



**Ejercicio 1** Encontrar el resultado o expresar en forma reducida cada uno de los siguientes números.

$$1. 2 + 3 \times 4 - ((-3)2 + 4 - 3 \times 5) = 31$$

$$2. 4(3 - 5) - 3^2 2^3 + 7\sqrt{25} = -45$$

$$3. \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

$$4. 4\left(\frac{1}{3}\right) = \frac{4}{3}$$

$$5. \frac{-3}{5}\left(\frac{2}{3} - 1\right) - \frac{4}{3} = -\frac{17}{15}$$

$$6. \left(1 + \frac{1}{2}\right)^2 = \frac{9}{4}$$

$$7. \frac{\frac{1}{2}}{\frac{1}{3} + \frac{3}{4}} = \frac{6}{13}$$

$$8. \left(\frac{1}{3} + \frac{4}{5}\right)\left(\frac{1}{4} - \frac{3}{2}\right) = -\frac{17}{12}$$

$$9. \left(\frac{1}{5} - \frac{2}{3}\right)^3 = -\frac{7^3}{15^3}$$

$$10. \left(\frac{2^3}{3^3}\right)^4 \left(\frac{3}{4}\right)^2 = \frac{2^8}{3^{10}}$$

$$11. \left(\frac{1/3}{2/5}\right)^{-2} = \frac{36}{25}$$

$$12. \frac{7\frac{3}{2}}{7\frac{2}{2}} = \frac{1}{7}$$

$$13. 3! + \frac{1}{3!} = \frac{37}{6}$$

$$14. \frac{5!}{2!+3!} = 15$$

$$15. \frac{6!}{2!3!} = 60$$

**Ejercicio 2** Calcular simplificando la respuesta lo más posible. Expresar el resultado como una sola fracción reducida.

$$1. \frac{3}{5} - \frac{4}{3} = -\frac{11}{15}$$

$$2. \frac{3}{5} \times \frac{4}{3} \times \frac{5}{2} = 2$$

$$3. \frac{1+\frac{3}{2}}{\frac{3}{4}-1} = -10$$

$$4. \frac{3}{4(x+1)} - \frac{7}{2(x-1)} = -\frac{11x+17}{4(x-1)(x+1)} = -\frac{11x+17}{4(x^2-1)}$$

$$5. \frac{x^2-4}{x+1} \div \frac{x+2}{3x-5} = \frac{(x-2)(3x-5)}{x+1}$$

$$6. \frac{xy}{yz} - \frac{y}{z} = \frac{x-y}{z}$$

$$7. \frac{A}{x-1} + \frac{B}{x+2} = \frac{(A+B)x+2A-B}{(x-1)(x+2)}$$

$$8. \frac{x}{yz} + \frac{y}{z} = \frac{x+y^2}{yz}$$

$$9. \frac{3x}{5y} + \frac{4x}{2y^2} = \frac{6xy+20x}{10y^2} = \frac{x(3y+10)}{5y^2}$$

$$10. \frac{\frac{x}{y}+1}{1-\frac{y}{x}} = \frac{x^2+yx}{xy-y^2}$$

$$11. \frac{x+\frac{y}{z}}{\frac{y}{z}-z} = \frac{xz+y}{y-z^2}$$

$$12. \left(\frac{x}{y}\right)^3 \left(\frac{xy^2}{z}\right)^4 = \frac{x^7 y^5}{z^4}$$

**Ejercicio 3** Simplificar los siguientes radicales (se asumen,  $a, b, x, y$  positivos)

$$1. \sqrt{32}\sqrt{2} = 8$$

$$2. \sqrt[3]{24} = 2\sqrt[3]{3}$$

$$3. 10\sqrt[3]{5^2} = 2 \times 5^{5/3} = 2\sqrt[3]{5^5}$$

$$4. \sqrt{\frac{243}{125}} = \frac{9\sqrt{3}}{5\sqrt{5}}$$

$$5. \sqrt[3]{5}\sqrt[3]{5^2} = 5$$

$$6. \frac{\sqrt[6]{4^5}}{\sqrt[6]{9^3}} = \frac{4^{5/6}}{3}$$

$$7. \frac{\sqrt[3]{-2}}{\sqrt[3]{54}} = -\frac{1}{3}$$

$$8. \frac{1}{\sqrt[3]{x^4}} = x^{-4/3}$$

$$9. \frac{\sqrt[4]{32x^4}}{\sqrt[4]{2}} = 2x$$

$$10. \sqrt{xy}\sqrt{x^3y} = x^2y$$

$$11. \sqrt{16a^4b^3} = 4a^2b\sqrt{b}$$

$$12. \frac{\sqrt[5]{96a^6}}{\sqrt[5]{3a}} = 2a$$



## Ejercicio 4 *Calcular:*

$$1. \sqrt{112} - 2\sqrt{63} + \frac{3}{5}\sqrt{175} = \sqrt{7}$$

$$3. 2\sqrt{25} - \sqrt[4]{16} + 3\sqrt[5]{32} - \sqrt[3]{-8} = 16$$

$$2. \frac{\sqrt{2}\sqrt[4]{2}\sqrt[6]{2}}{2^4\sqrt[3]{8}} + \frac{2^5}{4^2} = 2^{-49/12} + 2$$

