$$P := \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}, \qquad D := \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Diagonalizacíón

Sean

$$P := \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}, \qquad D := \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Se tiene que:

$$AP = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = PD$$

Diagonalizacíón

Sean

$$P := \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}, \qquad D := \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Se tiene que:

$$AP = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = PD$$

Entonces

$$AP = PD, \qquad A = PDP^{-1}$$

Cálculo de e^{At} por el método de diagonalización

$$\mathrm{e}^{At} = \sum_{k=0}^{\infty} \frac{A^k t^k}{k!} = \sum_{k=0}^{\infty} \frac{PD^k P^{-1} t^k}{k!} = P \sum_{k=0}^{\infty} \frac{D^k t^k}{k!} P^{-1} = P\mathrm{e}^{Dt} P^{-1}$$

$$\boxed{\mathrm{e}^{At} = P\mathrm{e}^{Dt} P^{-1}}$$

Cálculo de e^{At} por el método de diagonalización

$$\mathrm{e}^{At} = \sum_{k=0}^{\infty} \frac{A^k t^k}{k!} = \sum_{k=0}^{\infty} \frac{PD^k P^{-1} t^k}{k!} = P \sum_{k=0}^{\infty} \frac{D^k t^k}{k!} P^{-1} = P\mathrm{e}^{Dt} P^{-1}$$

$$\boxed{\mathrm{e}^{At} = P\mathrm{e}^{Dt} P^{-1}}$$

$$\mathrm{e}^{Dt} = \begin{bmatrix} \mathrm{e}^{\lambda_1 t} & 0 & 0 \\ 0 & \mathrm{e}^{\lambda_2 t} & 0 \\ 0 & 0 & \mathrm{e}^{\lambda_3 t} \end{bmatrix} = \begin{bmatrix} \mathrm{e}^{-t} & 0 & 0 \\ 0 & \mathrm{e}^{-2t} & 0 \\ 0 & 0 & \mathrm{e}^{-3t} \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}, \qquad \det(P) = -2, \qquad \operatorname{cof}(P) = \begin{bmatrix} -6 & 6 & -2 \\ -5 & 8 & -3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$P^{-1} = \frac{\operatorname{adj}(P)}{\det(P)} = \frac{(\operatorname{cof}(P))^{\intercal}}{\det(P)} = \begin{bmatrix} 3 & \frac{5}{2} & \frac{1}{2} \\ -3 & -4 & -1 \\ 1 & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Cálculo de e^{At} por el método de diagonalización

$$
\mathrm{e}^{At} = P\mathrm{e}^{Dt}P^{-1}
$$

$$
= \begin{bmatrix} 3e_1(t) - 3e_2(t) + e_3(t) & \frac{5}{2}e_1(t) - 4e_2(t) + \frac{3}{2}e_3(t) & \frac{1}{2}e_1(t) - e_2(t) + \frac{1}{2}e_3(t) \\ -3e_1(t) + 6e_2(t) - 3e_3(t) & -\frac{5}{2}e_1(t) + 8e_2(t) - \frac{9}{2}e_3(t) & -\frac{1}{2}e_1(t) + 2e_2(t) - \frac{3}{2}e_3(t) \\ 3e_1(t) - 12e_2(t) + 9e_3(t) & \frac{5}{2}e_1(t) - 16e_2(t) + \frac{27}{2}e_3(t) & \frac{1}{2}e_1(t) - 4e_2(t) + \frac{9}{2}e_3(t) \end{bmatrix}
$$

donde

$$
\begin{aligned}
e_1(t) &= \mathrm{e}^{\lambda_1 t} = \mathrm{e}^{-t} \\
e_2(t) &= \mathrm{e}^{\lambda_2 t} = \mathrm{e}^{-2t} \\
e_3(t) &= \mathrm{e}^{\lambda_3 t} = \mathrm{e}^{-3t}
\end{aligned}
$$

SyC - Hoja 5 - Ej. 2 (solamente para la 1ra. matriz)

2) Calcule $\mathbf{e}^{\mathbf{At}}$ para los siguientes valores de A:

$$\begin{bmatrix} 3 & 0 & -2 \\ -1 & 2 & 2 \\ -1 & 1 & 4 \end{bmatrix}, \qquad \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}, \qquad \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Sugerencia: usar diferentes métodos (Laplace, Cayley-Hamilton).

