Serie de Fourier

$x(t)$ función periódica, con período T

$$X[k] = \frac{1}{T}\int_{t_o}^{t_o+T} x(t)e^{-jk\frac{2\pi}{T}t}dt \qquad x(t) = \sum_{k=-\infty}^{+\infty} X[k]e^{jk\frac{2\pi}{T}t}$$

para cualquier t_o.

$$\frac{1}{T}\int_{t_o}^{t_o+T} x(t)y^*(t)dt = \sum_{k=-\infty}^{\infty} X[k]Y^*[k]$$

$$Im[x(t)] = 0 \Longleftrightarrow X[-k] = X^*[k]$$

$$x(-t) = x^*(t) \Longleftrightarrow Im[X[k]] = 0$$

$$Re[x(t)] = 0 \Longleftrightarrow X[-k] = -X^*[k]$$

$$x(-t) = -x^*(t) \Longleftrightarrow Re[X[k]] = 0$$

Función	Transformada
$ax(t) + by(t)$	$aX[k] + bY[k]$
$f(t-\tau)$	$e^{-j2\pi k\tau/T}X[k]$
$e^{j2\pi mt/T}x(t)$	$X[k-m]$
$\frac{1}{T}\int_0^T x(t-\tau)y(\tau)d\tau$	$X[k]Y[k]$
$x(t)y(t)$	$X[k] * Y[k]$
$\sum_n \Pi\left(\frac{t-nT}{\tau}\right)$	$\frac{\tau}{T}\mathrm{sinc}(k\tau/T)$

Identidades trigonométricas
$e^{j\theta} = \cos\theta + j\sin\theta$
$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$
$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$
$\cos\theta = \sin(\theta + 90°)$
$\sin\theta = \cos(\theta - 90°)$
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1/2(1 + \cos 2\theta)$
$\cos^3\theta = 1/4(3\cos\theta + \cos 3\theta)$
$\sin^2\theta = 1/2(1 - \cos 2\theta)$
$\sin^3\theta = 1/4(3\sin\theta - \sin 3\theta)$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$
$\sin\alpha\sin\beta = 1/2\cos(\alpha-\beta) - 1/2\cos(\alpha+\beta)$
$\cos\alpha\cos\beta = 1/2\cos(\alpha-\beta) + 1/2\cos(\alpha+\beta)$
$\sin\alpha\cos\beta = 1/2\sin(\alpha-\beta) + 1/2\sin(\alpha+\beta)$

Algunas funciones útiles	
$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-m)^2}{2\sigma^2}}$	Distribución de Gauss
$Q(k) = \frac{1}{\sqrt{2\pi}}\int_k^\infty e^{-\lambda^2/2}d\lambda$	Cola Gaussiana
$\mathrm{sinc}\ t = \frac{\sin\pi t}{\pi t}$	Sinc
$\mathrm{sign}\ t = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$	Signo
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$	Escalón
$\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \lvert t\rvert < \frac{\tau}{2} \\ 0 & \lvert t\rvert > \frac{\tau}{2} \end{cases}$	Rectángulo
$\Lambda\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{\lvert t\rvert}{\tau} & \lvert t\rvert < \tau \\ 0 & \lvert t\rvert > \tau \end{cases}$	Triángulo

Transformada de Fourier

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \qquad x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$$

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$$

$$Im[x(t)] = 0 \Longleftrightarrow X(f) = X^*(f)$$

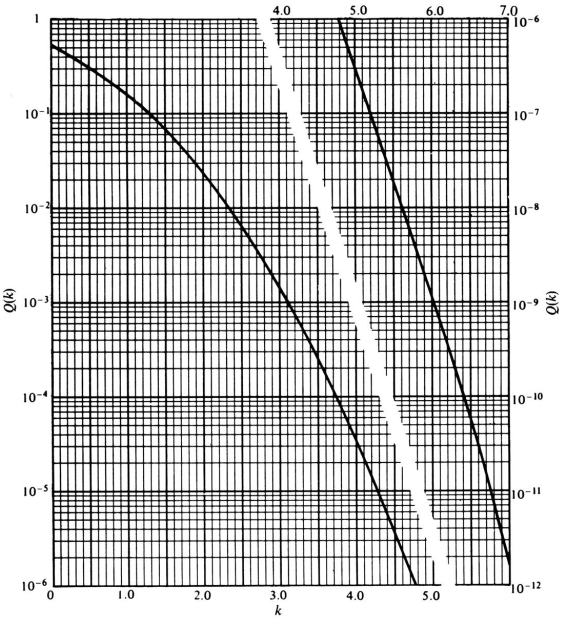
$$x(-t) = x^*(t) \Longleftrightarrow Im[X(f)] = 0$$

