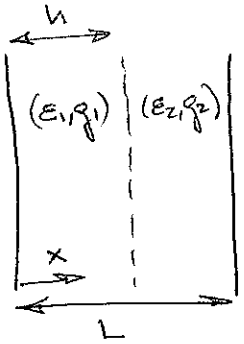


1)

$\vec{J} = g\vec{E}$ (ohm)

$\vec{D} = \varepsilon\vec{E}$ (lineal, isotrópico, homogéneo)

$$\underbrace{\nabla\cdot\vec{J}}_{=\,\nabla\cdot g\vec{E}\,=\,\frac{g}{\varepsilon}\nabla\cdot\vec{D}} = -\frac{\partial\rho}{\partial t} \longrightarrow \frac{g}{\varepsilon}\underbrace{\nabla\cdot\vec{D}}_{=\rho} = -\frac{\partial\rho}{\partial t}$$

$$\Rightarrow \frac{\partial\rho}{\partial t} = -\frac{g}{\varepsilon}\rho \to \rho(x,t) = \underbrace{\rho(x,0)}_{=0} e^{-\frac{g}{\varepsilon}t} \Rightarrow \rho = 0\ \forall x,t.$$

inicialmente, sólo hay carga libre en las placas conductoras

C.B. $x=0$

$$\underbrace{\hat{n}}_{=\hat{x}}\cdot\Big(\underbrace{\vec{D}\big|_{x=0^+}}_{=\vec{D}_1=\varepsilon_1\vec{E}_1} - \underbrace{\vec{D}\big|_{x=0^-}}_{=0}\Big) = \sigma_L\big|_{x=0}$$

$$\Rightarrow \boxed{E_1 = \frac{\sigma_L(x=0,t)}{\varepsilon_1}} \quad (1)$$

$$\underbrace{\hat{n}}_{=\hat{x}}\cdot\Big(\underbrace{\vec{J}\big|_{x=0^+}}_{=\vec{J}_1=g_1\vec{E}_1} - \underbrace{\vec{J}\big|_{x=0^-}}_{=0}\Big) = -\dot{\sigma}_L(x=0,t) \Rightarrow \boxed{E_1 = -\frac{\dot{\sigma}_L}{g_1}} \quad (2)$$

De (1) y (2): $\dot{\sigma}_L = -\frac{g_1}{\varepsilon_1}\sigma_L \to \sigma_L(x=0,t) = \underbrace{\sigma_L(x=0,t=0)}_{=\frac{Q}{A}} e^{-\frac{g_1}{\varepsilon_1}t}$

$$\Rightarrow \vec{J}_1 = g_1\vec{E}_1 \overset{(1)}{=} \left(\frac{g_1}{\varepsilon_1}\frac{Q}{A}e^{-\frac{g_1}{\varepsilon_1}t}\right)\hat{x}$$

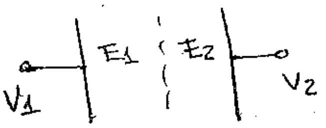
$$\Rightarrow \boxed{\vec{J}_1(x,t) = \left(\frac{g_1}{\varepsilon_1}\frac{Q}{A}e^{-\frac{g_1}{\varepsilon_1}t}\right)\hat{x}}$$

2) Analogamente, de C.B. en $x=L$: $J_2(x,t) = \frac{g_2}{\varepsilon_2}\frac{Q}{A}e^{-\frac{g_2}{\varepsilon_2}t}$

Como $\sigma_L\big|_{x=h} = 0\ \forall t$ (no se acumula carga libre en la interfase)

$$\underbrace{\hat{n}}_{=\hat{x}}\cdot\left(\vec{J}_2 - \vec{J}_1\right)\Big|_h = 0 \Rightarrow J_2 = J_1\ \forall t \Longleftrightarrow \frac{g_2}{\varepsilon_2} = \frac{g_1}{\varepsilon_1} \Longleftrightarrow \boxed{\frac{\varepsilon_2}{\varepsilon_1} = \frac{g_2}{g_1}}$$

3)