Polinomio característico de la matriz A

$$A \in \mathcal{M}_{n\times n}, \quad n = 3, \quad A = \begin{bmatrix} 3 & 0 & -2 \\ -1 & 2 & 2 \\ -1 & 1 & 4 \end{bmatrix}, \quad s\mathrm{I}-A = \begin{bmatrix} s-3 & 0 & 2 \\ 1 & s-2 & -2 \\ 1 & -1 & s-4 \end{bmatrix}$$

$$\begin{aligned} p_A(s) := \det(s\mathrm{I} - A) &= (s-3)(s-2)(s-4) - 2 - 2(s-2) - 2(s-3) \\ &= (s-3)(s-2)(s-4) - 2(1 + s - 2 + s - 3) \\ &= (s-3)(s-2)(s-4) - 2(2s-4) \\ &= (s-3)(s-2)(s-4) - 4(s-2) \\ &= (s-2)[(s-3)(s-4) - 4] \\ &= (s-2)\left(s^2 - 7s + 8\right) \\ &= (s-\lambda_1)(s-\lambda_2)(s-\lambda_3) \end{aligned}$$

$$\lambda_1 := 2 \qquad \lambda_2 := \frac{7+\sqrt{17}}{2} \approx 5,5616 \qquad \lambda_3 := \frac{7-\sqrt{17}}{2} \approx 1,4384$$

$$\mathrm{gr}\,[p_A] = n = 3$$

Cálculo de e^{At} por Cayley-Hamiltion

$$\mathrm{e}^{At} = R_t\left(A\right) = \alpha_0(t) + \alpha_1(t)A + \cdots + \alpha_{n-1}(t)A^{n-1}$$

$$\mathrm{e}^{\lambda t} = R_t\left(\lambda\right) = \alpha_0(t) + \alpha_1(t)\lambda + \cdots + \alpha_{n-1}(t)\lambda^{n-1}$$

Cálculo de e^{At} por Cayley-Hamiltion

$$\mathrm{e}^{At} = R_t\left(A\right) = \alpha_0(t) + \alpha_1(t)A + \cdots + \alpha_{n-1}(t)A^{n-1}$$

$$\mathrm{e}^{\lambda t} = R_t\left(\lambda\right) = \alpha_0(t) + \alpha_1(t)\lambda + \cdots + \alpha_{n-1}(t)\lambda^{n-1}$$

$$\begin{cases} \mathrm{e}^{\lambda_1 t} = \alpha_0(t) + \lambda_1\alpha_1(t) + \lambda_1^2\alpha_2(t) \\ \mathrm{e}^{\lambda_2 t} = \alpha_0(t) + \lambda_2\alpha_1(t) + \lambda_2^2\alpha_2(t) \\ \mathrm{e}^{\lambda_3 t} = \alpha_0(t) + \lambda_3\alpha_1(t) + \lambda_3^2\alpha_2(t) \end{cases}$$

Cálculo de e^{At} por Cayley-Hamiltion

$$\mathrm{e}^{At} = R_t\left(A\right) = \alpha_0(t) + \alpha_1(t)A + \cdots + \alpha_{n-1}(t)A^{n-1}$$

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$$\begin{cases} \mathrm{e}^{\lambda_1 t} = \alpha_0(t) + \lambda_1\alpha_1(t) + \lambda_1^2\alpha_2(t) \\ \mathrm{e}^{\lambda_2 t} = \alpha_0(t) + \lambda_2\alpha_1(t) + \lambda_2^2\alpha_2(t) \\ \mathrm{e}^{\lambda_3 t} = \alpha_0(t) + \lambda_3\alpha_1(t) + \lambda_3^2\alpha_2(t) \end{cases}$$

