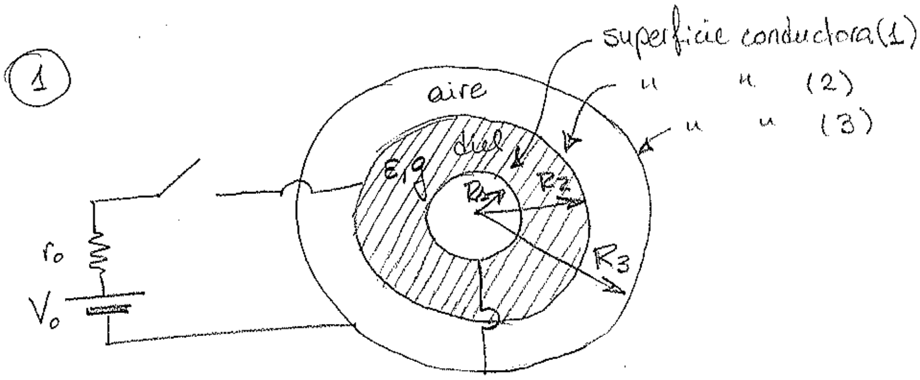
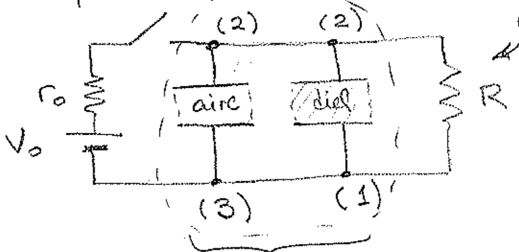
EXAMEN ELECTROMAGNETISMO — SOLUCIÓN — 22/02/2016.-
INST. DE FÍSICA, FAC. DE INGENIERÍA.

(1)



Circuito equivalente:



R: resistencia debida a la conductividad del material entre los radios R_1 y R_2

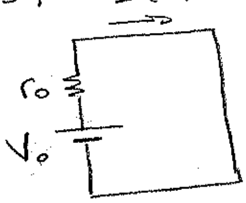
$\Rightarrow V_C = V_{23} = V_{21} = V_R$

a)

$V_C(0) = \frac{q_{eq}(0)}{C_{eq}} = 0$ "esferas inicialmente descargadas"

$(t=0)$ $I(0) = ?$

$\approx$



$V_0 - r_0 I(0) = 0$

$$\Rightarrow \boxed{I(0) = \frac{V_0}{r_0}}$$

b) Dos capacitores esféricos, uno de radios R_1 y R_2 relleno de diel. y el otro de radios R_2 y R_3 vacío, en paralelo (ver fig. del circ. equiv.)

$C_{aire} = \dfrac{4\pi\varepsilon_0}{\frac{1}{R_2} - \frac{1}{R_3}}$ $\quad C_{diel} = \dfrac{4\pi\varepsilon}{\frac{1}{R_1} - \frac{1}{R_2}}$ $\quad \Rightarrow C_{eq} = C_{aire} + C_{diel}$

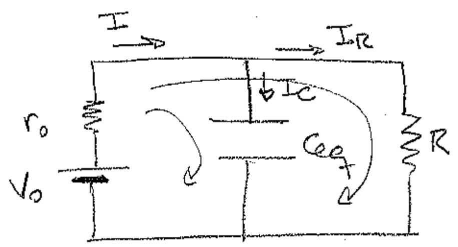
$$\boxed{C_{eq} = 4\pi\left[\frac{\varepsilon_0}{\left(\frac{1}{R_2} - \frac{1}{R_3}\right)} + \frac{\varepsilon}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}\right]}$$

