Escalado Multidimensional. (Ej 2 Práctico 4)

$$Q = -\frac{1}{2} P D^{(2)} P.$$

Suponga que $Q = (PX)(PX)'$

$= \tilde{X}\tilde{X}'$

descomp. espectral. $= U \Lambda U'$

Q simétrica y es semidefinida positiva ($\lambda \geq 0$ $\forall \lambda$ valor propio)

$= Y Y'$ donde $Y = U\Lambda^{1/2}$

siendo $U = (u_1 | u_2 | \cdots | u_n)$

donde $\{u_1, u_2, \ldots, u_n\}$ son vectores propios de Q.

los valores propios > 0 de $\tilde{X}\tilde{X}'$ y de $\tilde{X}'\tilde{X}$ coinciden.

vectores propios

$\tilde{X}\tilde{X}'$: $u_1, u_2, \ldots, u_n \in \mathbb{R}^n$

$\tilde{X}'\tilde{X}$: loadings. $a_1, a_2, \ldots, a_p \in \mathbb{R}^p$.

comp. principales. $z_1 = \tilde{X}a_1,\ z_2 = \tilde{X}a_2 \ldots z_p = \tilde{X}a_p \in \mathbb{R}^n$

si a_1 es vector propio de $\tilde{X}'\tilde{X}$ asociado a λ_1

$$\boxed{\tilde{X}'\tilde{X} a_1 = \lambda_1 a_1}$$

$$\Rightarrow \tilde{X}\tilde{X}'(\tilde{X} a_1) = \tilde{X}(\lambda_1 a_1)$$

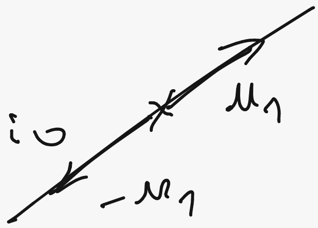
$$\Rightarrow \boxed{\tilde{X}\tilde{X}'(\tilde{X} a_1) = \lambda_1 \tilde{X} a_1.}$$

$\Rightarrow$ $\tilde{X} a_1$ es vector propio de $\tilde{X}\tilde{X}' = Q$ asociado a λ_1

unitario ($\|u_1\| = 1$).

$$\hookrightarrow \underbrace{\tilde{X} a_1}_{z_1 \leftarrow \text{no unitario}} \parallel u_1$$

$$\Rightarrow u_1 = \frac{z_1}{\sqrt{\lambda_1}}$$



2 sentidos para dirigir el subespacio propio.

Cuando hacemos escalado multidimensional lo que obtenemos son las coordenadas de la proyección de los individuos en el plano factorial.

$$Q = U \Lambda U' \qquad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

symétrique

$$= \begin{pmatrix} \boxed{u_1} & \boxed{u_2} \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \boxed{u_1} & \boxed{u_2} \end{pmatrix}'$$

$$= \begin{pmatrix} \boxed{u_1} & \boxed{u_2} \end{pmatrix} \begin{pmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{pmatrix} \begin{pmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{pmatrix} \begin{pmatrix} \boxed{u_1} & \boxed{u_2} \end{pmatrix}'$$

$$= \begin{pmatrix} \boxed{\sqrt{\lambda_1} u_1} & \boxed{\sqrt{\lambda_2} u_2} \end{pmatrix} \begin{pmatrix} \boxed{\sqrt{\lambda_1} u_1} & \boxed{\sqrt{\lambda_2} u_2} \end{pmatrix}'$$

$$= \begin{pmatrix} \boxed{z_1} & \boxed{z_2} \end{pmatrix} \begin{pmatrix} \boxed{z_1} & \boxed{z_2} \end{pmatrix}'$$

$$= Z \, Z'.$$

Obs: $Q = YY'$

Si $\tilde{Y} = YA$, A orthogonal. (A matriz de rotación).