Resolviendo el anterior sistema de ecuaciones:

$$\begin{cases} \alpha_0(t) \approx -4\mathrm{e}^{\lambda_1 t} + 0,1959\mathrm{e}^{\lambda_2 t} + 4,8041\mathrm{e}^{\lambda_3 t} \\ \alpha_1(t) \approx 3,5\mathrm{e}^{\lambda_1 t} - 0,2342\mathrm{e}^{\lambda_2 t} - 3,2658\mathrm{e}^{\lambda_3 t} \\ \alpha_2(t) \approx -0,5\mathrm{e}^{\lambda_1 t} + 0,0681\mathrm{e}^{\lambda_2 t} + 0,4319\mathrm{e}^{\lambda_3 t} \end{cases}$$

$$\boxed{\mathrm{e}^{At} = \alpha_0\mathrm{I} + \alpha_1(t)A + \alpha_2(t)A^2}$$

Cálculo de e^{At} por Cayley-Hamiltion

$\mathrm{e}^{At} \approx$

$$\begin{bmatrix} e_1(t) + 0{,}2425e_2(t) - 0{,}2426e_3(t) & e_1(t) - 0{,}1362e_2(t) - 0{,}8638e_3(t) & -0{,}4851e_2(t) + 0{,}485e_3(t) \\ -0{,}2425e_2(t) + 0{,}2425e_3(t) & 0{,}1362e_2(t) + 0{,}8638e_3(t) & 0{,}4851e_2(t) - 0{,}485e_3(t) \\ 0{,}5e_1(t) - 0{,}3106e_2(t) - 0{,}1894e_3(t) & 0{,}5e_1(t) + 0{,}1744e_2(t) - 0{,}6744e_3(t) & 0{,}6213e_2(t) + 0{,}3788e_3(t) \end{bmatrix}$$

donde

$$e_1(t) = \mathrm{e}^{\lambda_1 t}$$

$$e_2(t) = \mathrm{e}^{\lambda_2 t}$$

$$e_3(t) = \mathrm{e}^{\lambda_3 t}$$

Cálculo de e^{At} por Laplace

$$\mathrm{e}^{At} = \mathcal{L}^{-1}\left\{(s\mathrm{I} - A)^{-1}\right\}$$

$$(s\mathrm{I} - A)^{-1} = \frac{\mathrm{adj}\,(s\mathrm{I} - A)}{\det\,(s\mathrm{I} - A)} = \frac{(\mathrm{cof}\,(s\mathrm{I} - A))^{\mathsf{T}}}{\det\,(s\mathrm{I} - A)}$$

$$s\mathrm{I} - A = \begin{bmatrix} s-3 & 0 & 2 \\ 1 & s-2 & -2 \\ 1 & -1 & s-4 \end{bmatrix}$$

Cálculo de e^{At} por Laplace

$$\mathrm{e}^{At} = \mathcal{L}^{-1}\left\{(s\mathrm{I} - A)^{-1}\right\}$$

$$(s\mathrm{I} - A)^{-1} = \frac{\operatorname{adj}(s\mathrm{I} - A)}{\det(s\mathrm{I} - A)} = \frac{(\operatorname{cof}(s\mathrm{I} - A))^{\mathsf{T}}}{\det(s\mathrm{I} - A)}$$

$$s\mathrm{I} - A = \begin{bmatrix} s-3 & 0 & 2 \\ 1 & s-2 & -2 \\ 1 & -1 & s-4 \end{bmatrix}$$

$$\operatorname{cof}(s\mathrm{I} - A) = \begin{bmatrix} s^2 - 6s + 6 & -s + 2 & -s + 1 \\ -2 & s^2 - 7s + 10 & s - 3 \\ -2s + 4 & 2s - 4 & s^2 - 5s + 6 \end{bmatrix}$$