(Obs: $RC_{diel} = \varepsilon/g$

Consid. cascarón de radio r y espesor dr ($R_1 \leq r \leq R_2$), la resistencia debido a ese cascarón está dada por:

$$dR = \frac{1}{g}\frac{dr}{(4\pi r^2)} \Rightarrow \boxed{R = \int_{R_1}^{R_2} \frac{1}{g}\frac{1}{4\pi}\frac{dr}{r^2} = \left(\frac{1}{4\pi g}\right)\left(-\frac{1}{r}\right)\Big|_{R_1}^{R_2} = \frac{1}{4\pi g}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$)

c) $V_{23} = V_C = ?$



$I = I_C + I_R \quad (i)$ ← nodos

$V_0 - r_0 I - V_C = 0 \quad (ii)$ ← mallas

$V_0 - r_0 I - R I_R = 0 \quad (iii)$

De (ii) → $\boxed{I = \dfrac{V_0 - V_C}{r_0}}$

$V_R = V_C$

$R I_R = V_C \rightarrow \boxed{I_R = \dfrac{V_C}{R}}$

$V_C = \dfrac{q}{C_{eq}} \rightarrow \dot{V}_C = \dfrac{I_C}{C_{eq}} \rightarrow \boxed{I_C = C_{eq} \dot{V}_C}$

Sust. en (i): $I = I_C + I_R$

$$\frac{V_0 - V_C}{r_0} = C_{eq}\dot{V}_C + \frac{V_C}{R}$$

$$\boxed{\dot{V}_C + \frac{V_C}{C_{eq}}\left(\frac{1}{R} + \frac{1}{r_0}\right) - \frac{V_0}{C_{eq} r_0} = 0}$$

SGNH = SGH + SPNH ⟶ $V_C^{(P)} = \dfrac{V_0}{r_0} \dfrac{1}{\left(\frac{1}{R} + \frac{1}{r_0}\right)} = \dfrac{V_0 R}{(r_0 + R)}$

$\dot{V}_C + \left(\dfrac{r_0 + R}{C_{eq} R r_0}\right) V_C = 0 \rightarrow \dfrac{dV_C}{V_C} = -\left(\dfrac{r_0 + R}{C_{eq} R r_0}\right) dt \quad \left(\text{Sea } \tau := \dfrac{R r_0}{(r_0 + R)} C_{eq}\right)$

$\Rightarrow \ln V_C(t) = -\dfrac{1}{\tau} dt + cte \rightarrow \boxed{V_C^{(GH)}(t) = A e^{-t/\tau}}$

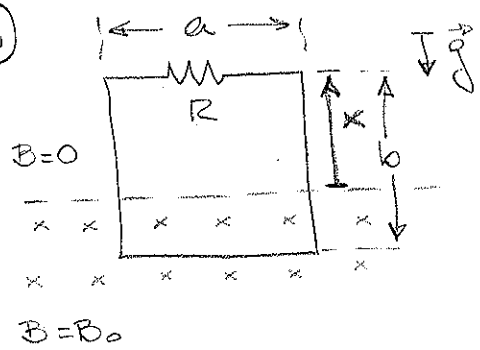
$\Rightarrow$ SGNH: $V_C(t) = A e^{-t/\tau} + \dfrac{V_0 R}{(r_0 + R)}$

C.I. $V_C(0) = 0 \quad (\text{xq } q(0) = 0) \Rightarrow A = -\left(\dfrac{R}{r_0 + R}\right) V_0$

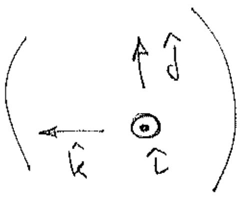
$\Rightarrow \boxed{V_C(t) = \left(\dfrac{R}{r_0 + R}\right) V_0 \left(1 - e^{-t/\tau}\right)}$ con $\boxed{\tau = \dfrac{R r_0}{r_0 + R} C_{eq}}$

— x —

(2)



$\downarrow \vec{g}$



$\vec{x} = x\hat{j}$ c.I.

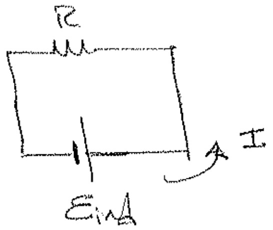
$\vec{v} = v\hat{j}$ $(v(0)=0)$

$\vec{B} = B_0(-\hat{i})$

$$\Phi_B = \int_S \vec{B}\cdot d\vec{A} = B_0(-\hat{i})\cdot(b-x)a\hat{i} = -B_0(b-x)a$$

$$\mathcal{E}_{ind} = -\frac{d\phi_B}{dt} = -B_0 a v$$

a) $\mathcal{E}_{ind} - RI = 0$



$\Rightarrow -B_0 a v - RI = 0$

$$\boxed{I = -\frac{B_0 a}{R} v} \quad (i)$$

2ª ley Newton:

según $\hat{j}$) $\boxed{M\dot{v} = (B_0 a)I - Mg}$ (ii)

