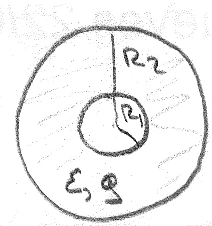
1er Parcial Electromagnetismo

1



$R_2 = 2R_1$

La resistencia de este sistema es: $R = \dfrac{R_2 - R_1}{4\pi g R_1 R_2}$ (ver. práctico 5)

En este caso $\boxed{R = \dfrac{1}{8\pi g R_1}}$

2 La capacitancia de dicho sistema es:

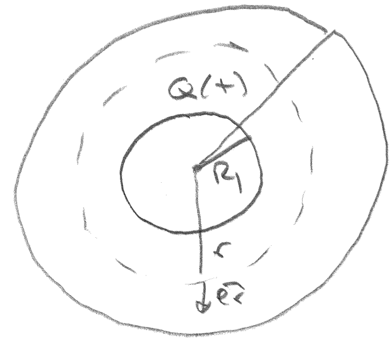
$$C = \frac{4\pi \varepsilon R_1 R_2}{R_2 - R_1} \quad \text{(ver práctico 4)}$$

En este caso $\boxed{C = 8\pi\varepsilon R_1}$

La energía disipada por efecto Joule hasta alcanzarse la condición electrostática es la energía almacenada en el instante inicial

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_0^2}{16\pi\varepsilon R_1}$$

3 $\vec{E}(r,t) = \dfrac{Q(t)}{4\pi\varepsilon}\dfrac{1}{r^2}\,\hat{e}_r$



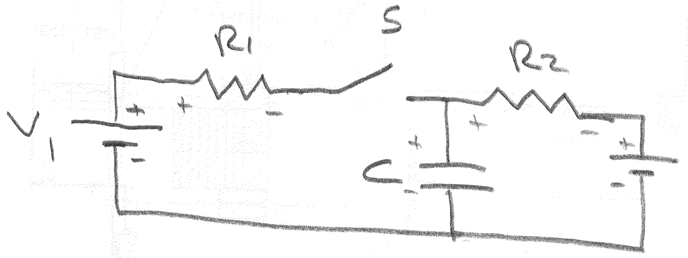
$\nabla \cdot \vec{J} = -\dfrac{\partial \rho}{\partial t} \rightarrow \dfrac{g}{\varepsilon}\sigma(t) = -\dot{\sigma}(t)$

$$\sigma(t) = \sigma_0 e^{-\frac{g}{\varepsilon} t} \quad \text{con} \quad \sigma_0 = \frac{Q_0}{4\pi R_1^2}$$

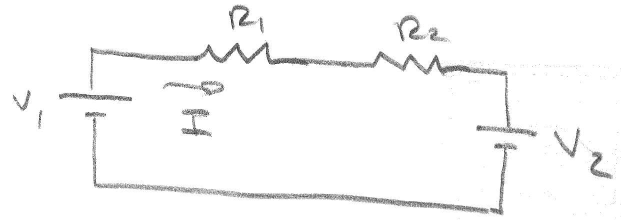
$$\Rightarrow \quad Q(t) = \sigma(t)\, 4\pi R_1^2 \Rightarrow Q(t) = Q_0 e^{-\frac{g}{\varepsilon} t}$$

$$\Rightarrow \vec{E}(\vec{r},t) = \frac{Q_0 e^{-\frac{g}{\varepsilon}t}}{4\pi\varepsilon} \frac{1}{r^2} \hat{e_r} \Rightarrow \boxed{\vec{J}(\vec{r},t) = \frac{Q_0 g}{4\pi\varepsilon} \frac{1}{r^2} e^{-\frac{g}{\varepsilon}t} \hat{e_r}}$$

$\boxed{4}$



antes de abrir S tengo:



$$I = \frac{|V_1 - V_2|}{R_1 + R_2}$$

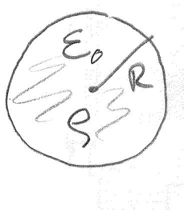
al abrir S:

en el instate inicial

$$I'(t=0) = \frac{|V_1 - V_2|}{R_1 + R_2}$$

xq' la corriente es continua

$\boxed{5}$

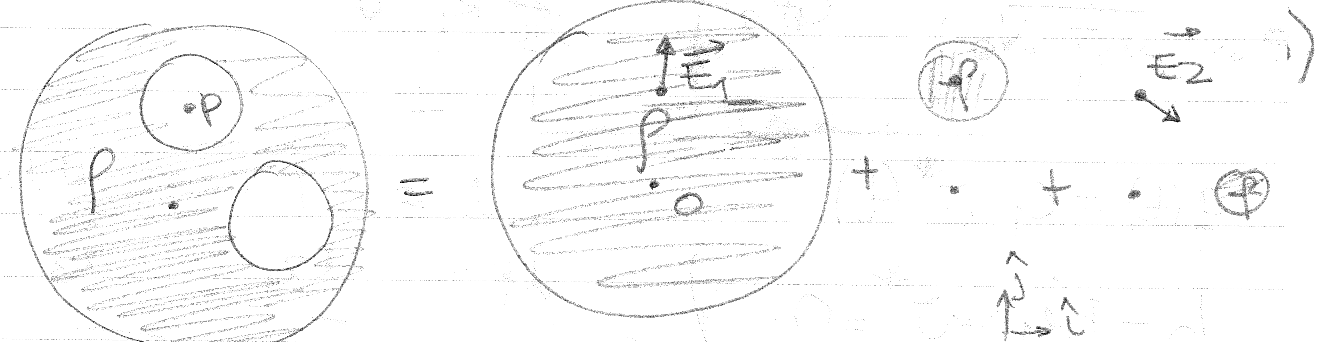


$$V_\infty = 0$$

$$\vec{E} = \begin{cases} \dfrac{\rho r}{3\varepsilon_0} & 0 \le r \le R \\ \dfrac{\rho R^3}{3\varepsilon_0} \dfrac{1}{r^2} & R \le r \end{cases}$$

$$V_0 - V_\infty = \int^{\infty} \vec{E} \cdot \vec{dr} = \frac{\rho}{3\varepsilon_0} \left[\int^{R} r\, dr + \int^{\infty} R^3 \, dr \right] = \frac{\rho R^2}{}$$