$$Re[x(t)] = 0 \Longleftrightarrow X(-f) = -X^*(f)$$

$$x(-t) = -x^*(t) \Longleftrightarrow Re[X(f)] = 0$$

Función	Transformada
$ax(t) + by(t)$	$aX(f) + bY(f)$
$(x * y)(t)$	$X(f)Y(f)$
$x(t)y(t)$	$(X * Y)(f)$
$x^*(t)$	$X^*(-f)$
$X(t)$	$x(-f)$
$x(t-t_d)$	$X(f)e^{-j2\pi ft_d}$
$x(t)e^{j\omega_c t}$	$X(f-f_c)$
$x(t)\cos(\omega_c t + \phi)$	$\frac{X(f-f_c)e^{j\phi} + X(f+f_c)e^{-j\phi}}{2}$
$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{f}{\alpha}\right)$
$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
$\int_{-\infty}^{t} x(\lambda)d\lambda$	$\frac{1}{j2\pi f}X(f) + \frac{X(0)}{2}\delta(f)$
$t^n x(t)$	$(-j2\pi)^{-n}\frac{d^n X(f)}{df^n}$
$\delta(t-t_d)$	$e^{-j2\pi ft_d}$
$e^{j(\omega_c t+\phi)}$	$e^{j\phi}\delta(f-f_c)$
$e^{-\pi(at)^2}$	$\frac{1}{a}e^{-\pi(f/a)^2}$
$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$
$\mathrm{sign}\ t$	$\frac{1}{j\pi f}$
$u(t)$	$\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$\Pi\left(\frac{t}{\tau}\right)$	$\tau\ \mathrm{sinc}\ f\tau$
$\Lambda\left(\frac{t}{\tau}\right)$	$\tau\ \mathrm{sinc}^2\ f\tau$

Cola Gaussiana



Cuando $k > 3$ la siguiente es una buena aproximación

$$Q(k) \approx \frac{1}{\sqrt{2\pi}k}e^{-k^2/2}$$

DTFT

$$X(e^{j\theta}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-jn\theta} \qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})e^{j\theta n}d\theta$$

$$\sum_{n=-\infty}^{+\infty} x_1[n]x_2^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2^*(e^{j\theta})d\theta$$

$$Im[x[n]] = 0 \Longleftrightarrow X(e^{-j\theta}) = X^*(e^{j\theta})$$

$$x[-n] = x^*[n] \Longleftrightarrow Im[X(e^{j\theta})] = 0$$

$$Re[x[n]] = 0 \Longleftrightarrow X(e^{-j\theta}) = -X^*(e^{j\theta})$$

$$x[-n] = -x^*[n] \Longleftrightarrow Re[X(e^{j\theta})] = 0$$

Función	Transformada
$a_1x[n] + a_2y[n]$	$a_1X(e^{j\theta}) + a_2Y(e^{j\theta})$
$(x * y)[n]$	$X(e^{j\theta})Y(e^{j\theta})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\lambda})Y(e^{j(\theta-\lambda)})d\lambda$
$x^*[n]$	$X^*(e^{-j\theta})$
$x[n-n_o]$	$X(e^{j\theta})e^{-jn_o\theta}$
$x[n]e^{jn\theta_o}$	$X(e^{j(\theta-\theta_o)})$
$n\,x[n]$	$j\frac{d\,X(e^{j\theta})}{d\theta}$
$x[-n]$	$X(e^{-j\theta})$
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-jn_o\theta}$
1	$\sum_{k=-\infty}^{+\infty} 2\pi\delta(\theta+2\pi k)$
$a^nu[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\theta}}$
$u[n]$	$\frac{1}{1-e^{-j\theta}} + \sum_{k=-\infty}^{\infty}\pi\delta(\theta+2\pi k)$
$(n+1)a^nu[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\theta})^2}$
$\frac{r^n\sin\theta_o(n+1)}{\sin\theta_o}u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\theta_o e^{-j\theta}+r^2e^{-j2\theta}}$
$\frac{\sin\theta_o n}{\pi n}$	$\sum_k \Pi\left(\frac{\theta+2\pi k}{2\theta_o}\right)$
$x[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otro n} \end{cases}$	$\frac{\sin(\theta(M+1)/2)}{\sin(\theta/2)}e^{-j\theta M/2}$
$e^{j\theta_o n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\theta-\theta_o+2\pi k)$