Cálculo de e^{At} por Laplace

$$\mathrm{e}^{At} = \mathcal{L}^{-1}\left\{(s\mathrm{I} - A)^{-1}\right\}$$

$$(s\mathrm{I} - A)^{-1} = \frac{\operatorname{adj}(s\mathrm{I} - A)}{\det(s\mathrm{I} - A)} = \frac{(\operatorname{cof}(s\mathrm{I} - A))^{\mathsf{T}}}{\det(s\mathrm{I} - A)}$$

$$s\mathrm{I} - A = \begin{bmatrix} s-3 & 0 & 2 \\ 1 & s-2 & -2 \\ 1 & -1 & s-4 \end{bmatrix}$$

$$\operatorname{cof}(s\mathrm{I} - A) = \begin{bmatrix} s^2 - 6s + 6 & -s + 2 & -s + 1 \\ -2 & s^2 - 7s + 10 & s - 3 \\ -2s + 4 & 2s - 4 & s^2 - 5s + 6 \end{bmatrix}$$

$$\det(s\mathrm{I} - A) = (s - \lambda_1)(s - \lambda_2)(s - \lambda_3)$$

Cálculo de e^{At} por Laplace (otra forma de invertir $s\mathrm{I} - A$)

$$\mathrm{e}^{At} = \mathcal{L}^{-1}\left\{(s\mathrm{I} - A)^{-1}\right\}$$

$$(s\mathrm{I} - A)^{-1} = \frac{B(s)}{d(s)}, \qquad \begin{array}{l} B(s) = s^{n-1}B_0 + s^{n-2}B_1 + \ldots + sB_{n-2} + B_{n-1} \\ d(s) = \det(s\mathrm{I} - A) = s^n + d_1 s^{n-1} + \cdots + d_n \end{array}$$

$$\begin{cases} B_0 = \mathrm{I} \\ B_1 = B_0 A + d_1 \mathrm{I} \\ B_2 = B_1 A + d_2 \mathrm{I} \\ \vdots \\ B_{n-1} = B_{n-2} A + d_{n-1} \mathrm{I} \\ 0 = B_{n-1} A + d_n \mathrm{I} \end{cases}$$

Cálculo de e^{At} por Laplace (otra forma de invertir $s\mathrm{I} - A$)

$$e^{At} = \mathcal{L}^{-1}\left\{(s\mathrm{I} - A)^{-1}\right\}$$

$$(s\mathrm{I} - A)^{-1} = \frac{B(s)}{d(s)}, \qquad \begin{aligned} B(s) &= s^{n-1}B_0 + s^{n-2}B_1 + \ldots + sB_{n-2} + B_{n-1} \\ d(s) &= \det(s\mathrm{I} - A) = s^n + d_1 s^{n-1} + \cdots + d_n \end{aligned}$$

$$\begin{cases} B_0 = \mathrm{I} \\ B_1 = B_0 A + d_1 \mathrm{I} \\ B_2 = B_1 A + d_2 \mathrm{I} \\ \vdots \\ B_{n-1} = B_{n-2} A + d_{n-1} \mathrm{I} \\ 0 = B_{n-1} A + d_n \mathrm{I} \end{cases}$$

$$B(s) = s^2 B_0 + sB_1 + B_2 = s^2\mathrm{I} + s\left(A + d_1\mathrm{I}\right) + \left(A^2 + d_1 A + d_2\mathrm{I}\right)$$

$$\begin{aligned} d(s) &= \det(s\mathrm{I} - A) = (s - \lambda_1)(s - \lambda_2)(s - \lambda_3) \\ &= s^3 \underbrace{-(\lambda_1 + \lambda_2 + \lambda_3)}_{d_1} s^2 + \underbrace{(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)}_{d_2} s \underbrace{-\lambda_1\lambda_2\lambda_3}_{d_3} \end{aligned}$$

Cálculo de e^{At} por Laplace

Queda pendiente antitransformar

$$\frac{(\operatorname{cof}(s\mathrm{I}-A))^{\mathsf{T}}}{d(s)} = \frac{B(s)}{d(s)} = \frac{\begin{bmatrix} s^2-6s+6 & -2 & -2s+4 \\ -s+2 & s^2-7s+10 & 2s-4 \\ -s+1 & s-3 & s^2-5s+6 \end{bmatrix}}{(s-\lambda_1)(s-\lambda_2)(s-\lambda_3)}$$

para obtener e^{At} y verificar que coincide con el resultado anteriormente obtenido mediante Cayley-Hamilton.

