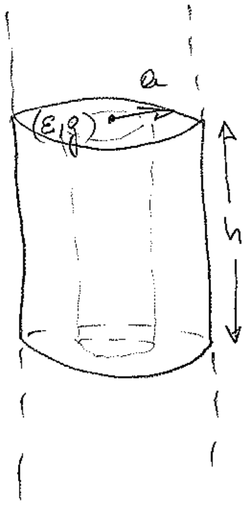
SOLUCIÓN EXAMEN ELECTROMAGNETISMO 23/12/2015.-

(1)



$$\rho(r,0) = \rho_0 \frac{a}{r} \qquad \begin{cases} \vec{D} = \varepsilon \vec{E} \\ \vec{J} = g \vec{E} \end{cases}$$

a) $$\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0 \longrightarrow \frac{g}{\varepsilon}\underbrace{(\nabla\cdot\vec{D})}_{\rho} + \frac{\partial\rho}{\partial t} = 0$$

$$\Rightarrow \boxed{\rho(r,t) = \rho(r,0)\, e^{-\frac{g}{\varepsilon}t} = \left(\frac{\rho_0 a}{r}\right) e^{-\frac{g}{\varepsilon}t}}$$

$$\int_V \nabla\cdot\vec{J}\, dv = -\int_V \frac{\partial\rho}{\partial t}\, dv \longrightarrow \oint_S \vec{J}\cdot\hat{n}\, da = -\int_V \frac{\partial\rho}{\partial t}\, dv$$

$$J\,(2\pi r h) = \cancel{-}\int_0^r (2\pi \cancel{r} h\, dr)\left[\frac{\rho_0 a}{\cancel{r}}\left(\cancel{-}\frac{g}{\varepsilon}\right) e^{-\frac{g}{\varepsilon}t}\right]$$

$$J\,(\cancel{2\pi}\cancel{r}\cancel{h}) = \cancel{2\pi h}\,\frac{g}{\varepsilon}\, e^{-\frac{g}{\varepsilon}t} \rho_0 a \underbrace{\int_0^r dr}_{\cancel{r}} = \frac{\rho_0 a g}{\varepsilon} e^{-\frac{g}{\varepsilon}t}$$

$$\Rightarrow \boxed{J = \frac{\rho_0 a g}{\varepsilon} e^{-\frac{g}{\varepsilon}t}} \quad (*)$$

b) $$P(t) = \int_V \underbrace{\vec{J}\cdot\vec{E}}_{\frac{J^2}{g}}\, dv = \frac{1}{g}\, \frac{\rho_0^2 a^2 g^{2}}{\varepsilon^2} e^{-\frac{2g}{\varepsilon}t} \underbrace{(\pi a^2 h)}_{\text{vol}} = \frac{\pi h \rho_0^2 a^4 g}{\varepsilon^2} e^{-\frac{2g}{\varepsilon}t}$$

J cte en V

$$E_{dis} = \int_0^\infty P(t)\, dt = \frac{\pi h \rho_0^2 a^4 g}{\varepsilon^2}\left(-\frac{\varepsilon}{2g}\right) e^{-\frac{2g}{\varepsilon}t}\Big|_0^{+\infty} = \frac{\pi h \rho_0^2 a^4 \cancel{g}}{\varepsilon^{\cancel{2}}}\left(-\frac{\cancel{\varepsilon}}{2\cancel{g}}\right)(0-1)$$

$$\Rightarrow \boxed{\frac{E_{dis}}{h} = \frac{\pi}{2}\frac{\rho_0^2 a^4}{\varepsilon}}$$

(*) Alternativamente, $\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

En coords. cilíndricas (geom. axial) $\vec{J} = J\,\hat{e}_r$ $\quad \nabla\cdot\vec{J} = \frac{1}{r}\frac{d(rJ)}{dr}$

