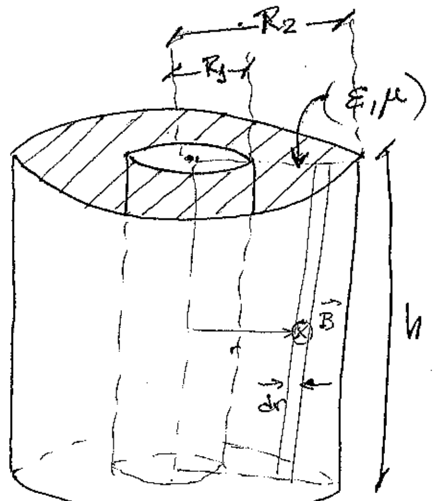
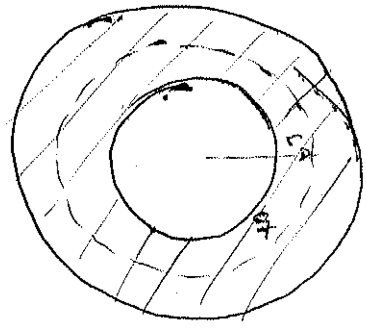
2º PARCIAL ELECTROMAGNETISMO 3/12/15.-

1



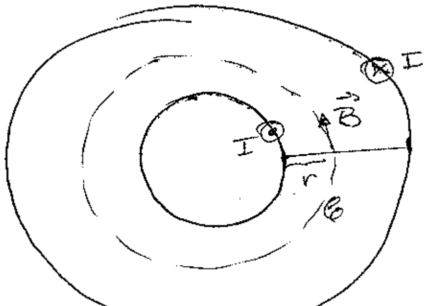


Gauss $\varepsilon E(2\pi r h) = q \to \vec{E}(r) = \frac{q}{2\pi\varepsilon r h}\hat{e}_r$

$V = \int \vec{E}\cdot d\vec{r} = \frac{q}{2\pi\varepsilon h}\ln\left(\frac{R_2}{R_1}\right) = \frac{q}{C_0}$

$\Rightarrow \boxed{C = \frac{C_0}{h} = \frac{2\pi\varepsilon}{\ln(R_2/R_1)}}$

por unidad de longitud



Ampère $\oint \frac{\vec{B}}{\mu}\cdot d\vec{l} = I$

$\mathcal{C}$ $\vec{B}(r) = \frac{\mu I}{2\pi r}\hat{e}_\varphi$; $\hat{e}_\varphi$ antihorario

$\Phi_B = \int_S \vec{B}\cdot\hat{n}\, da = \int_{R_1}^{R_2}\frac{\mu I}{2\pi r}(h\,dr) = \frac{\mu I h}{2\pi}\ln(R_2/R_1)$

$L_0 = \frac{d\Phi_B}{dI} = \frac{\mu h}{2\pi}\ln(R_2/R_1) \to \boxed{L = \frac{L_0}{h} = \frac{\mu}{2\pi}\ln(R_2/R_1)}$

$\Rightarrow \boxed{LC = \frac{\mu}{2\pi}\cancel{\ln(R_2/R_1)}\,\frac{2\pi\varepsilon}{\cancel{\ln(R_2/R_1)}} = \mu\varepsilon}$

—— x ——

Obs: alternativamente

$u_B = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2\mu}\frac{\mu^2 I^2}{4\pi^2 r^2}$

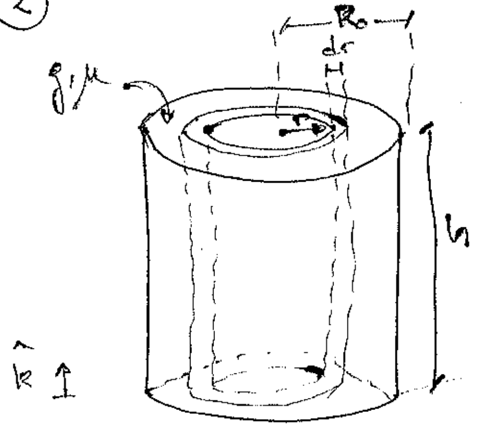
$U_B = \int_{R_1}^{R_2} u_B(h\,2\pi r\,dr) = \int_{R_1}^{R_2}\frac{1}{2}\mu I^2 h\frac{2\pi r}{(2\pi)^2 r^2}\,dr = \frac{1}{2}\frac{\mu I^2}{2\pi}h\ln r\Big|_{R_1}^{R_2} = \frac{1}{2}\frac{\mu I^2}{2\pi}h\ln\left(\frac{R_2}{R_1}\right)$