$$A'A = AA' = I.$$

$$\begin{aligned}
\tilde{Y}\tilde{Y}' &= (YA)(YA)' \\
&= YAA'Y' \\
&= YIY' \\
&= YY' = Q.
\end{aligned}$$

$$\begin{bmatrix} x_{1j} \\ x_{2j} \\ \vdots \\ x_{nj} \end{bmatrix}$$

col. j

$$\tilde{x}_{1j} = \frac{x_{1j} - \bar{x}_j}{\sigma_j}$$

$$\bar{x}_j = \frac{x_{1j} + x_{2j} + \dots + x_{nj}}{n}$$

$$\sigma_j = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}$$

(desvío)

R no usa σ_j pero $s_j = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}$

(desvío corregido)

Ejercicio 5 Práctico 4.

$$E = \{ x_1, x_2, \ldots, x_n \}. \qquad X = \begin{pmatrix} \boxed{x_1} \\ \boxed{x_2} \\ \vdots \\ \boxed{x_n} \end{pmatrix}$$

$$d^2_{n+1,i} = d^2(\underset{\substack{\uparrow \\ \text{individuo} \\ n+1}}{x_{n+1}}, \underset{\substack{\uparrow \\ \text{individuo} \\ i}}{x_i}) = \|x_{n+1} - x_i\|^2$$

$$= \|x_{n+1}\|^2 + \|x_i\|^2 - 2\, x_{n+1}' x_i$$

$$= x_{n+1}' x_{n+1} + x_i' x_i - 2 x_{n+1}' x_i$$

$$d = \begin{pmatrix} d^2_{n+1,1} \\ \vdots \\ d^2_{n+1,n} \end{pmatrix} = \|x_{n+1}\|^2 \underset{\substack{\uparrow \\ \text{vector} \\ n \text{ dimensional} \\ \text{de } 1\text{'s}}}{1_n} + \begin{pmatrix} x_1' x_1 \\ x_2' x_2 \\ \vdots \\ x_n' x_n \end{pmatrix} - 2 X x_{n+1}$$

$\left(\begin{array}{c} Q = XX' \\ \text{con } X = U\Lambda^{1/2} \end{array} \right)$

$$\Rightarrow \quad d = \|x_{n+1}\|^2 1_n + \underbrace{\text{diag}(Q)}_{q} - 2 X x_{n+1}$$

Multiplico por X'

$$\Rightarrow X'd = \|x_{n+1}\|^2 X' 1_n + X' q - 2 X' X x_{n+1}$$

$$\Rightarrow 2 X' X x_{n+1} = X'(q - d) + \|x_{n+1}\|^2 \underbrace{X' 1_n}_{= \vec{0}}$$

$$\Rightarrow \quad x_{n+1} = \frac{1}{2} (X'X)^{-1} X' (q - d)$$

$$x_{n+1} = \frac{1}{2} \Lambda^{-1} X' (q - d)$$

Recordar las fórmulas:

* contribución individuo x_i al eje k:

$$\frac{p_i \, c_{ik}^2}{\lambda_k} \qquad c_{ik} = \langle x_i, a_k \rangle$$

$$X a_k = z_k = \begin{pmatrix} c_{1k} \\ c_{2k} \\ \vdots \\ c_{nk} \end{pmatrix}, \quad \|z_k\|^2 = \lambda_k$$

* contribución variable x_j al eje k:

$$\frac{d_{jk}^2}{\lambda_k} = a_{jk}^2 \qquad d_{jk} = \langle x_j, v_k \rangle$$

$$X' v_k = w_k = \begin{pmatrix} d_{1k} \\ d_{2k} \\ \vdots \\ d_{pk} \end{pmatrix} \quad \|w_k\|^2 = \lambda_k$$

* calidad individuo x_i sobre eje k:

$$\cos^2(\theta_{ik}) = \frac{p_i \, c_{ik}^2}{\|x_i\|^2}$$

* calidad variable x_j sobre eje k:

$$\cos^2(\theta_{jk}) = \frac{d_{jk}^2}{\|x_j\|^2}$$