$V_1 > V_2$ En estado estacionario $\dot{\sigma}_L\big|_h = 0 \Rightarrow \underline{J_1 = J_2}$

$$\Rightarrow g_1E_1 = g_2E_2 \to E_2 = \frac{g_1}{g_2}E_1$$

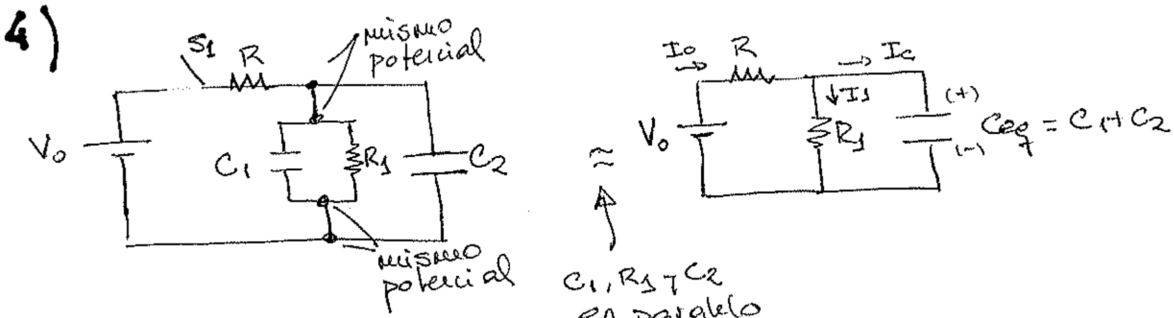
$$V_1 - V_2 = \int_0^h E_1\,dx + \int_h^L E_2\,dx = E_1 h + E_2(L-h) = E_1 h + \frac{g_1}{g_2}E_1(L-h)$$

$$E_1\left(h + \frac{g_1}{g_2}(L-h)\right) = V_1 - V_2$$

$$\Rightarrow \boxed{J_1 = g_1E_1 = \frac{g_1(V_1-V_2)}{h + \frac{g_1}{g_2}(L-h)}}$$

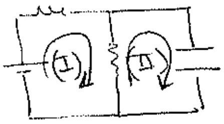
— x —

4)



C_1, R_1 y C_2 en paralelo

mallas



$$V_0 - RI_0 - R_1 I_1 = 0 \quad (I)$$

$$-V_C + R_1 I_1 = 0 \quad (II)$$

nodos $\quad I_0 = I_1 + I_c \quad (III)$

De (II): $V_c = R_1 I_1 \rightarrow Q = R_1 C_{eq} I_1$, con $V_c = \dfrac{Q}{C_{eq}}$

$$\boxed{I_c = \dot{Q} = R_1 C_{eq} \dot{I}_1}$$

De (I): $V_0 = RI_0 + R_1 I_1 \rightarrow \boxed{I_0 = \dfrac{V_0}{R} - \dfrac{R_1}{R} I_1}$

sust. en (III):

$$I_1 = I_0 - I_c = \frac{V_0}{R} - \frac{R_1}{R} I_1 - R_1 C_{eq} \dot{I}_1$$

$$\boxed{R_1 C_{eq} \dot{I}_1 + \left(1 + \frac{R_1}{R}\right) I_1 = \frac{V_0}{R}}$$

SGNH = SGH + SPNH

SGH) $R_1 C_{eq} \dot{I}_1 = -\left(\dfrac{R+R_1}{R}\right) I_1 \rightarrow \dot{I}_1 = -\left(\dfrac{R+R_1}{RR_1 C_{eq}}\right) I_1 \rightarrow I_1(t) = A e^{-t/\tau}$; $\tau = \dfrac{RR_1 C_{eq}}{R+R_1}$

SPNH) $\left(\dfrac{R+R_1}{\cancel{R}}\right) I_1 = \dfrac{V_0}{\cancel{R}} \rightarrow I_1 = \dfrac{V_0}{R+R_1}$

SGNH)

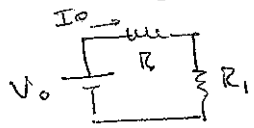
$$\Rightarrow \quad I_1(t) = A e^{-t/\tau} + \frac{V_0}{R+R_1}$$