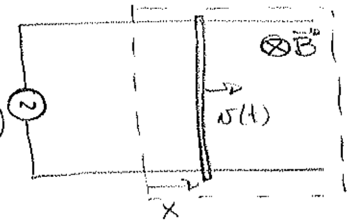
$$\Rightarrow \frac{1}{r}\frac{d(rJ)}{dr} = -\rho_0 \frac{a}{r}\left(-\frac{g}{\varepsilon}\right) e^{-\frac{g}{\varepsilon}t}$$

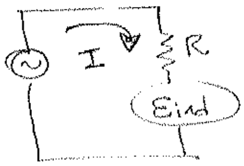
$$\int_0^r \frac{d(rJ)}{dr}\, dr = \int_0^r \left[\frac{\rho_0 a g}{\varepsilon} e^{-\frac{g}{\varepsilon}t}\right] dr$$

$$\cancel{r} J = \left[\frac{\rho_0 a g}{\varepsilon} e^{-\frac{g}{\varepsilon}t}\right] \cancel{r} \longrightarrow \boxed{J = \frac{\rho_0 a g}{\varepsilon} e^{-\frac{g}{\varepsilon}t}}$$

(2) a) posición (1):

$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t)$





$\vec{F} = I\vec{l}\times\vec{B}$

$m\dot{v} = IlB \rightarrow I = \dfrac{m\dot{v}}{lB}$ (1)

$\mathcal{E} + \mathcal{E}_{ind} - RI = 0$ (2)

$\mathcal{E}_{ind} = -\dfrac{d\phi_B}{dt} = \dfrac{d(Blx)}{dt} = -Blv$ (3)

Sust. (1) y (3) en (2): $\mathcal{E} - Blv - \dfrac{Rm}{lB}\dot{v} = 0$

En régimen, $\mathcal{E} = \mathrm{Re}\{\mathcal{E}_0 e^{j\omega t}\}$

$\Rightarrow v = \mathrm{Re}\{v_0 e^{j(\omega t + \phi)}\} = \mathrm{Re}\{\tilde{v}_0 e^{j\omega t}\}$ con $\tilde{v}_0 = v_0 e^{j\phi}$

$\Rightarrow \mathcal{E}_0 - Bl\tilde{v}_0 - j\omega\dfrac{Rm}{lB}\tilde{v}_0 = 0$

$\mathcal{E}_0 - \left(Bl + j\dfrac{\omega Rm}{lB}\right)\tilde{v}_0 = 0 \rightarrow \tilde{v}_0 = \dfrac{\mathcal{E}_0}{Bl + j\dfrac{\omega Rm}{lB}} = \dfrac{\mathcal{E}_0 Bl}{B^2l^2 + j\omega Rm}$

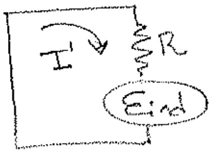
$$\boxed{v_0 = |\tilde{v}_0| = \sqrt{\tilde{v}_0 \cdot \tilde{v}_0^*} = \dfrac{\mathcal{E}_0 Bl}{\sqrt{B^4l^4 + \omega^2R^2m^2}}}$$

$$\boxed{\mathrm{tg}\,\phi = -\dfrac{\omega Rm}{B^2l^2} \longrightarrow \phi = \mathrm{arctg}\left(\dfrac{-\omega Rm}{B^2l^2}\right) = -\mathrm{arctg}\left(\dfrac{\omega Rm}{B^2l^2}\right)}$$

$$\Rightarrow \boxed{v(t) = \dfrac{\mathcal{E}_0 Bl}{\sqrt{B^4l^4 + \omega^2R^2m^2}}\cos\left[\omega t - \mathrm{arctg}\left(\dfrac{\omega Rm}{B^2l^2}\right)\right]}$$

b) $\mathrm{arctg}\left(\dfrac{\omega Rm}{B^2l^2}\right) = \dfrac{\pi}{4} \longrightarrow \dfrac{\omega Rm}{B^2l^2} = \mathrm{tg}\left(\dfrac{\pi}{4}\right) = 1 \longrightarrow \boxed{\omega = \dfrac{B^2l^2}{Rm}}$

c) posición (2):



