Problema 4

Solución:

1) Ecuaciones

$$\mathrm{M} * \ddot{\mathrm{H}} = \mathrm{P}_4 * A - M * g$$

$$\mathrm{m} * \ddot{x} = P_3 * a - K * V - b * \dot{x} \approx 0$$

$$P_1 = P_3 + \rho * g * x$$

$$P_2 = P_4 + \rho * g * H$$

$$\mathrm{Q} = \gamma * \sqrt{P_1 - P_2}$$

$$\mathrm{Q} = \mathrm{A} * \dot{H}$$

$$VL = a * x + c * L + A * H$$

$$x = -\frac{A}{a} * H$$

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$$P_1 = \rho * g * x + \left(\frac{K}{a}\right) * V + \left(\frac{b}{a}\right) * \dot{x}$$

$$P_1 = \rho * g * \left(\frac{(VL - c * L)}{a} - \left(\frac{A}{a}\right) * H\right) + \left(\frac{K}{a}\right) * V + \left(\frac{b * A}{a^2}\right) * \dot{x}$$

$$P_2 = \rho * g * H + \left(\frac{M}{A}\right) * \ddot{\mathrm{H}} + \left(\frac{M}{A}\right) * g$$

$$\left(\frac{A}{\gamma}\right)^2 * \dot{H}^2 = (P_1 - P_2) * sgn(P_1 - P_2) \qquad \beta = sgn(P_1 - P_2)$$

$$\beta * \left(\frac{A}{\gamma}\right)^2 * \dot{H}^2 = (P_1 - P_2)$$

$$\beta * \left(\frac{A}{\gamma}\right)^2 * \dot{H}^2 = \left[\rho * g * \left(\frac{(VL - c * L)}{a} - \left(\frac{A}{a}\right) * H\right) + \left(\frac{K}{a}\right) * V - \left(\frac{b * A^2}{a^3}\right) * \dot{H}\right] - \left[\rho * g * H + \left(\frac{M}{A}\right) * \ddot{\mathrm{H}} + \left(\frac{M}{A}\right) * g\right]$$

$$\left(\frac{M}{A}\right) * \ddot{H} + \beta\left(\frac{A}{\gamma}\right)^2 * \dot{H}^2 + \left(\frac{bA^2}{a^3}\right) * \dot{H} + \rho g\left(\frac{A+a}{a}\right)H = \left(\frac{K}{a}\right) * V + \rho g\left(\frac{VL-cL}{a}\right) - \left(\frac{M}{A}\right)g$$

2) Posición de equilibrio:

$H = H_0,\ V = V_{0,}\quad \dot{H}_0 = 0,\ \ddot{H}_0 = 0$

$$V = \rho g\left(\frac{A+a}{K}\right)H - \rho g\left(\frac{VL-cL}{K}\right) - \left(\frac{M}{A}\right)g$$

3) Modelo en pequeña señal

$H = H_0 + h,\ \ V = V_0 + v$

$$\ddot{H} = -\beta\left(\frac{A^3}{M_0\gamma^2}\right)\dot{H}^2 - \left(\frac{bA^3}{M_0a^3}\right) * \dot{H} - \rho g\left(\frac{A+a}{M_0a}\right)AH + \left(\frac{KA}{M_0a}\right)V + \rho g\left(\frac{VL-cL}{M_0a}\right)A - g$$

$$\ddot{h} = -\left(\frac{bA^3}{M_0a^3}\right) * \dot{h} - \rho g\left(\frac{A+a}{M_0a}\right)A * h + \left(\frac{KA}{M_0a}\right) * v$$

$$\frac{H(s)}{V(s)} = \frac{\left(\frac{KA}{M_0a}\right)}{s^2 + \left(\frac{bA^3}{M_0a^3}\right)s + \rho g\left(\frac{A+a}{M_0a}\right)A}$$