C.I. $I_1(0) = 0$ (porque en $t=0$ C_1 y C_2 descargados, $V_1, V_2 = 0$, se comportan como cables)

$$\Rightarrow A = -\frac{V_0}{R+R_1}$$

$$\Rightarrow \boxed{I_1(t) = \frac{V_0}{R+R_1}\left(1 - e^{-t/\tau}\right)} \; ; \; \boxed{\tau = \frac{RR_1(C_1+C_2)}{R+R_1}}$$

Obs: a tiempos largos, los capacitores se cargan totalmente y se abre allí el circuito, sólo circularía corriente por $R_1 \Rightarrow I_1(t \to \infty) = I_0$



$$V_0 - (R+R_1) I_0 = 0 \rightarrow I_1(t \to \infty) = \frac{V_0}{R+R_1}$$

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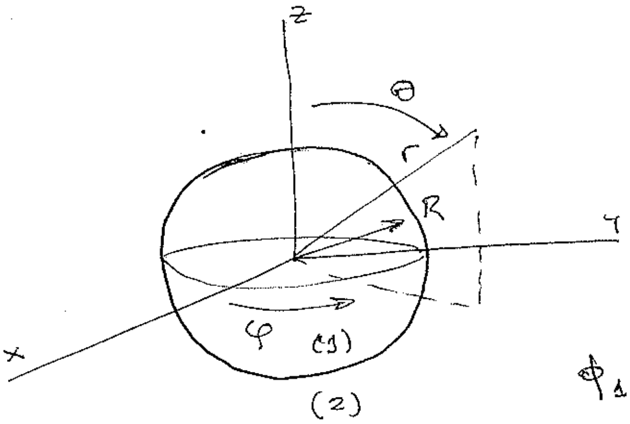
5)



coords. esféricas

$\phi = \phi(r,\theta,\cancel{\varphi}) = \phi(r,\theta)$

simetría azimutal

$\nabla^2 \phi = 0$

$\phi_1(r,\theta) = A_1 + \frac{B_1}{r} + \left(C_1 r + \frac{D_1}{r^2}\right)\cos\theta \quad ; \; r \leq R$

$\phi_2(r,\theta) = A_2 + \frac{B_2}{r} + \left(C_2 r + \frac{D_2}{r^2}\right)\cos\theta \quad ; \; r \geq R$

$\phi(R,\theta) = V(\theta) = V_0 \cos\theta$

afuera (región 2) $\phi_2 \xrightarrow[r\to\infty]{} 0$ $\left.\begin{matrix} A_2 = 0 \\ C_2 = 0 \end{matrix}\right\}$ $\phi_2(r,\theta) = \frac{B_2}{r} + \frac{D_2}{r^2}\cos\theta$

adentro (región 1) ϕ_1 acotado $r \to 0$ $\left.\begin{matrix} B_1 = 0 \\ D_1 = 0 \end{matrix}\right\}$ $\phi_1(r,\theta) = A_1 + C_1 r\cos\theta$

C.B. $r = R$

$\phi_1(R) = \phi_2(R)$
(Ambos deben valer $V(\theta)$ en $r = R$)

$\phi_1(R) = A_1 + C_1 R\cos\theta = V_0\cos\theta \rightarrow \boxed{A_1 = 0}\ \boxed{C_1 = \frac{V_0}{R}}$

$\phi_2(R) = \frac{B_2}{R} + \frac{D_2}{R^2}\cos\theta = V_0\cos\theta \rightarrow \boxed{B_2 = 0}\ \boxed{D_2 = V_0 R^2}$

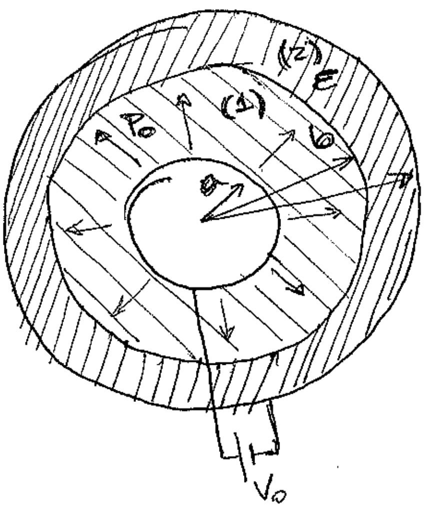
$\Rightarrow \begin{cases} \phi_1(r,\theta) = \frac{V_0}{R} r\cos\theta \\ \phi_2(r,\theta) = \frac{V_0 R^2}{r^2}\cos\theta \end{cases}$