$\vec{F}_g = (-Mg)\hat{j}$

$\vec{F}_B = I\,\vec{a}\times\vec{B}$

$= I(-\hat{k})\times B_0(-\hat{i})$

$= (IB_0 a)\hat{j}$

Sust. (i) en (ii):

$$M\dot{v} + B_0 a\left(+\frac{B_0 a}{R}v\right) + Mg = 0$$

$$\boxed{\dot{v} + \frac{(B_0 a)^2}{MR}v + g = 0}$$

SGNH = SGH + SPNH $\longrightarrow \frac{(B_0 a)^2}{MR}v = -g \longrightarrow v^{(P)} = -\frac{MR}{(B_0 a)^2}g$

$$\dot{v} = -\frac{(B_0 a)^2}{MR}v \longrightarrow v(t)^{(GH)} = Ae^{-t/\tau} \quad \text{con} \quad \boxed{\tau = \frac{MR}{(B_0 a)^2}}$$

$\Rightarrow v(t) = Ae^{-t/\tau} - \tau g$

CI $v(0) = 0 \longrightarrow A = \tau g$

$$\Rightarrow \boxed{v(t) = \tau g\left[e^{-t/\tau} - 1\right] = -\tau g\left(1 - e^{-t/\tau}\right)}$$

obs: como $\vec{v} = v\hat{j}$, el sgn. (-) implica que el circuito cae, es decir x (medido como en el dibujo) decrece en el tiempo.

En (i):

$$I = -\frac{B_0 a v}{R} = +\frac{\cancel{B_0 a}}{\cancel{R}}\left(+\frac{M\cancel{R}}{(B_0 a)^{\cancel{2}}}\right)g\left(1 - e^{-t/\tau}\right)$$

$$\boxed{I(t) = \frac{M}{B_0 a}g\left(1 - e^{-t/\tau}\right)}$$

b) $\varepsilon_{ind} - L\frac{dI}{dt} = 0$

$$-B_0 a v - L\dot{I} = 0 \Rightarrow \dot{I} = -\frac{B_0 a}{L} v \quad (iii)$$

2ª ley Newton:

$$\hat{j}) \quad M\dot{v} = (B_0 a) I - Mg \quad (*)$$

$$\frac{d}{dt} \quad M\ddot{v} = (B_0 a)\dot{I} \quad (iv)$$

Sust. (iii) en (iv):

$$M\ddot{v} = (B_0 a)\left(-\frac{B_0 a}{L} v\right)$$

$$\left.\begin{array}{l} \ddot{v} = -\frac{(B_0 a)^2}{ML} v \\ \text{C.I. } v(0) = 0 \end{array}\right\} \Rightarrow v(t) = \underset{?}{\underline{A}} \operatorname{sen}(\omega t) \quad \text{con } \boxed{\omega = \frac{B_0 a}{\sqrt{ML}}}$$

En (*) usando que $I(0) = 0$:

$$M\dot{v}(0) = (B_0 a) I(0)^{=0} - Mg$$

$$\cancel{M}(A\omega) = -\cancel{M}g \Rightarrow A = -\frac{g}{\omega}$$

$$\Rightarrow \boxed{v(t) = -\frac{g}{\omega}\operatorname{sen}(\omega t)} \quad (\vec{v} = v(t)\hat{j})$$