$\mathcal{E}_{ind} - RI' = 0$

$-Blv' - RI' = 0$ (5)

$\vec{F} = I'\vec{l}\times\vec{B}$

$m\dot{v}' = I'lB \rightarrow I' = \dfrac{m\dot{v}'}{lB}$ (4)

Sust. (4) en (5):

$-Blv' - \dfrac{Rm\dot{v}'}{lB} = 0 \longrightarrow \displaystyle\int_{v'(0)}^{v'(t)} \dfrac{dv'}{v'} = -\int_0^t \dfrac{l^2B^2}{Rm}dt \rightarrow \ln\left[\dfrac{v'}{v'(0)}\right] = -\dfrac{l^2B^2}{Rm}t$

$\Rightarrow v'(t) = v'(0)\, e^{-\frac{l^2B^2}{Rm}t}$ (6)

De a) y b) $\rightarrow v(t) = \dfrac{\mathcal{E}_0}{\sqrt{2}Bl}\cos(\omega t - \pi/4)$ para $\omega = \dfrac{B^2l^2}{Rm}$

« v_{max}

C.I.

$v'(0) = \dfrac{\mathcal{E}_0}{\sqrt{2}Bl}$ $\Rightarrow$ En (6): $v'(t) = \dfrac{\mathcal{E}_0}{\sqrt{2}Bl}e^{-\frac{l^2B^2}{Rm}t}$

Finalmente, sust. en (4): $I' = \dfrac{m\dot{v}'}{lB} = \dfrac{\cancel{m}}{\cancel{lB}}\dfrac{\mathcal{E}_0}{\sqrt{2}\cancel{Bl}}\left(-\dfrac{\cancel{l^2B^2}}{R\cancel{m}}\right)e^{-\frac{l^2B^2}{Rm}t}$

$$\Rightarrow \boxed{I'(t) = -\dfrac{\mathcal{E}_0}{\sqrt{2}R}e^{-\frac{l^2B^2}{Rm}t}}$$

$\rightarrow$ el signo (−) es porque el sentido de I' es el opuesto al considerado.

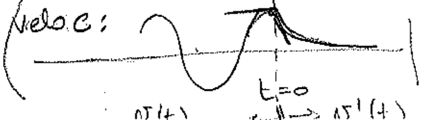
Obs: Se produce un salto en la corriente, instantáneamente, al pasar la llave de (1) a (2).

Cierro llave a posic. (2) en $t=0$.

Antes, $I = \dfrac{m\dot{v}}{lB} \Rightarrow I(0^-) = 0$ ($v_{máx}$)

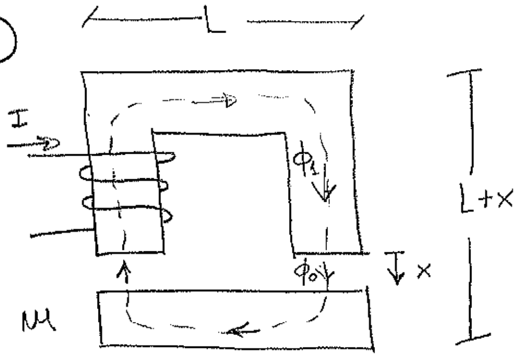
Desp., $I'(0) = -\dfrac{\mathcal{E}_0}{\sqrt{2}R}$

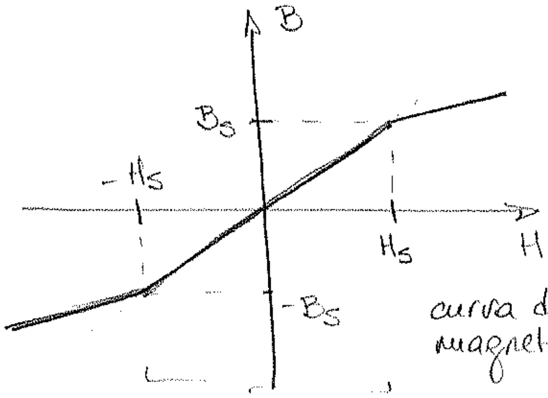
Veloc:



la veloc. es cont en $t=0$ pero no derivable

③





a) $NI = \phi_1\left(\frac{4L}{\mu S}\right) + \phi_0\left(\frac{2x}{\mu_0 S}\right)$ (1)