$\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_L$; $\hat{n} = \hat{e}_r$

$-\varepsilon_0 \left.\frac{\partial\phi_2}{\partial r}\right|_R - \left(-\varepsilon_0 \left.\frac{\partial\phi_1}{\partial r}\right|_R\right) = \sigma_L \rightarrow \sigma_L(\theta) = \varepsilon_0 \left\{ \left.\frac{\partial\phi_1}{\partial r}\right|_R - \left.\frac{\partial\phi_2}{\partial r}\right|_R \right\}$

$\Rightarrow \boxed{\sigma_L(\theta) = \varepsilon_0 \left\{ \frac{V_0}{R}\cos\theta + \frac{2V_0 \cancel{R^2}}{R^{\cancel{3}}}\cos\theta \right\} = \frac{3\varepsilon_0 V_0 \cos\theta}{R}}$

6)



// $\sigma_{2p}\big|_b = \overset{?}{\vec{P_2}}\big|_b \cdot (-\hat{e}_r)$ (I)

// región (1) $a<r<b$

$\vec{P_1} = P_0\,\hat{e}_r = \frac{\varepsilon_0 V_0}{2a}\,\hat{e}_r$

$\vec{D_1} = \varepsilon_0 \vec{E_1} + \vec{P_1}$

$\boxed{\vec{E_1} = \frac{\vec{D_1}}{\varepsilon_0} - \frac{\vec{P_1}}{\varepsilon_0} = \frac{\vec{D_1}}{\varepsilon_0} - \frac{\cancel{\varepsilon_0} V_0\,\hat{e}_r}{\cancel{\varepsilon_0}\,2a}}$ (II)

// región (2) $b<r<c$

$\underbrace{\vec{D_2}}_{\substack{(\text{lineal})\ \| \\ \varepsilon \vec{E_2}}} = \varepsilon_0 \vec{E_2} + \vec{P_2} \Rightarrow \vec{P_2} = (\underbrace{\varepsilon}_{2\varepsilon_0} - \varepsilon_0)\vec{E_2}$

$\Rightarrow \boxed{\vec{P_2} = \varepsilon_0 \vec{E_2}}$ (III)

Gauss dieléctrico

carga libre en $r=a$

región (1):

$D_1(r)(4\pi r^2) = q \longrightarrow \vec{D_1}(r) = \frac{q}{4\pi r^2}\,\hat{e}_r$

$\Rightarrow$ En (II): $\boxed{\vec{E_1} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{e}_r - \frac{V_0}{2a}\,\hat{e}_r}$

región (2):

$D_2(r)(4\pi r^2) = q \longrightarrow \underbrace{\vec{D_2}(r)}_{2\varepsilon_0 \vec{E_2}} = \frac{q}{4\pi\, r^2}\,\hat{e}_r \Rightarrow \boxed{\vec{E_2} = \frac{q}{4\pi(2\varepsilon_0)r^2}\,\hat{e}_r}$

// $V_0 = \int_a^{b=2a} E_1\,dr + \int_{b=2a}^{c=3b=6a} E_2\,dr = \frac{q}{4\pi\varepsilon_0}\underbrace{\int_a^{2a}\frac{dr}{r^2}}_{-\frac{1}{r}\big|_a^{2a}} - \frac{V_0}{\cancel{2a}}\underbrace{(2a-a)}_{\cancel{a}} + \frac{q}{4\pi 2\varepsilon_0}\int_{2a}^{6a}\frac{dr}{r^2}$

$V_0 = \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{2a}+\frac{1}{a}\right) - \frac{V_0}{2} + \frac{q}{4\pi 2\varepsilon_0}\left(-\frac{1}{6a}+\frac{1}{2a}\right)$

$= \frac{q}{4\pi\varepsilon_0}\frac{1}{2a} - \frac{V_0}{2} + \frac{q}{4\pi 2\varepsilon_0}\frac{1}{3a}$