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $M \in \mathcal{M}_{n \times n}$:

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k$$

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $M \in \mathcal{M}_{n\times n}$:

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k$$

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k = F_t\left(M\right) p_A\left(M\right) + R_t\left(M\right) \quad \text{donde} \quad \mathrm{gr}\left[R_t\right] = n-1$$

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $M \in \mathcal{M}_{n\times n}$:

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k$$

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k = F_t\left(M\right) p_A\left(M\right) + R_t\left(M\right) \quad \text{donde} \quad \mathrm{gr}\left[R_t\right] = n-1$$

Para $M = A$:

$$\mathrm{e}^{At} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} A^k = F_t\left(A\right) \underbrace{p_A\left(A\right)}_{0\in\mathcal{M}_{n\times n}} + R_t\left(A\right) \quad \text{donde} \quad \mathrm{gr}\left[R_t\right] = n-1$$

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $M \in \mathcal{M}_{n\times n}$:

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k$$

$$\mathrm{e}^{Mt} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} M^k = F_t(M)\, p_A(M) + R_t(M) \quad \text{donde} \quad \mathrm{gr}\,[R_t] = n-1$$

Para $M = A$:

$$\mathrm{e}^{At} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} A^k = F_t(A)\, \underbrace{p_A(A)}_{0\in\mathcal{M}_{n\times n}} + R_t(A) \quad \text{donde} \quad \mathrm{gr}\,[R_t] = n-1$$

$$\mathrm{e}^{At} = R_t(A) = \alpha_0(t) + \alpha_1(t)A + \cdots + \alpha_{n-1}(t)A^{n-1}$$

¿Cómo hallamos los $\alpha_0(t), \cdots, \alpha_{n-1}(t)$?

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $s \in \mathbb{C}$:

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k$$

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $s \in \mathbb{C}$:

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k$$

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k = F_t(s)\, p_A(s) + R_t(s) \quad \text{donde} \quad \mathrm{gr}\left[R_t\right] = n - 1$$

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $s \in \mathbb{C}$:

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k$$

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k = F_t(s)\, p_A(s) + R_t(s) \quad \text{donde} \quad \mathrm{gr}[R_t] = n-1$$

Si λ es un valor propio de A, para $s = \lambda$:

$$\mathrm{e}^{\lambda t} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} \lambda^k = F_t(\lambda) \underbrace{p_A(\lambda)}_{0 \in \mathbb{C}} + R_t(\lambda) \quad \text{donde} \quad \mathrm{gr}[R_t] = n-1$$

Cálculo de e^{At} por Cayley-Hamiltion (repaso de teórico)

Para cualquier $s \in \mathbb{C}$:

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k$$

$$\mathrm{e}^{st} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} s^k = F_t(s)\, p_A(s) + R_t(s) \quad \text{donde} \quad \mathrm{gr}\left[R_t\right] = n-1$$

Si λ es un valor propio de A, para $s = \lambda$:

$$\mathrm{e}^{\lambda t} = \sum_{k=0}^{+\infty} \frac{t^k}{k!} \lambda^k = F_t(\lambda)\, \underbrace{p_A(\lambda)}_{0 \in \mathbb{C}} + R_t(\lambda) \quad \text{donde} \quad \mathrm{gr}\left[R_t\right] = n-1$$

$$\mathrm{e}^{\lambda t} = R_t(\lambda) = \alpha_0(t) + \alpha_1(t)\lambda + \cdots + \alpha_{n-1}(t)\lambda^{n-1}$$

Si existen n valores propios distintos, existen n ecuaciones como la última, de donde se pueden calcular los $\alpha_0(t), \cdots, \alpha_{n-1}(t)$.