

The Fluidyne engine

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The Fluidyne is a two-part hot-air engine which has the peculiarity that both its power piston and displacer are liquids. Both parts operate in tandem with the common working gas (air) transferring energy from the displacer to the piston side, from which work is extracted. We describe analytically the thermodynamics of the Fluidyne engine using the approach previously developed for the Stirling engine. We obtain explicit expressions for the amplitude of the power piston movement and for the working gas temperatures and pressure as functions of the engine parameters. We also study numerically the power and efficiency of the engine in terms of the phase shift between the motions of the piston and displacer. © 2019 American Association of Physics Teachers.

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I. INTRODUCTION

The Fluidyne engine was invented by Colin West in 1969.¹ It is a Stirling machine with one or more liquid pistons. Usually, it contains air as a working gas and either two liquid pistons or one liquid piston and a displacer. The Fluidyne engine operates at a low frequency, typically ~ 1 Hz, and close to atmospheric pressure. The most common application of the liquid-piston system is in irrigation pumping, particularly for irrigation or drainage pumping in places where electric power may not be available.² Nowadays, many commercial setups are available for specific applications in agriculture, building services, drinking water and sanitation.³

The basic principle of a Fluidyne engine is the fact that air expands when heated and contracts when cooled. It is possible to find many interesting videos that present the construction and operation of this type of engine for demonstration and teaching purposes, on the internet. However, it is not easy to find the theoretical material that explains its thermodynamics without an overly complicated technical discussion of the Stirling cycle. In the present paper, we study the thermodynamics of a particular type of Fluidyne engine that can be seen as a simple Stirling engine with one free liquid piston and displacer.^{2,4,5}

Recently, we developed an alternative theoretical model⁶ for the usual Stirling cycle.⁷ The main characteristic of that approach is the introduction of a polytropic process,⁸ for which $PV^\beta = \text{constant}$, as a way to represent the exchange of heat with the environment. The assumption of a polytropic process allows modeling different dependencies of the working gas temperature on its volume. This means that the polytropic index β can be used as an additional degree of freedom that could be adjusted to the experimental data of a real operating engine. In particular, we remark that all the discussion presented remains unchanged if β is replaced by $\gamma = c_p/c_v$ because this last is a special case of the model.

Our alternative model provides analytic expressions for the pressure, temperatures of the working gas, and the work and heat exchanged with the heat reservoirs. The theoretical pressure-volume diagram achieved a closer agreement with the experimental one than the standard analysis. Due to the generality of the analytic expressions obtained, they can be adapted to any type of Stirling engine. In the present paper, we use the mentioned model to study the thermodynamics of the liquid-piston Stirling engine, “the Fluidyne engine.”

The paper is organized as follows. In Sec. II, we present the liquid-piston Stirling engine and we study in detail the dynamics of the liquid piston with the help of the results of the thermodynamic model. In Sec. III, we obtain the pressure-volume and the work-efficiency diagrams for the Fluidyne engine. In Sec. IV, we present the main conclusions.

II. LIQUID PISTON STIRLING ENGINE

The Fluidyne engine consists of two U-tubes partially filled with liquid and connected with a tube of negligible caliber as shown in Fig. 1. One end of the tube containing the piston liquid is open to the atmosphere. The three connected sections also contain the working gas. Figure 1 shows a snapshot picture of the engine working with its two liquids in motion modifying the gas volume. In this figure, the liquids in the left and right U-tubes are, respectively, the power piston and the displacer of the engine. While the gas volume changes due to the motion of the liquids, it maintains a uniform pressure determined by the action of the heat reservoir, represented by the torch in the figure. Two zones can be distinguished by their temperature in the volume occupied by the gas and labeled with the numbers 1 and 2 in Fig. 1. Zone 2 is the hot zone where the gas is in contact with the reservoir at the external temperature T_h , and zone 1 is the cool zone where the gas is in contact with the reservoir at the external temperature T_c . Then, the working gas is never completely in either the hot or cold zone of the engine. However, the upper connection between the zones imposes the same pressure P in both. (Note that the diameter of the upper connection is relatively small but still very large compared with the mean free path of the gas molecules, this condition ensures that there is no effusion process involved.)⁹ Consequently, the gas density must be different in each zone to keep the same pressure with different temperatures. The left side of the liquid piston is the free zone, here the pressure is the atmospheric pressure P_0 and the temperature is also T_c .