De (*): $(B_0 a) I = M\dot{v} + Mg$

$$(B_0 a) I = M\left(-\frac{g}{\cancel{\omega}}\right)\cancel{\omega}\cos(\omega t) + Mg$$

$$\boxed{I(t) = \left(\frac{Mg}{B_0 a}\right)\left[1 - \cos(\omega t)\right]}$$

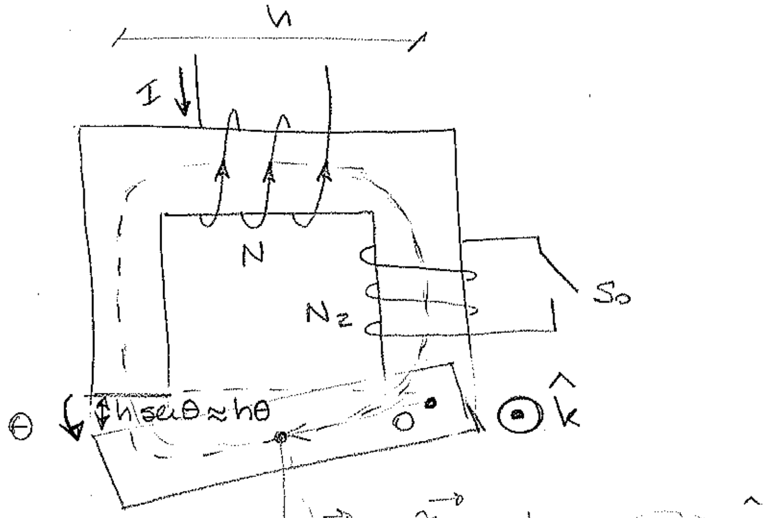
(Usando que $\cos^2\alpha + \operatorname{sen}^2\alpha = 1$ y $\cos\alpha = \cos^2\left(\frac{\alpha}{2}\right) - \operatorname{sen}^2\left(\frac{\alpha}{2}\right)$

también se puede escribir:

$$\boxed{I(t) = 2\left(\frac{Mg}{B_0 a}\right)\operatorname{sen}^2\left(\frac{\omega t}{2}\right)}$$
)

—— x ——

(3)



$$\vec{\tau}_g = \frac{h}{2} mg \cos\theta \, \hat{k}$$
$\cos\theta \simeq 1$

Al cerrar la llave S_0, se cortocircuita el bobinado de N_2 vueltas, la fem inducida debe ser nula y ésto obliga a que se mantenga el flujo constante en el circuito (para una corriente I dada).

N_2 $V_2 = 0 \Rightarrow \frac{d\phi}{dt} = 0 \Rightarrow \phi = cte$

* Ampère: $\oint \vec{H} \cdot \vec{dl} = NI$ (recorro la curva en stdo. antihorario)

$$\left(\underset{(\text{núcleo})}{H} = \frac{B}{\mu} = \frac{\phi}{\mu S} \; ; \; \underset{(\text{entrehierro})}{H_e} = \frac{B}{\mu_0} = \frac{\phi}{\mu_0 S} \right)$$

$$\Rightarrow \frac{\phi}{\mu S}(4h) + \frac{\phi}{\mu_0 S}(\theta h) = NI$$

$$\phi \left[4h\mu_0 + h\theta\mu \right] = \mu_0 \mu S N I$$

$$\boxed{\phi = \frac{\mu_0 \mu S N I}{4h\mu_0 + h\theta\mu}}$$

* $$U = \frac{1}{2}\int_V \vec{H} \cdot \vec{B}\, dV = \frac{1}{2}\int_{V_{\text{núcleo}}} \vec{H} \cdot \vec{B}\, dV + \frac{1}{2}\int_{V_{\text{entrehierro}}} \vec{H} \cdot \vec{B}\, dV$$

$$U = \frac{1}{2}\frac{\phi^2}{\mu S}(4h) + \frac{1}{2}\frac{\phi^2}{\mu_0 S}(\theta h)$$

$$\vec{\tau}_{mag} = -\left.\frac{dU}{d\theta}\right|_{\phi = cte} \hat{k}$$

La mínima corriente para que la rama no se desprenda ($\theta = 0$) es cuando el torque magnético iguala (en módulo) al torque del peso.

según $\hat{k}$) $$\tau_{mag} = -\frac{1}{2}\frac{(\phi|_{\theta=0})^2 h}{\mu_0 S} = -\frac{1}{2}\frac{\mu_0^{\cancel{2}}\mu^2 S^{\cancel{2}} N^2 I^2 h}{(4h\mu_0)^2 \cancel{\mu_0}\cancel{S}}$$

$$\vec{\tau}_g = \frac{h}{2} mg\,\hat{k} \longleftarrow \text{torque del peso}$$

$$\Rightarrow -\frac{1}{2}\frac{\mu_0\mu^2 S N^2 h I^2}{(4h\mu_0)^2} + \frac{h}{2}mg = 0$$