$V_0 + \frac{V_0}{2} = \frac{q}{4\pi\varepsilon_0}\frac{1}{2a}\frac{4}{3} \rightarrow \boxed{\frac{q}{4\pi\varepsilon_0} = \cancel{2}a\,\frac{3}{4}\,\frac{3V_0}{\cancel{2}} = \frac{9}{4}V_0 a}$

En (III)

$\Rightarrow \vec{P_2} = \varepsilon_0 \vec{E_2} = \varepsilon_0\left\{\frac{q}{4\pi\varepsilon_0 2}\frac{1}{r^2}\,\hat{e}_r\right\}$

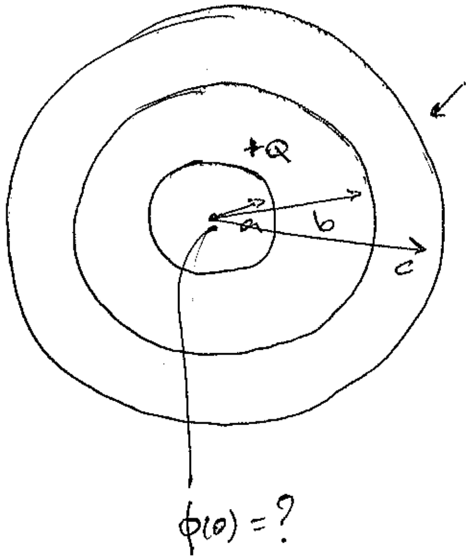
$= \varepsilon_0\left\{\frac{9}{4}\frac{V_0 a}{2r^2}\,\hat{e}_r\right\} \Rightarrow \boxed{\vec{P_2} = \frac{9}{8}\frac{\varepsilon_0 V_0 a}{r^2}\,\hat{e}_r}$

sust en (I):

$\boxed{\sigma_{2p}\big|_b = \vec{P_2}\big|_{r=b=2a}\cdot(-\hat{e}_r) = -\frac{9}{8}\frac{\varepsilon_0 V_0 a}{(2a)^2} = -\frac{9}{32}\frac{\varepsilon_0 V_0}{a}}$

—— x ——

7)



$b = 2a$
$c = 3b = 6a$

$$\phi(0) - \phi(\infty) = \int_0^\infty E\,dr$$

Gauss en c/región (ver (6))

$$\vec{E}(r) = \begin{cases} 0\ \hat{e}_r & ;\ 0<r<a \\ \left(\dfrac{Q}{4\pi\varepsilon_0 r^2} - \dfrac{\rho_0}{\varepsilon_0}\right)\hat{e}_r & ;\ a<r<b=2a \\ \dfrac{Q}{4\pi \underset{= 2\varepsilon_0}{\varepsilon} r^2} & ;\ 2a=b<r<c=6a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & ;\ r>c=6a \end{cases}$$

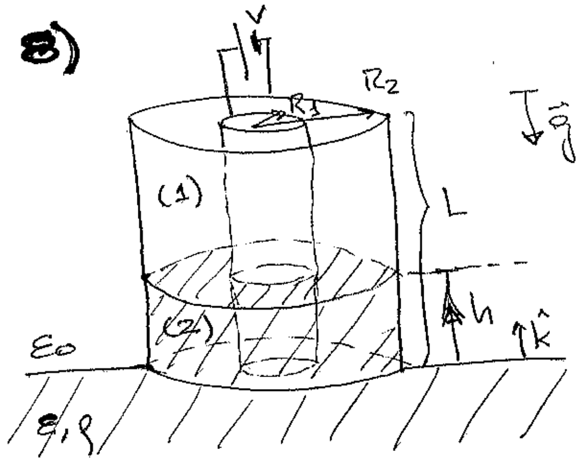
$$\Rightarrow \phi(0) - \underset{=0}{\phi(\infty)} = \int_a^{2a}\left[\frac{Q}{4\pi\varepsilon_0 r^2} - \frac{\rho_0}{\varepsilon_0}\right]dr + \int_{2a}^{6a}\frac{Q}{4\pi(2\varepsilon_0)r^2}\,dr + \int_{6a}^{\infty}\frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\underbrace{\left(-\frac{1}{r}\Big|_a^{2a}\right)}_{=\frac{1}{a}-\frac{1}{2a}=\frac{1}{a}\underbrace{\left(1-\frac{1}{2}\right)}_{=\frac{1}{2}}} - \frac{\rho_0}{\varepsilon_0}a + \frac{Q}{4\pi(2\varepsilon_0)}\underbrace{\left(-\frac{1}{r}\Big|_{2a}^{6a}\right)}_{=\frac{1}{2a}-\frac{1}{6a}=\frac{1}{2}\frac{1}{a}\underbrace{\left(1-\frac{1}{3}\right)}_{=\frac{2}{3}}} + \frac{Q}{4\pi\varepsilon_0}\underbrace{\left(-\frac{1}{r}\Big|_{6a}^{\infty}\right)}_{=\frac{1}{6a}}$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\Big\{\underbrace{\frac{1}{2}+\frac{1}{6}+\frac{1}{6}}_{=\frac{3+1+1}{6}=\frac{5}{6}}\Big\} - \frac{\rho_0 a}{\varepsilon_0}$$