In order to start the engine, we must set in motion externally both the piston and the displacer. One way to produce this initial motion starting from the static situation, where both liquids are at rest, is that an external agent performs the following three steps: first, it rotates the engine a small angle with respect to an axis perpendicular to the main plane of the

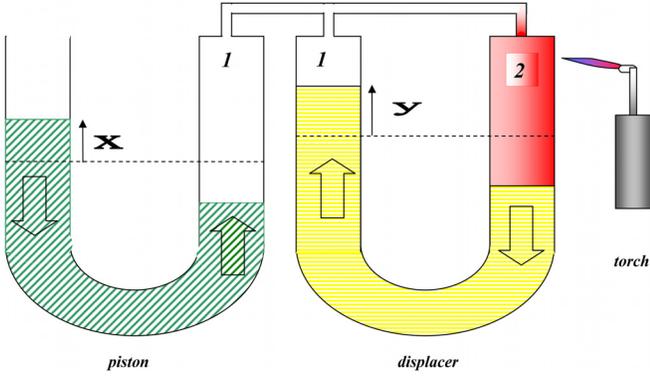


Fig. 1. Simple model of a Fluidyne engine. The piston and displacer are liquids and the cylinders are two U-tubes. The working gas moves in zones 1 and 2 that are, respectively, the cold and hot zones. Both zones have the same pressure but the open branch of the left U-tube is subjected to atmospheric pressure. x and y indicate the displacement of the piston and the displacer from their equilibrium positions. The open arrows indicate the movements of the piston and displacer. The torch works as a heat reservoir.

engine (the plane of Fig. 1), second, it reverses the rotation to return the tubes to the initial position, and third, it temporarily manipulates the pressure over the left side of the liquid piston in such a way as to obtain the desired relative movement between the displacer and piston and simultaneously going back to the pressure P_0 . After these steps, the Fluidyne engine is in the situation shown by Fig. 1, where both liquids are moving inside their U-tubes, and the only external agents that interact with the gas are the heat reservoirs.

With the liquids in motion, the mechanism of energy transfer between the hot and cold zones works without any type of one-way valves and can be qualitatively understood as follows. Initially, we focus on the movement of the displacer and suppose the piston to be still. When the displacer is at the center of the U-tube, suppose that the air volumes are more or less the same in the hot and cold zones. When the displacer liquid moves towards the cold (hot) zone, air is displaced, through the upper connecting tube, towards the hot (cold) zone and a greater quantity of air increases (decreases) its temperature and consequently the air pressure increases (decreases). The air pressure of the right side of the liquid piston (see Fig. 1) depends on the displacer's motion but the left side remains at a constant atmospheric pressure. If we now incorporate the motion of the piston, the net motion of air between zones 1 and 2 depends on the phase of the relative motions of the displacer and piston fluids, which results in a cyclical transfer of energy from the high temperature zone 2 to the piston liquid of zone 1 (with a small amount going into sustaining the oscillation of the displacer liquid against dissipative losses). The phase of this motion is a crucial parameter for the efficiency and power delivered by the engine, and is treated in the analysis that follows.

When the displacer liquid is set into oscillation in its U-tube, then the gas above this liquid is transferred back and forth between the hot and cold zones. Both sides of the displacer tube have the same pressure, even when the engine is working, therefore if the displacer behaves as an ideal fluid, any initial oscillation never decreases. In what follows, we assume that the displacer behaves as an ideal fluid subject to the restoring force of gravity, and any oscillation has a natural frequency given by

$$\omega_d = \sqrt{\frac{2g}{L_d}}, \quad (1)$$

where g is the gravity acceleration and L_d is the displacer's length. Then, the amplitude of the oscillation is given by

$$y = y_0 \sin \omega_d t, \quad (2)$$

where y_0 is the maximum amplitude and we take a vanishing initial phase. If the displacer is not an ideal fluid, it is always possible to use part of the energy produced by the engine to compensate losses and to maintain the original oscillation. As we shall see, ω_d will be the operating frequency of the engine.