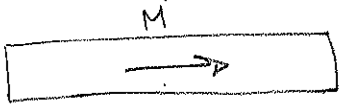
$$\frac{1}{\cancel{2}}\frac{\mu_0\mu^2 S N^2 h I^2}{(4h\mu_0)^2} = \frac{h}{\cancel{2}}mg$$

$$I^2 = \frac{mg\cancel{h}(4h\mu_0)^2}{\mu_0\mu^2 S N^2 \cancel{h}} = \frac{mg}{\mu^2 S N^2}\frac{\mu_0^{\cancel{2}}}{\cancel{\mu_0}}(4h)^2$$

$$\boxed{I = \frac{4h}{\mu N}\sqrt{\frac{\mu_0 mg}{S}}}$$

b)

$\vec{M}$



$$\vec{H}_{\substack{\text{material}\\ \text{con } M}} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

∮ Ampère

$$NI = \oint_{\mathcal{C}} \vec{H}\cdot d\vec{l} \overset{\text{recorro antih.}}{=} \underbrace{\left(\frac{\phi}{\mu S}\right)3h}_{H_{3h}} + \underbrace{\left(\frac{\phi}{\mu_0 S}\right)h\theta}_{H_e} + \underbrace{\left(\frac{B}{\mu_0} - M\right)h}_{H_M}$$

tengo que tratar por separado la barra de abajo (respecto a las otras 3) porque tiene $\neq$ valor de H

$$\Rightarrow \phi\left(\frac{3}{\mu} + \frac{\theta}{\mu_0} + \frac{1}{\mu_0}\right) = (NI + Mh)\frac{S}{h}$$

$$\phi = (NI + Mh)\frac{S}{h}\frac{\mu\mu_0}{(3\mu_0 + (1+\theta)\mu)} \qquad (\#)$$

$$U = \frac{1}{2}\int_{\substack{\text{núcleo}\\(\sin \vec{M})}} \vec{H}\cdot\vec{B}\, dV + \frac{1}{2}\int_{\text{entrehierro}} \vec{H}\cdot\vec{B}\, dV + \frac{1}{2}\int_{\substack{\text{material}\\ \text{con } \vec{M}}} \vec{H}\cdot\vec{B}\, dV$$

$$= \frac{1}{2}\frac{\phi^2}{\mu S}(3h) + \underbrace{\frac{1}{2}\frac{\phi^2}{\mu_0 S}(h\theta)}_{\substack{\text{Único término que depende de } \theta \\ \text{en forma explícita}}} + \frac{1}{2}\left(\frac{\phi}{S}\right)\left(\frac{\phi}{\mu_0 S} - M\right)hS$$

$$\vec{\tau}_{\text{magnético}} = -\left.\frac{dU}{d\theta}\right|_{\phi=\text{cte}} \hat{k} = \left(-\frac{1}{2}\frac{\phi^2}{\mu_0 S}h\right)\hat{k}$$

Para $\phi|_{\theta=0}$ debe cancelar al torque del peso:

$$-\frac{1}{\cancel{2}}\frac{\left(\phi|_{\theta=0}\right)^2 h}{\mu_0 S} + \frac{h}{\cancel{2}} mg = 0$$

$$\left(\phi|_{\theta=0}\right)^2 = \mu_0 mg S$$

De (#) para $\theta = 0$;

$$\left[(NI + Mh)\frac{S}{h}\frac{\mu\mu_0}{(3\mu_0+\mu)}\right]^2 = \mu_0 mg S$$

$$(NI + Mh)^2 = \cancel{\mu_0} mg \cancel{S} \frac{h^2}{S^{\cancel{2}}} \frac{(3\mu_0+\mu)^2}{\mu^2 \mu_0^{\cancel{2}}}$$

$$NI = \sqrt{\frac{(3\mu_0+\mu)^2 mg h^2}{\mu^2 \mu_0 S}} - Mh$$

$$\boxed{I = \frac{(3\mu_0+\mu)h}{\mu N}\sqrt{\frac{mg}{\mu_0 S}} - \frac{Mh}{N}}$$