$$\Rightarrow \boxed{\phi(0) = \frac{5}{6}\frac{Q}{4\pi\varepsilon_0 a} - \frac{\rho_0}{\varepsilon_0}a = \frac{5Q}{24\pi\varepsilon_0 a} - \frac{\rho_0 a}{\varepsilon_0}}$$

8)



En equilibrio: $\vec{F}_g + \vec{F}_E = 0$ (I)

$$\boxed{\vec{F}_g = \rho g h \left[\pi (R_2^2 - R_1^2)\right](-\hat{k})} \text{ peso}$$

$$\vec{F}_E = \left.\frac{\partial U}{\partial h}\right|_V \hat{k}$$

$$U = \frac{1}{2} Q V = \frac{1}{2} C_{eq} V^2 \; ; \; C_{eq} = C_1 + C_2$$
(paralelo)

Capacitor C_1

Gauss $E_1(r)(2\pi r)(L-h) = \frac{q_1}{\varepsilon_0} \rightarrow \vec{E}_1(r) = \frac{q_1}{2\pi\varepsilon_0 (L-h)} \frac{1}{r} \hat{e}_r$

$$V = -\int_{(-)}^{(+)} \vec{E} \cdot d\vec{r} = \int_{(+)}^{(-)} \frac{q_1}{2\pi\varepsilon_0 (L-h)} \frac{1}{r} dr = \frac{q_1}{2\pi\varepsilon_0 (L-h)} \int_{R_1}^{R_2} \frac{dr}{r} = \underbrace{\frac{\ln(R_2/R_1)}{2\pi\varepsilon_0 (L-h)}}_{= \frac{1}{C_1}} q_1$$

$$\Rightarrow \boxed{C_1 = \frac{2\pi\varepsilon_0 (L-h)}{\ln(R_2/R_1)}}$$

De forma similar en (2): $\boxed{C_2 = \frac{2\pi \varepsilon h}{\ln(R_2/R_1)}}$

$$\Rightarrow C_{eq} = C_1 + C_2 = \frac{2\pi\varepsilon_0 (L-h) + 2\pi\varepsilon h}{\ln(R_2/R_1)} = \frac{2\pi h(\varepsilon - \varepsilon_0) - 2\pi\varepsilon_0 L}{\ln(R_2/R_1)}$$

$$U = \frac{1}{2} C_{eq} V^2$$

$$\Rightarrow \vec{F}_E = \left.\frac{\partial U}{\partial h}\right|_V \hat{k} = \frac{1}{\cancel{2}} V^2 \frac{\cancel{2}\pi(\varepsilon - \varepsilon_0)}{\ln(R_2/R_1)} \hat{k} \rightarrow \boxed{\vec{F}_E = \frac{\pi V^2 (\varepsilon - \varepsilon_0)}{\ln(R_2/R_1)} \hat{k}} \text{ fuerza eléctrica}$$

De (I) (condición de equilibrio):

$$\rho g h \cancel{\pi} (R_2^2 - R_1^2) = \frac{\cancel{\pi} V^2 (\varepsilon - \varepsilon_0)}{\ln(R_2/R_1)} \rightarrow \boxed{h = \frac{V^2 (\varepsilon - \varepsilon_0)}{\rho g (R_2^2 - R_1^2) \ln(R_2/R_1)}}$$

—— ∝ ——