To study the dynamics of the power piston, we observe that the pressures on the right side P and left side P_0 give rise to a net force on the liquid piston proportional to $P - P_0$. We also consider an external force that dissipates all the useful power delivered by the working gas. In a real engine, the power piston should be coupled to a crankshaft to transfer the useful power. We opt for this simplified model in order to obtain a closed analytical solution; however, the real system can be addressed numerically. Therefore, the power piston dynamics is determined by Newton's second law applied to the displacement x of the liquid piston (see Fig. 1)

$$\rho a L_p \ddot{x} = -b\dot{x} - 2g\rho a x + (P - P_0)a, \quad (3)$$

where ρ , a , and L_p are the piston liquid density, cross-sectional area, and length of the fluid, respectively, and b is a damping coefficient.

To obtain the working gas pressure P , we assume the following simplifying hypotheses for the working gas: (i) the gas behaves as a classical ideal gas. (ii) The gas has a uniform temperature in each zone namely T_1 and T_2 . (iii) The mass of gas inside the thin connecting tubes is negligible, i.e., all the gas is inside the U-tubes, where the displacer and the piston move. (iv) The expansion and compression processes are treated as polytropic processes. For an ideal gas with a constant number of moles, the polytropic process may be defined through the relation $PV^\beta = \text{constant}$, where β is the polytropic index with typical values such that $1 < \beta \leq \gamma$ with $\gamma = 1.4$ for dry air at room temperature. In such a context, we have obtained in Ref. 6 analytic expressions for the pressure and temperatures of the working gas as functions of the gas volume, the external temperatures, and the polytropic index β that characterizes the heat absorption process. The gas pressure is given by

$$P = P_i \left(\frac{\alpha V_{10} + V_{20}}{\alpha V_1 + V_2} \right) \left(\frac{V_0}{V} \right)^{\beta-1}, \quad (4)$$

and the temperatures T_1 and T_2 are

$$T_1 = T_c \left(\frac{V_0}{V} \right)^{\beta-1}, \quad (5)$$

$$T_2 = T_h \left(\frac{V_0}{V} \right)^{\beta-1}, \quad (6)$$

where α is the ratio between the temperatures of the reservoirs

$$\alpha \equiv T_h/T_c, \quad (7)$$

V_1 is the gas volume of the cool zone with temperature T_1 , V_2 is the gas volume of the hot zone with temperature T_2 , $V = V_1 + V_2$ is the total gas volume, V_{10} , V_{20} , and V_0 are the initial values of V_1 , V_2 , and V , respectively, and P_i is the initial value of P . It is interesting to note that Eqs. (4)–(6) for the isothermal case $\beta = 1$ coincide with the so-called Schmidt solution, published in 1871.¹⁰ For the case $\beta = \gamma$, the same equations describe the classical adiabatic solution.^{11,12} Both examples show the adaptability of polytropic processes for describing the thermodynamics of this type of engines.

Assuming that both U-tubes have the same cross section area a , the volumes of the zones 1 and 2 can be expressed as (see Fig. 1)

$$V_1 = V_{10} + a(x - y), \quad (8)$$

$$V_2 = V_{20} + ay, \quad (9)$$

where the displacements x and y are taken from the initial (equilibrium) position, i.e., $x = y = 0$ give the initial volumes in Eqs. (8) and (9). The total volume is then

$$V = V_0 + ax. \quad (10)$$

The explicit dependence of P on x and y , from Eqs. (4), (8), (9) and (10), is found to be

$$P = \frac{P_i}{1 + \frac{\alpha a}{\alpha V_{10} + V_{20}} \left[x + \left(\frac{1}{\alpha} - 1 \right) y \right]} \left(\frac{1}{1 + \frac{a}{V_0} x} \right)^{\beta-1}. \quad (11)$$