Fórmula de Poisson

$$\sum_{k=-\infty}^{+\infty} e^{jk\alpha t} = \frac{2\pi}{\alpha}\sum_{k=-\infty}^{+\infty}\delta\left(t - k\frac{2\pi}{\alpha}\right)$$

Desigualdad de Schwarz

$$\left|\int_a^b v(\lambda)w^*(\lambda)d\lambda\right|^2 \le \int_a^b |v(\lambda)|^2 d\lambda \int_a^b |w(\lambda)|^2 d\lambda$$

La igualdad se da si $v(\lambda) = K \cdot w(\lambda)$ con K constante.

Transformada Z unilateral

$$X(z) = \sum_{n=0}^{+\infty} x[n]z^{-n}$$

$$x[n] \longleftrightarrow X_u(z)$$

$$x[n+n_o] \longleftrightarrow z^{n_o}X_u(z) - x[0]z^{n_o} - \cdots - x[n_o-1]z$$

$$x[n-n_o] \longleftrightarrow z^{-n_o}X_u(z) + x[-1]z^{-n_o+1} + \cdots + x[-n_o]$$

Fórmula 1

$$e^{j\pi} + 1 = 0$$

DFT

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn} \qquad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-kn}$$

donde $W_N = e^{-j2\pi/N}$

$$\sum_{n=0}^{N-1} x[n]y^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]Y^*[k]$$

$$Im[x[n]] = 0 \Longleftrightarrow X[-n \bmod N] = X^*[k]$$

$$x[-n \bmod N] = x^*[n] \Longleftrightarrow Im[X[k]] = 0$$

$$Re[x[n]] = 0 \Longleftrightarrow X[-n \bmod N] = -X^*[k]$$

$$x[-n \bmod N] = -x^*[n] \Longleftrightarrow Re[X[k]] = 0$$

Función	Transformada
$ax[n] + by[n]$	$aX[k] + bY[k]$
$X[n]$	$Nx[-k \bmod N]$
$x[(n-m) \bmod N]$	$W_N^{km}X[k]$
$W_N^{-ln}x[n]$	$X[(k-l) \bmod N]$
$\sum_{m=0}^{N-1} x[m]y[(n-m) \bmod N]$	$X[k]Y[k]$
$x[n]y[n]$	$\frac{1}{N}\sum_{l=0}^{N-1} X[l]Y[(k-l) \bmod N]$
$x^*[n]$	$X^*[-k \bmod N]$
$x^*[-n \bmod N]$	$X^*[k]$

Transformada Z

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n} \qquad x(n) = \frac{1}{2\pi j}\oint_C X(z)z^{n-1}dz$$

donde C es una curva antihoraria en la región de convergencia y que envuelve al origen.

Secuencia	Transformada Z	ROC
$ax[n] + by[n]$	$aX(z) + bY(z)$	contiene $R_x \cap R_y$
$x[n-n_o]$	$z^{-n_o}X(z)$	R_x, quizá ± 0 ó ∞
$z_o^n x[n]$	$X(z/z_o)$	$\lvert z_o\rvert R_x$
$n^k x[n]$	$\left(-z\frac{d}{dz}\right)^k X(z)$	R_x, quizá ± 0 ó ∞
$x^*[n]$	$X^*(z^*)$	R_x
$x[-n]$	$X(1/z)$	$1/R_x$
$(x * y)[n]$	$X(z)Y(z)$	contiene $R_x \cap R_y$
$\delta[n]$	1	$\forall z$
$a^nu[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^nu[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$\delta[n-n_o]$	z^{-n_o}	$\forall z$ excepto 0 ó ∞
$\cos(\omega_o n)u[n]$	$\frac{1-z^{-1}\cos\omega_o}{1-2z^{-1}\cos\omega_o+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(\omega_o n)u[n]$	$\frac{z^{-1}\sin\omega_o}{1-2z^{-1}\cos\omega_o+z^{-2}}$	$\lvert z\rvert > 1$

Serie Geométrica

$$\sum_{k=N_1}^{N_2}\alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha} \quad \text{si } N_2 > N_1 \text{ y } \alpha \ne 1$$