$\phi_1 = \phi_0$ (2) ($B_1 = B_0$, secc. unif S)

trabajo en región de la curva en que $B = \mu H$ con $\mu = \frac{B_S}{H_S}$ ($B \le B_S$)

Sust (2) en (1):

$$NI = \phi_0\left\{\frac{4L}{\mu S}+\frac{2x}{\mu_0 S}\right\} \Rightarrow \phi_0 = \frac{NI}{\left(\frac{4L}{\mu S}+\frac{2x}{\mu_0 S}\right)} = \frac{NI\mu\mu_0 S}{(4L\mu_0+2x\mu)}$$

$$B_0 = \frac{\phi_0}{S} = \boxed{\frac{NI\mu\mu_0}{4L\mu_0+2x\mu}} = \frac{NI\frac{B_S}{H_S}\mu_0}{4L\mu_0+2x\frac{B_S}{H_S}} = \boxed{\frac{NIB_S\mu_0}{4L\mu_0H_S+2xB_S}}$$

b) $m\vec{g}+\vec{F}=0$

según $\hat{j}$ ($\downarrow\hat{j}$): $mg+F=0$ (3)

$\vec{F} = \left(\frac{\partial U}{\partial x}\right)_I \hat{j}$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\overbrace{\left(N\frac{d\phi}{dI}\right)}^{L}I^2 = \frac{1}{2}\frac{N^2\mu\mu_0 S}{(4L\mu_0+2x\mu)}I^2 \quad (*)$$

$$\Rightarrow \left(\frac{\partial U}{\partial x}\right)_I = \frac{1}{\cancel{2}}N^2\mu_0\mu S I^2\left(\frac{-\cancel{2}\mu}{(4L\mu_0+2x\mu)^2}\right) = -\frac{N^2\mu_0\mu^2 S I^2}{(4\mu_0 L+2\mu x)^2}$$

$$\Rightarrow \text{En (3)}: \ mg - \frac{N^2\mu_0\mu^2SI^2}{(4\mu_0L+2\mu x)^2}=0 \Rightarrow I^2 = \frac{mg}{N^2\mu_0\mu^2 S}(4\mu_0L+2\mu x)^2$$

$$\Rightarrow I^2_{min} \underset{(x=0)}{=} \frac{mg}{N^2\mu_0\mu^2S}(4\mu_0L)^2 = \frac{mg\,16\mu_0^{\cancel{2}}L^2}{N^2\cancel{\mu_0}\mu^2S} \Rightarrow \boxed{I_{min} = \frac{4L}{N\mu}\sqrt{\frac{mg\mu_0}{S}}} = \boxed{\frac{4H_SL}{NB_S}\sqrt{\frac{mg\mu_0}{S}}}$$

— x —

(*) Alternativamente,

$$U = \int\frac{1}{2}B\cdot H\,dV = \frac{1}{2}\frac{B_1^2}{\mu}(4LS) + \frac{1}{2}\frac{B_0^2}{\mu_0}(2xS) \overset{B_1=B_0}{=} \frac{1}{2}B_0^2S\left(\frac{4L}{\mu}+\frac{2x}{\mu_0}\right) = \frac{1}{2}\frac{B^2S}{\mu\mu_0}(4L\mu_0+2x\mu)$$

$$\underset{B_0 \text{ de a)}}{=} \frac{1}{2}\left[\frac{N^2I^2\mu^{\cancel{2}}\mu_0^{\cancel{2}}}{(4L\mu_0+2x\mu)^{\cancel{2}}}\right]\frac{S}{\cancel{\mu\mu_0}}\cancel{(4L\mu_0+2x\mu)} = \frac{1}{2}\frac{N^2\mu\mu_0 S I^2}{(4L\mu_0+2x\mu)}$$