It is clear now that Eq. (3) has a non-linear dependence on both x and y . However, the engine operates in a closed regenerative cycle, which presupposes only one characteristic frequency. Then, only the periodic solutions of Eq. (3), with ω_d as a characteristic frequency, will be useful. Here, $\alpha \geq 1$ and we further assume that the engine geometry is such that both $ax/V_{10} \ll 1$ and $ay/V_{10} \ll 1$ are satisfied so that these magnitudes can be treated as perturbations in Eq. (11). Expanding Eq. (11) around the unperturbed volumes and substituting in Eq. (3), we get the following linear equation:

$$\ddot{x} + \frac{\xi}{\omega_d} \dot{x} + \omega_p^2 x = \frac{P_i - P_0}{\rho L_p} + \frac{a P_i (\alpha - 1)}{\rho L_p (\alpha V_{10} + V_{20})} y, \quad (12)$$

where

$$\xi = \frac{b \omega_d}{\rho L_p a}, \quad (13)$$

$$\omega_p^2 = \frac{2g}{L_p} + \frac{a P_i}{\rho L_p} \left(\frac{\beta - 1}{V_0} + \frac{\alpha}{\alpha V_{10} + V_{20}} \right), \quad (14)$$

and y is given by Eq. (2).

Equation (12) describes a linearly damped oscillator with external forcing. The first term on the right-hand side produces a constant shift from the equilibrium position, which is equivalent to a redefinition of the volume V_{10} . Therefore, from now on, we assume $P_i = P_0$ in order to ignore it and we will concentrate on the periodic solution.

When the engine achieves the steady state motion, the solution of Eq. (12) that determines its dynamics is

$$x = x_0 \sin(\omega_d t - \phi), \quad (15)$$

where

$$x_0 = \frac{a P_0 (\alpha - 1)}{\rho L_p \xi (\alpha V_{10} + V_{20})} y_0 \sin \phi \quad (16)$$

and

$$\phi = \arctan \left(\frac{\xi}{\omega_p^2 - \omega_d^2} \right). \quad (17)$$

From Eq. (16), it is clear that the engine does not work if $\alpha = 1$, that is, in order to function the engine needs heat reservoirs with $T_h > T_c$, see Eq. (7). Moreover, when $\alpha \rightarrow \infty$, then x_0 tends asymptotically to a finite value. On the other hand, if $\omega_p = \omega_d$ in Eq. (17), the power piston is in resonance with the displacer, $\phi = \pi/2$, and the amplitude x_0 is maximum.

III. FLUIDYNE THERMODYNAMICS

To obtain the pressure-volume diagram for the Fluidyne engine, we use Eqs. (2), (11), and (15). This is shown in Fig. 2 for two characteristic values of ϕ that will be explained below. The area inside such smooth closed curves represents the total work W of the cycle, whose expression is

$$W = \oint P dV = a \int_0^{2\pi/\omega_d} P \dot{x} dt, \quad (18)$$

where the last equality follows from Eq. (10). Here, W is the work available for overcoming mechanical friction losses and for providing useful power.

The differential equation for the heat absorbed by the working gas in the polytropic process was obtained in the general case of a Stirling engine in Ref. 6 as follows:

$$dQ = \frac{P}{\gamma - 1} \left[(\gamma - \beta) dV + (\alpha - 1) \frac{V_1 dV_2 - V_2 dV_1}{\alpha V_1 + V_2} \right], \quad (19)$$

where V_1 , V_2 , and V are defined by Eqs. (8)–(10) and

$$\gamma = \frac{c_P}{c_V} \quad (20)$$

is the quotient of the specific heats of the gas at constant pressure c_P and constant volume c_V . In this paper, we take the polytropic index such that $0 \leq \beta \leq \gamma = 1.4$.