$U_B = \frac{1}{2}L_0 I^2$ $\Rightarrow L_0 = \frac{\mu h\ln(R_2/R_1)}{2\pi}$

$\Rightarrow L = \frac{L_0}{h} = \frac{\mu}{2\pi}\ln(R_2/R_1)$ –

—— x ——

(2)



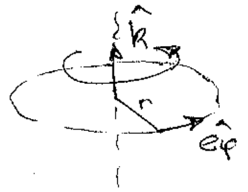
$$B = B_o \operatorname{sen}(\omega t)\,\hat{k}$$

$$R_r = \left(\frac{1}{g}\right)\frac{(2\pi r)}{(h\,dr)}$$ ⇐ resistencia en cascarón cil. de radio r y espesor dr

$$\Phi_B(t) = B_o(\pi r^2)\operatorname{sen}(\omega t)$$ ⇐ flujo a través de πr^2

$$\mathcal{E}_r = -\frac{d\Phi_B}{dt} = (-)\,B_o(\pi r^2)\,\omega\cos(\omega t)$$

→ significa que la corriente inducida va a ir según $-\hat{e}_\varphi$



$$P_{r\,dis}(t) = \mathcal{E}_r I = \frac{\mathcal{E}_r^2}{R}$$

$$P_{dis}(t) = \left(\frac{g h\,dr}{2\pi \cancel{r}}\right) B_o^2 \pi^2 \cancel{r^4} r^3 \omega^2 \cos(\omega t)$$

$$\Rightarrow P_{dis}(t) = \frac{g h}{2\cancel{\pi}} B_o^2 \pi^{\cancel{2}} \omega^2 \cos^2(\omega t) \underbrace{\int_0^{R_o} r^3 dr}_{= \frac{R_o^4}{4}} = \frac{g h B_o^2 \pi \omega^2 R_o^4}{8}\cos^2(\omega t) \quad (*)$$

$$\boxed{\langle P_{dis}\rangle} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{\omega}{2\pi}\,\frac{g h B_o^2 \pi \omega^2 R_o^4}{8} \underbrace{\int_0^{2\pi/\omega}\cos^2(\omega t)\,dt}_{= \frac{1}{2}\frac{2\pi}{\omega}} = \boxed{\frac{g h B_o^2 \pi \omega^2 R_o^4}{16}}$$

— x —

Obs: alternativamente:

$$P_{dis}(t) = \int \underbrace{\vec{J}}_{= g\vec{E}}\cdot\vec{E}\,dv = \int g E^2\,dv$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\underbrace{\int_S \nabla\times\vec{E}\cdot d\vec{a}}_{= \oint_C \vec{E}\cdot d\vec{l}} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{a} \qquad \left(\mathcal{E} = -\frac{d\Phi_B}{dt}\ ✓\right)$$

$$E\,(2\pi r) = -\frac{d}{dt}B(t)\,(\pi r^2)$$

$$E(r,t) = \omega B_o \cos(\omega t)\frac{\cancel{\pi} r^{\cancel{2}}}{2\cancel{\pi}\cancel{r}}$$

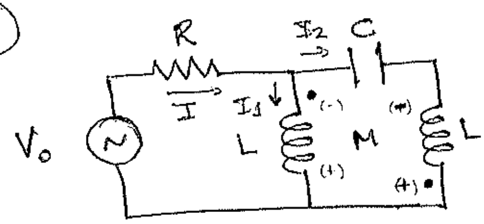
$$gE^2 = \frac{g\omega^2 B_o^2 \cos^2(\omega t)\, r^2}{4}$$

$$\Rightarrow P_{dis}(t) = \int_0^{R_o} \frac{g\omega^2 B_o^2}{\cancel{4}\,2}\cos^2(\omega t)\, r^2 (\cancel{2}\pi r\,dr\,h)$$

$$= \frac{g h B_o^2 \pi \omega^2 R_o^4}{8}\cos^2(\omega t) \quad (\text{idem } *)$$

— x —

③



$$\begin{cases} I = I_1 + I_2 \\ V_0 - RI - j\omega L I_1 + j\omega M I_2 = 0 \\ V_0 - \frac{1}{j\omega C} I_2 - j\omega L I_2 + j\omega M I_1 = 0 \end{cases}$$