⑥ SUPERPOSICIÓN:



$$\vec{E}_p = \vec{E}_1 + 0 + \vec{E}_2$$

lo que quiero

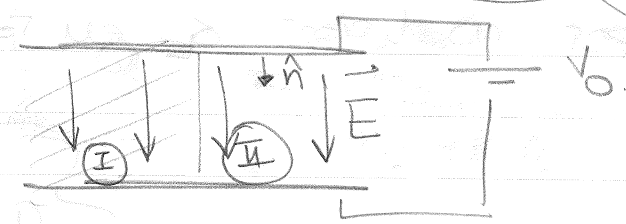
Utilizando ley de GAUSS: $\vec{E} = \begin{cases} \dfrac{\rho r}{3\varepsilon_0} & r<a \\ \dfrac{\rho a^3}{3r^2\varepsilon_0} & r>a \end{cases}$

$$\Rightarrow \vec{E}_1 = \frac{\rho d}{3\varepsilon_0}\hat{j},$$

$$\vec{E}_2 = \frac{\rho a^3}{3(\sqrt{2}d)^2\varepsilon_0} \cdot \frac{\hat{i}-\hat{j}}{\sqrt{2}}$$

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 \Rightarrow |E_p| = \frac{\rho}{6\varepsilon_0 d^2}\sqrt{\frac{a^6}{2} + \left(2d^3 - \frac{a}{\sqrt{2}}\right)^2}$$

⑦

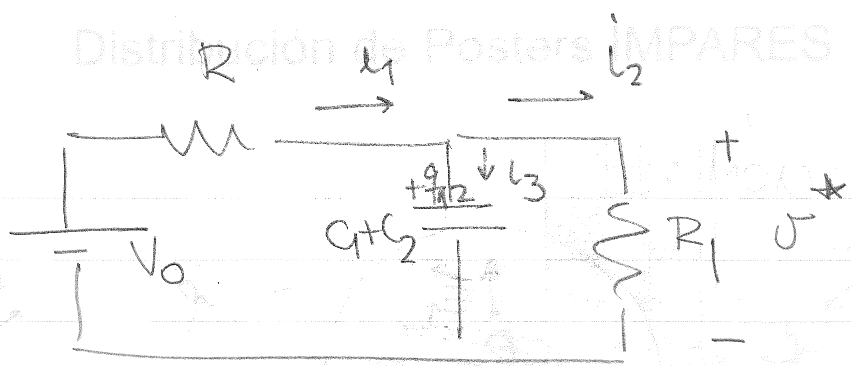


$\vec{E}$ uniforme $\Rightarrow E = \dfrac{V_0}{d}$

$$Q_L = \int \sigma_L \qquad \sigma_L = \vec{D}\cdot\hat{n} = \begin{cases} (\varepsilon_0\vec{E} + \vec{P_0})\cdot\hat{u} = \varepsilon_0\dfrac{V_0}{d} + P_0 & \text{I} \\ \varepsilon\vec{E} = \varepsilon\dfrac{V_0}{d} & \text{II} \end{cases}$$

$$Q_L = \frac{ab}{2}\cdot\left[(\varepsilon+\varepsilon_0)\frac{V_0}{d} + P_0\right]$$

(8)



$q(t) = C_1 \cdot v^*(t)$.

$$\left.\begin{aligned} &V_0 - R i_1 - v^* = 0 \\ &R_1 i_2 = v^* \\ &q_{12} = (C_1+C_2)\, v^* = 0 \\ &i_3 = \dot{q}_{12} \\ &i_1 = i_2 + i_3 \end{aligned}\right\} \quad \underbrace{\frac{V_0 - v^*}{R}}_{i_1} = \underbrace{\frac{v^*}{R_1}}_{i_2} + \underbrace{(C_1+C_2)\dot{v}^*}_{i_3}$$

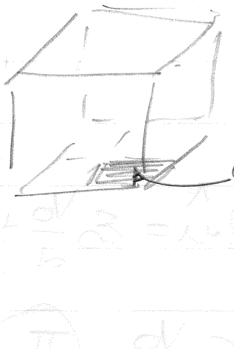
$$\left\{\begin{aligned} &\frac{V_0}{R(C_1+C_2)} = \dot{v}^* + \frac{1}{C_1+C_2}\underbrace{\left(\frac{1}{R_1}+\frac{1}{R}\right)}_{\frac{R+R_1}{R \cdot R_1}} v^* \\ &v^*(t=0) = 0 \end{aligned}\right.$$

<u>SOL</u>: $v^*(t) = \frac{R_1}{R_1+R} V_0 \left[1 - e^{-t/\tau}\right]$, $\tau = \frac{R R_1 (C_1+C_2)}{R+R_1}$

$q(t) = \frac{C_1 R_1}{R_1+R} V_0 \left(1 - e^{-t/\tau}\right)$

(9) D es continuo por no haber q en r=b.

(10)



$\phi_E = \frac{1}{4}\phi_{lado} = \frac{1}{24}\underbrace{\phi_{cubo}}_{\frac{q}{\varepsilon_0}\ \text{"GAUSS}} = \frac{q}{24\varepsilon_0}$