It is important to emphasize that Eq. (19) together with Eqs. (4), (5) and (6), determine the Fluidyne thermodynamics. These four equations depend on the parameter α (the temperature ratio) and they have finite asymptotic values when $\alpha \rightarrow \infty$. This means that after a certain value of α , no matter how much we increase the temperature ratio, the pressure and the absorbed heat are bounded, and this in turn explains why the useful work and the efficiency in any Stirling engine are asymptotically bounded. Let us call Q_{in} the heat absorbed by the gas in the cycle with the convention $Q_{in} > 0$; similarly, we call $Q_{out} < 0$ the heat rejected. They can be calculated numerically using Eq. (19) and the definitions

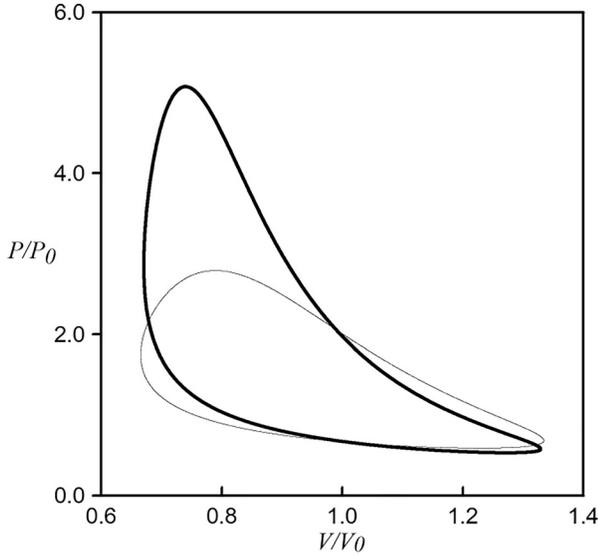


Fig. 2. Dimensionless pressure-volume diagrams for the gas of the Fluidyne engine, the thick line for $\phi=0.59\pi$ and the thin line for $\phi=0.43\pi$. The parameters are $\alpha=10$, $\beta=1.25$, $V_{10}=1.0$, $V_{20}=0.5$, $y_0=0.60$, and $x_0=0.51 \sin \phi$.

$$Q_{in} = \int_{C_{in}} dQ, \quad (21)$$

$$Q_{out} = \int_{C_{out}} dQ, \quad (22)$$

where C_{in} and C_{out} refer to the paths where $dQ > 0$ and $dQ < 0$, respectively. As the internal energy change in the entire cycle vanishes, the total work verifies $W = Q_{in} + Q_{out}$. Therefore, the efficiency η defined as W/Q_{in} can be written as

$$\eta = 1 + Q_{out}/Q_{in}. \quad (23)$$

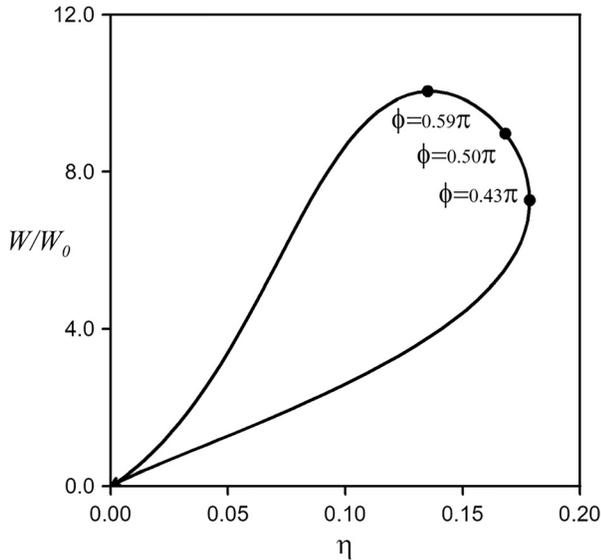


Fig. 3. Dimensionless work as a function of the efficiency, where $W_0 = P_0 V_{10}$ and the angle ϕ varying between 0 and π . The dots on the curve indicate the values of ϕ for the maximum efficiency ($\phi=0.43\pi$), resonance ($\phi=0.50\pi$), and maximum ($\phi=0.59\pi$) work. The values of the parameters are the same as in Fig. 2.