C tq $I = 0$ $\quad (I_2 = -I_1)$

$$\begin{cases} V_0 - (j\omega L + j\omega M) I_1 = 0 \\ V_0 - \left(\frac{j}{\omega C} - j\omega L - j\omega M\right) I_1 = 0 \end{cases}$$

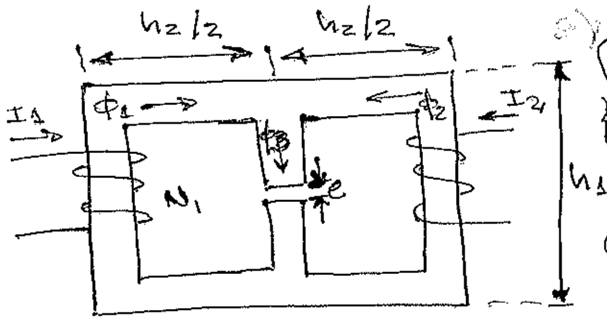
$$\Rightarrow j\omega L + j\omega M = \frac{j}{\omega C} - j\omega L - j\omega M$$

$$2j\omega L + 2j\omega \underbrace{M}_{=\frac{L}{2}} = \frac{j}{\omega C}$$

$$2\omega L + \omega L = \frac{1}{\omega C} \to 3\omega L = \frac{1}{\omega C} \to \boxed{C = \frac{1}{3\omega^2 L}}$$

—— x ——

④



$$\phi_3 = \phi_1 + \phi_2$$

$$\begin{cases} N_1 I_1 = \phi_1\left(\frac{h_2+h_1}{\mu S}\right) + \phi_3\left(\frac{h_1-e}{\mu S} + \frac{e}{\mu_0 S}\right) \\ N_2 I_2 = \phi_2\left(\frac{h_2+h_1}{\mu S}\right) + \phi_3\left(\frac{h_1-e}{\mu S} + \frac{e}{\mu_0 S}\right) \end{cases}$$

(+)

$$N_1I_1 + N_2I_2 = \underbrace{(\phi_1+\phi_2)}_{=\phi_3}\left[\frac{h_2+h_1}{\mu S}\right] + 2\phi_3 \underbrace{\left[\frac{h_1-e}{\alpha\mu_0 S} + \frac{e}{\mu_0 S}\right]}_{=\frac{h_1+(\alpha-1)e}{\alpha\mu_0 S}}$$

$$N_1I_1 + N_2I_2 = \frac{\overset{=B_e S}{\phi_3}}{\alpha\mu_0 S}\left\{h_2 + \underbrace{h_1 + 2h_1}_{=3h_1} + 2(\alpha-1)e\right\}$$

$$\Rightarrow \boxed{B_e = \frac{\alpha\mu_0(N_1I_1 + N_2I_2)}{h_2 + 3h_1 + 2e(\alpha-1)}}$$

—— x ——

⑤ $e = 0$, $h_1 = \frac{h_2}{2} = h$ De 4) $\phi_3 = \frac{\mu S(N_1I_1 + N_2I_2)}{(2h + 3h)}$ $= 5h$

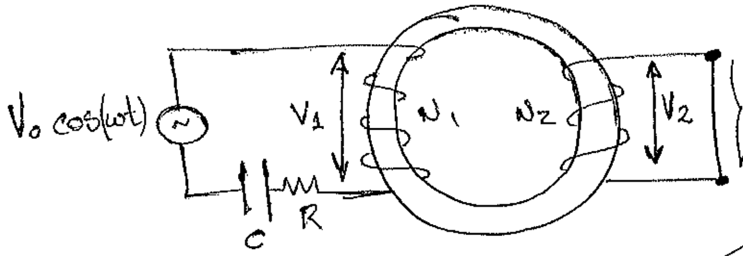
$$N_1I_1 = \phi_1\left(\frac{3h}{\mu S}\right) + \phi_3\left(\frac{h}{\mu S}\right) = \phi_1\left(\frac{3h}{\mu S}\right) + \left(\frac{\cancel{h}}{\cancel{\mu S}}\right)\frac{\cancel{\mu S}(N_1I_1 + N_2I_2)}{5\cancel{h}}$$

$$\Rightarrow \phi_1 = \frac{\mu S}{3h}\left[N_1I_1\left(1 - \frac{1}{5}\right) - \frac{N_2I_2}{5}\right] \longrightarrow M = \frac{d(N_1\phi_1)}{dI_2} = -\frac{\mu S}{3h}\frac{N_1N_2}{5}$$

$$\boxed{|M| = \frac{\mu S N_1 N_2}{15h}}$$

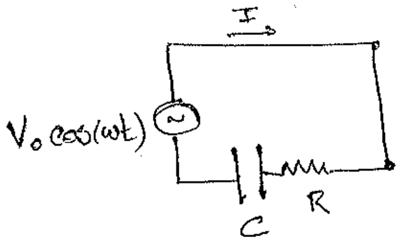
—— x ——

(6)