4) $\rho g\left(\frac{A+a}{a}\right)H_0 = \left(\frac{K}{a}\right) * V_0 + \rho g\left(\frac{VL-cL}{a}\right) - \left(\frac{M_0}{A}\right)g$

$$\rho g\left(\frac{A+a}{a}\right)H = \left(\frac{K}{a}\right) * V_0 + \rho g\left(\frac{VL-cL}{a}\right) - \left(\frac{M}{A}\right)g$$

$$\frac{\Delta H}{\Delta M} = -\frac{\left(\frac{a}{A}\right)g}{\rho g A}$$

5) Sistema realimentado

a) $\rho g\left(\frac{A+a}{a}\right)H_0 = \left(\frac{K}{a}\right) * V_0 + \rho g\left(\frac{VL-cL}{a}\right) - \left(\frac{M_0}{A}\right)g$

$$\rho g\left(\frac{A+a}{a}\right)H = \left(\frac{K}{a}\right)\left(V_0 - KcKs(H-H_0)\right) + \rho g\left(\frac{VL-cL}{a}\right) - \left(\frac{M}{A}\right)g$$

$$\rho g\left(\frac{A+a}{a}\right)(H-H_0) = -\left(\frac{K}{a}\right)KcKs(H-H_0) - \left(\frac{M-M_0}{A}\right)g$$

$$\left(\rho g\left(\frac{A+a}{a}\right) + \left(\frac{K}{a}\right)KcKs\right)(H-H_0) = -\left(\frac{M-M_0}{A}\right)g$$

$$\left(\rho g\left(\frac{A+a}{a}\right) + \left(\frac{K}{a}\right)KcKs\right)(H-H_0) = -\left(\frac{M-M_0}{A}\right)g$$

$$\frac{\Delta M}{\Delta H} = -\frac{\frac{a}{A}g}{\rho gA + KKcKs}$$

Relación:

$$R = \frac{\rho gA}{\rho gA + KKcKs}$$

b) V = cte

$$\ddot{H} = \frac{1}{M}\left(-\beta\left(\frac{A^3}{\gamma^2}\right)\dot{H}^2 - \left(\frac{bA^3}{a^3}\right) * \dot{H} - \rho g\left(\frac{A+a}{a}\right)AH + \left(\frac{KA}{a}\right)V + \rho g\left(\frac{VL-cL}{a}\right)A - Mg\right)$$

$$\ddot{h} = -\left(\frac{bA^3}{M_0 a^3}\right) * \dot{h} - \rho g\left(\frac{A+a}{M_0 a}\right)A * h + \left(\frac{KA}{M_0 a}\right) * v - g\Delta m$$

$$\frac{\Delta H(s)}{\Delta M(s)} = -\frac{\frac{g}{M_0}}{s^2 + \left(\frac{bA^3}{M_0 a^3}\right)s + \rho g\left(\frac{A+a}{M_0 a}\right)A}$$

Sistema realimentado

$$\ddot{h} = -\left(\frac{bA^3}{M_0 a^3}\right) * \dot{h} - \rho g\left(\frac{A+a}{M_0 a}\right) A * h + \left(\frac{KA}{M_0 a}\right) * (-KcKsh) - g\Delta m$$

$$\frac{\Delta H(s)}{\Delta M(s)} = -\frac{\frac{g}{M_0}}{s^2 + \left(\frac{bA^3}{M_0 a^3}\right) s + \left(\frac{\rho g A}{M_0 a}\right)(A + KKcKs)}$$

c) $\left(\frac{bA^3}{M_0 a^3}\right)^2 - 4\left(\frac{\rho g A}{M_0 a}\right)(A + KKcKs) = 0$

$$Kc = \frac{\left(\frac{bA^3}{M_0 a^3}\right)^2 - 4\left(\frac{\rho g A^2}{M_0 a}\right)}{4\left(\frac{\rho g A}{M_0 a}\right) KKs}$$

$$\frac{H(s)}{H_{R(s)}} = \frac{\left(\frac{KA}{M_0 a}\right)}{\left(s + \frac{bA^3}{2M_0 a^3}\right)^2}$$

d) $$\frac{H(s)}{H_{R(s)}} = \frac{\left(\frac{4KM_0 a^5}{bA^5}\right)}{\left(\left(\frac{2M_0 a^3}{bA^3}\right)s + 1\right)^2}$$

$$G = \frac{4KM_0 a^5}{bA^5}$$

$$T = \frac{2M_0 a^3}{bA^3}$$

$$h(t) = G\left(1 - e^{-\frac{t}{T}} - \frac{t}{T} t e^{-\frac{t}{T}}\right)$$

$$e^{-\frac{t}{T}}\left(1 + \frac{t}{T}\right) < 0.01$$

$$\boxed{t = 6.64T}$$