## Ejercicio 5 *Simplificar las siguientes expresiones (se asumen, $a, b, x, y, z$ positivos):*

$$1. a^2b^{-2}a^{-\frac{1}{3}}\frac{1}{b^4}(b^2)^{\frac{1}{2}} = a^{\frac{5}{3}}b^{-5}$$

$$2. \frac{a^{-3}b^3a^4}{a^{-2}a^5b^3} = \frac{1}{a^2} = a^{-2}$$

$$3. \frac{x^2y^{-\frac{5}{4}}z^3}{y^{-2}z^4x} = \frac{xy^{3/4}}{z}$$

## Ejercicio 6 *Factorizar las siguientes expresiones:*

$$1. 2x + 12x^3 = 2x(6x^2 + 1)$$

$$11. t^3 + 1 = (t + 1)(t^2 - t + 1)$$

$$2. 5ab - 8abc = (-8c + 5)ba$$

$$12. 4t^2 - 9s^2 = -(3s - 2t)(3s + 2t)$$

$$3. x^2 + 7x + 6 = (x + 1)(x + 6)$$

$$13. 4t^2 - 12t + 9 = (2t - 3)^2$$

$$4. x^2 - x - 6 = (x - 3)(x + 2)$$

$$14. x^3 - 27 = (x - 3)(x^2 + 3x + 9)$$

$$5. x^2 - 2x - 8 = (x - 4)(x + 2)$$

$$15. x^3 + 2x^2 + x = x(x + 1)^2$$

$$6. 2x^2 + 7x - 4 = (x + 4)\left(2x - \frac{1}{2}\right) = 2(x + 4)\left(2x - \frac{1}{2}\right)$$

$$16. x^3 - 4x^2 + 5x - 2 = (x - 2)(x - 1)^2$$

$$7. 9x^2 - 36 = 9(x - 2)(x + 2)$$

$$17. x^3 + 3x^2 - x - 3 = (x - 1)(x + 1)(x + 3)$$

$$8. 8x^2 + 10x + 3 = (2x + 1)\left(4x + \frac{3}{2}\right) = 8\left(x + \frac{1}{2}\right)\left(x + \frac{3}{4}\right)$$

$$18. x^3 - 2x^2 - 23x + 60 = (x - 4)(x - 3)(x + 5)$$

$$9. 6x^2 - 5x - 6 = (2x - 3)\left(3x + \frac{2}{3}\right) = 6\left(x - \frac{3}{2}\right)\left(x + \frac{2}{3}\right)$$

$$19. 2x^3 + 7x^2 - 5x - 4 = (x - 1)(x + 4)\left(2x + \frac{1}{2}\right) = 2(x - 1)(x + 4)\left(x + \frac{1}{2}\right)$$

$$10. x^2 + 10x + 25 = (x + 5)^2$$

$$20. x^3 - 3x^2 - 4x + 12 = (x - 3)(x - 2)(x + 2)$$

## Ejercicio 7 *1. Expresar en forma logarítmica las siguientes expresiones:*

$$a) 5^3 = 125 \Leftrightarrow \log_5 125 = 3$$

$$b) 3^{-2} = \frac{1}{9} \Leftrightarrow \log_3 \frac{1}{9} = -2$$

$$c) \left(\frac{1}{2}\right)^3 = \frac{1}{8} \Leftrightarrow \log_{\frac{1}{2}} \frac{1}{8} = 3$$

2. Hallar  $\log(32)$  en base 4 y  $\log(1/81)$  en base 27.

$$\blacksquare \log_4 32 = \log_4(2^5) = \frac{5}{2}$$

$$\blacksquare \log_{27}\left(\frac{1}{81}\right) = \log_{27}(3^{-4}) = \frac{-4}{3}$$

3. Expresar  $\log(75/16) - 2\log(5/9) + \log(32/243)$  en términos de  $\log(2)$  y  $\log(3)$ .



$$\begin{aligned} & \blacksquare \log(75/16) - 2\log(5/9) + \log(32/243) = \\ & \log\left(\frac{3 \times 5^2}{2^4}\right) + \log\left(\frac{9}{5}\right)^2 + \log\left(\frac{2}{3}\right)^5 = \\ & 2\log 5 + \log 3 - 4\log 2 + 4\log 3 - 2\log 5 + 5\log 2 - 5\log 3 = \log 2 \end{aligned}$$