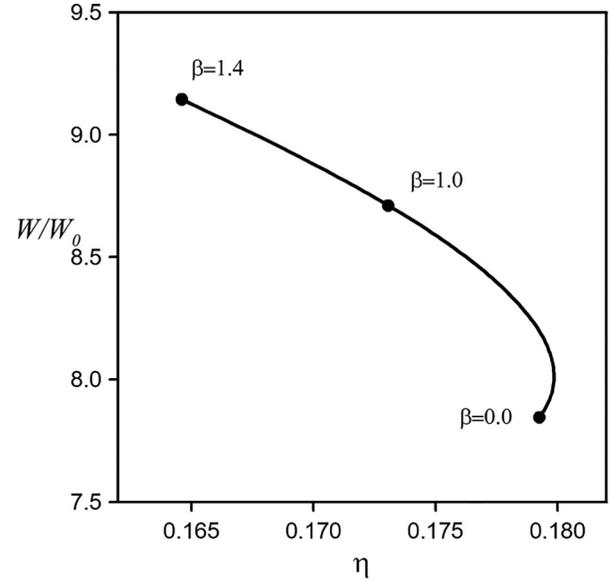


Fig. 4. Dimensionless work as a function of the efficiency when the polytropic index β varies between 0.0 and 1.4. The dots on the curve indicate three particular values of β . The other parameters are the same as those of previous figures.

We have numerically integrated Eqs. (18), (21) and (22) using the standard Simpson's rule. Figure 3 shows the relation between work and efficiency as functions of the phase ϕ . From this figure, it is clear that the phasing between the piston and the displacer plays a central role in the functioning of the engine; in particular, we observe that the maximum efficiency phase does not coincide with the maximum work phase. The PV diagrams in Fig. 2 correspond precisely to the phases of maximum work and maximum efficiency.

When $\phi = \pi/2$, the piston is in resonance with the displacer and this situation is intermediate between maximum work and maximum efficiency; there the engine is in a good performance zone, where the engine operates in a good compromise between power and efficiency.

Figure 4 shows the behavior of work and efficiency when the polytropic index β varies. This parameter depends on the gas initial thermodynamic conditions and in particular on its relative humidity.⁸ The figure shows that the dependence on β is stronger for the power than for the efficiency.

IV. CONCLUSIONS

Research on Stirling engines is one of the lines that contribute both to the rational use of energy and to sustainable development. In particular, the solar thermal conversion systems based on these engines are among the most interesting and promising research lines.¹³⁻¹⁸ This paper presents an unusual application of heat engine thermodynamics combined with the physics of oscillators at a level appropriate for advanced undergraduate students.

From a strictly technical point of view, we have proposed a theoretical model that describes the thermodynamics of a Stirling engine in a simple, precise, and natural way.⁶ The engine studied in this paper is a special type of the Stirling engine; therefore, we have applied the theoretical model to investigate its thermodynamics and dynamical evolution. The exchange of heat with the environment is modeled as a

polytropic process. We obtain analytic expressions for the piston amplitude, the working gas temperatures, and pressure. These quantities are expressed as functions of (i) the phase difference between the power piston and the displacer, ϕ ; (ii) the ratio of the temperatures of the heat reservoirs, α ; (iii) the polytropic index, β ; and (iv) the geometrical characteristics of the engine. We also study numerically the engine power and its efficiency as a function of ϕ and β and show that, when in resonance, the engine is working in a good performance regime. These results show the versatility of the thermodynamic model developed here to describe different types of Stirling engines and they encourage further work on this line.

In summary, the theoretical approach proposed describes analytically the Fluidyne engine thermodynamics and it contributes to an understanding of the subtle way through which this engine transforms heat into work.

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