$V_2 = 0$ (secundario cortocircuitado)

transf. ideal: $\dfrac{V_2}{N_2} = \dfrac{V_1}{N_1} \Rightarrow V_1 = 0$



$$V_0 - R\underset{\substack{\parallel \\ \frac{V_0}{Z}}}{(I)} - V_C = 0 \rightarrow V_C = V_0\left(1 - \frac{R}{Z}\right) = V_0\left(\frac{Z-R}{Z}\right)$$

$$Z = R + \frac{1}{j\omega C} = \frac{j\omega RC + 1}{j\omega C}$$

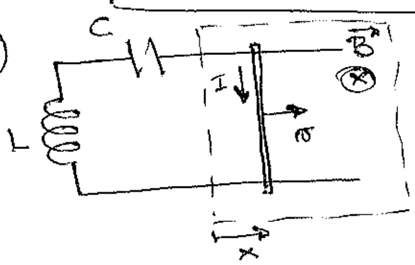
$$\frac{Z-R}{Z} = \frac{\cancel{j\omega RC} + 1 - \cancel{j\omega RC}}{j\omega C\left(\dfrac{j\omega RC+1}{j\omega C}\right)} = \frac{1 - j\omega RC}{1+\omega^2R^2C^2}$$

$$|V_C| = V_0\left|\frac{Z-R}{Z}\right| = \frac{V_0}{1+\omega^2R^2C^2}\sqrt{(1-j\omega RC)(1+j\omega RC)} = \frac{V_0\sqrt{1+\omega^2R^2C^2}}{(1+\omega^2R^2C^2)}$$

$$\Rightarrow \boxed{|V_C| = \frac{V_0}{\sqrt{1+\omega^2R^2C^2}}}$$

—— x ——

(7)



$I(0) = I_0$
$q(0) = 0$
$\dfrac{3B^2l^2}{m} = \dfrac{1}{C}$

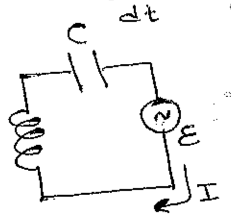
$$\vec{F} = I\vec{l} \times \vec{B}$$

$(\rightarrow)$ $m\dfrac{dv}{dt} = \dfrac{dq}{dt} lB$ (1)

$m[v(t) - \overset{0}{v(0)}] = [q(t) - \overset{0}{q(0)}]lB$

$m v(t) = q lB \rightarrow q = \dfrac{m}{lB}v$

$$\mathcal{E}_i = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(Blx) = -Blv(t)$$



$$-Blv - \frac{q}{C} - L\frac{dI}{dt} = 0 \quad (2)$$

Sust. en (2) y usando de (1) que $\dot{I} = \dfrac{m}{lB}\ddot{v}$:

$$-Blv - \frac{m}{lBC}v - L\frac{m}{lB}\ddot{v} = 0 \rightarrow \ddot{v} + \underbrace{\frac{lB}{mL}\left(Bl + \frac{m}{lBC}\right)}_{\frac{B^2l^2}{mL} + \frac{1}{LC}}v = 0$$

$$\frac{B^2l^2}{mL} + \frac{1}{LC} = \frac{B^2l^2}{mL} + \frac{3B^2l^2}{mL} = \frac{4B^2l^2}{mL} = \omega_0^2$$

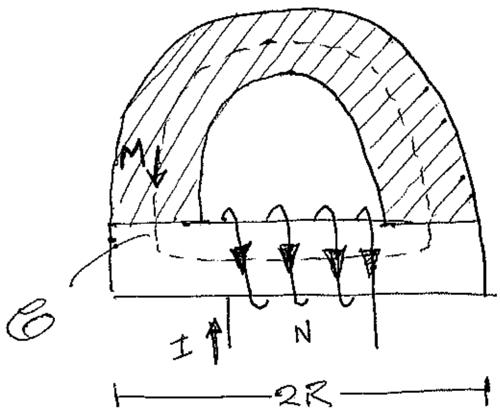
$\ddot{v} + \omega_0^2 v = 0 \rightarrow v(t) = v_m \operatorname{sen}(\omega_0 t)$ (vf. CI $v(0) = 0$)