4. ¿Se cumple que  $2\log(15/18) - \log(25/162) + \log(4/9) = \log(2)$ ? R: Sí, se cumple.

$$\begin{aligned} & \blacksquare 2\log(15/18) - \log(25/162) + \log(4/9) = \\ & 2\log\left(\frac{3 \times 5}{3 \times 3 \times 2}\right) - \log\left(\frac{5^2}{2 \times 3^4}\right) + \log\left(\frac{2^2}{3^2}\right) = \\ & 2\log 5 - 2\log 2 - 2\log 3 - 2\log 5 + \log 2 + 4\log 3 + 2\log 2 - 2\log 3 = \log 2 \end{aligned}$$

**Ejercicio 8** Simplificar las siguientes expresiones:

- $\log_2 16 + \log_2 32 = \log_2(2^4) + \log_2(2^5) = 4\log_2 2 + 5\log_2 2 = 9$
- $\log_3 81 - \log_3 9 = \log_3(3^4) - \log_3(3^2) = 4\log_3 3 - 2\log_3 3 = 2$
- $\log_2(1/8) - \log_3(1/9) = \log_2(2^{-3}) - \log_3(3^{-2}) = -3\log_2 2 + 2\log_3 3 = -1$
- $\log_3 729 - 2\log_3 3^3 + \log_4 2 = \log_3(3^6) - 6\log_3 3 + \log_4 2 = 6\log_3 3 - 6\log_3 3 + \frac{1}{2}\log_2 2 = \frac{1}{2}$
- $\frac{1}{2}\log_2 48 - \frac{2}{3}\log_3 27^9 = \frac{1}{2}\log_2(3 \times 2^4) - \frac{2}{3}\log_3(3^3)^9 =$   
 $= \frac{1}{2}\log_2 3 + 2\log_2 2 - \frac{2}{3}27\log_3 3 = \frac{1}{2}\log_2 3 - 16 = \frac{1}{2}\log_2\left(\frac{3}{2^{32}}\right)$
- $\log_6 24 + 2\log_6 3 = \log_6(6 \times 2^2) + \log_6(3^2) = \log_6 6 + 2(\log_6 2 + \log_6 3) = 3$

**Ejercicio 9** Escribir en términos de un único logaritmo:

- $3\ln(x) + \frac{1}{3}\ln(x+2) = \ln(x^3) + \ln(\sqrt[3]{x+2}) = \ln(x^3 \sqrt[3]{x+2})$
- $-3\ln(x) + 4\ln(y) + 5\ln(z) = \ln\left(\frac{y^4 z^5}{x^3}\right)$
- $5\ln(x+1) + 2\ln(x^3) - \ln(x) = \ln((x+1)^5 x^5) = 5\ln((x+1)x)$
- $\ln(\sqrt{8x+2}) - \ln(\sqrt{4-x}) + \ln(2) = \frac{1}{2}\ln\left(\frac{4(8x+2)}{4-x}\right)$
- $\log_3(a^2) + \log_3(9) - 5\log_3(x) = \log_3 \frac{(3a)^2}{x^5}$
- $\ln(x+8) - \ln(x) - \ln(8) = \ln \frac{x+8}{8x}$
- $\log_3(2x^{-1}) + \log_3(12x^5) + \log_3\left(\frac{2}{3}x^{-3}\right) = \log_3 16x$
- $\log(4y^3) + \log(5y^4) - \log(2y^5) = \log(10y^2) = 1 + 2\log y$

**Ejercicio 10** 1. Si  $p = e^x$  y  $q = e^y$  expresar  $e^{2y-3x}$  en función de  $p$  y de  $q$ .

$$\blacksquare e^{2y-3x} = \frac{(e^y)^2}{(e^x)^3} = \frac{q^2}{p^3}$$

2. Si  $\ln(a) = 2$  y  $\ln(b) = 4$ , calcular  $\ln\left(\frac{1}{\sqrt{ab}}\right)$ .



- $\ln\left(\frac{1}{\sqrt{ab}}\right) = -\frac{1}{2}(\ln a + \ln b) = -3$

3. Expandir las siguientes expresiones: a)  $\log(24(k+2)^3)$ , b)  $\log_3\left(\frac{243y^2}{\sqrt{2}}\right)$

a)  $\log(24(k+2)^3) = \log 24 + 3\log(k+2)$

b)  $\log_3\left(\frac{243y^2}{\sqrt{2}}\right) = \log_3 3^5 + 2\log_3 y - \frac{1}{2}\log_3 2 = 5 - \frac{1}{2}\log_3 2 + 2\log_3 y$