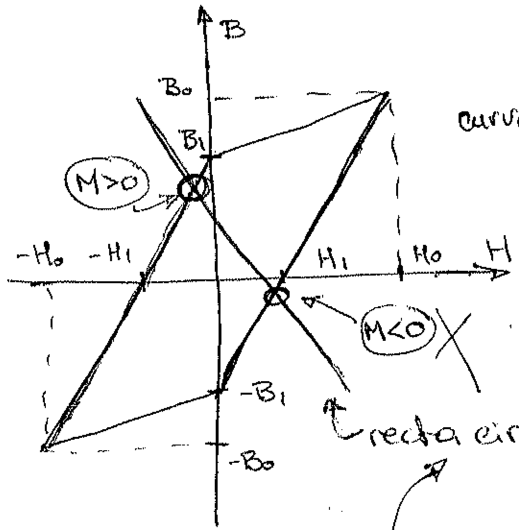
$\underset{(1)}{\dot{v}(0)} = I(0)\dfrac{lB}{m} = I_0\dfrac{lB}{m} \rightarrow \omega_0 v_m = I_0 lB$

$$\boxed{v_m = \frac{I_0 lB}{\omega_0 m} = I_0\frac{\cancel{lB}\sqrt{mL}}{m\, 2\cancel{Bl}} = \frac{I_0}{2}\sqrt{\frac{L}{m}}}$$

—— x ——

⑧





curva de histéresis

recta circuito (I)

Ampère: $\oint_{\mathcal{C}} \vec{H}\cdot d\vec{l} = NI$ (sentido antihorario)

$$\frac{B}{\mu}(2R) + H_m(\pi R) = NI \quad (1)$$

Cons. flujo $B_m S = B S \longrightarrow B = B_m \quad (2)$ (sección unif.)

sust. (2) en (1): $\dfrac{B_m}{\mu}(2R) + H_m(\pi R) = NI \Rightarrow \boxed{B_m = -\dfrac{\mu(\pi \cancel{R})}{2\cancel{R}} H_m + \dfrac{\mu N I}{2R}}$ (I)

recta B_m vs. H_m del circuito

De curva de histéresis:

$$\boxed{B_m = \frac{B_1}{H_1} H_m \oplus B_1} \quad (II)$$

M>0 determina recta (+)

Obs: $\vec{H_m} = \dfrac{\vec{B_m}}{\mu_0} - \vec{M}$

$\vec{M} = \dfrac{\vec{B_m}}{\mu_0} - \vec{H_m} \longrightarrow M = \dfrac{B_m}{\mu_0} - H_m > 0$ (stdo. antih.)

M>0

De (I): $H_m = \dfrac{NI}{\pi R} - \dfrac{B_m}{\mu}\dfrac{(2\cancel{R})}{\pi \cancel{R}}$

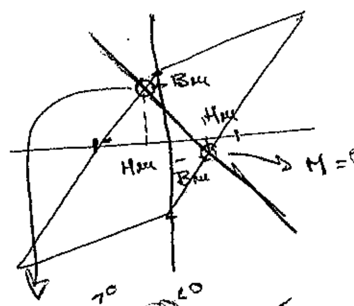
De (II): $H_m = \dfrac{H_1}{B_1}(B_m - B_1)$ (M>0)

igualando: $\dfrac{H_1}{B_1} B_m - H_1 = \dfrac{NI}{\pi R} - \dfrac{2 B_m}{\mu \pi}$, con $\dfrac{H_1}{B_1} = \dfrac{1}{\pi\mu}$

$$B_m\left(\frac{1}{\cancel{\pi}\mu} + \frac{2}{\cancel{\pi}\mu}\right) = H_1 + \frac{NI}{\cancel{\pi} R} \longrightarrow B_m\left(\frac{3}{\mu}\right) = \frac{B_1}{\mu} + \frac{NI}{R}$$

$$H_1 = \frac{B_1}{\pi\mu}$$

$$\boxed{B_m = \frac{B_1}{3} + \frac{\mu N I}{3R}}$$



$M = \dfrac{B_m}{\mu_0} - H_m < 0$ ✗

$M = \dfrac{B_m}{\mu_0} - H_m > 0$ ✓

———— x ————