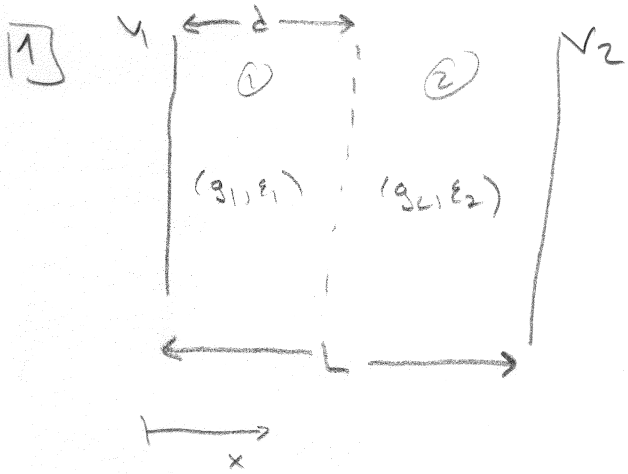
1



Vale Laplace en c/región

$\phi_1(x) = Ax + B$

$\phi_2(x) = Cx + D$

$\phi_1(0) = V_1$

$\phi_2(L) = V_2$

$\phi_1(d) = \phi_2(d)$

$B = V_1$

E. estado estacionario $\quad J_1 = J_2 \rightarrow g_1 E_1 = g_2 E_2$

$$g_1 A = g_2 C$$

$$Ad + V_1 = Cd + D$$

$$CL + D = V_2$$

$$C = \frac{g_1}{g_2} A \rightarrow D = V_2 - \frac{g_1}{g_2} LA$$

$$Ad + V_1 = \frac{g_1}{g_2} dA + V_2 - \frac{g_1}{g_2} LA$$

$$A\left[d + \frac{g_1}{g_2}(L-d)\right] = V_2 - V_1$$

$$A = \frac{(V_2 - V_1)\, g_2}{d g_2 + g_1(L-d)}$$

a) $$\vec{J} = \frac{(V_2 - V_1)\, g_2}{d g_2 + g_1(L-d)}\, g_1 (-\hat{x})$$

b) $$\phi_1(d) = \frac{(V_2 - V_1)\, g_2\, d}{d g_2 + g_1(L-d)} + V_1$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \vec{P}_1 = (\varepsilon_1 - \varepsilon_0)\,\vec{E}_1$$

Idem $\qquad \vec{P}_2 = (\varepsilon_2 - \varepsilon_0)\,\vec{E}_{2}$

$$\vec{E}_1 = \frac{(V_2 - V_1)\,g_2}{d g_2 + g_1 (L-d)}\,(-\hat{z})$$

$$\vec{E}_2 = \frac{(V_2 - V_1)\,g_1}{d g_2 + g_1 (L-d)}\,(-\hat{z})$$

$$\sigma_I^L = \vec{D}_1 \cdot (-\hat{z}) + \vec{D}_2 \cdot (\hat{z}) = \frac{V_2 - V_1}{d g_2 + g_1 (L-d)} \left\{ g_2 \varepsilon_1 (-\hat{z}) \cdot (-\hat{z}) + g_1 \varepsilon_2 (\hat{z}) \cdot (-\hat{z}) \right\}$$

$$\boxed{\sigma_I^L = \frac{(V_2 - V_1)}{d g_2 + g_1 (L-d)} \left(g_2 \varepsilon_1 - g_1 \varepsilon_2 \right)}$$

$$\sigma_p^{Iz} = \vec{P}_1 \cdot \hat{z} = \frac{(\varepsilon_1 - \varepsilon_0)(V_2 - V_1)(-1)}{d g_2 + g_1 (L-d)}\, g_2 \qquad \text{Izquierda}$$

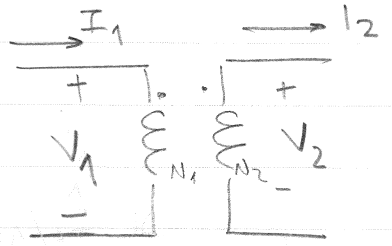
$$\sigma_p^{d} = \vec{P}_2 \cdot (-\hat{z}) = \frac{(V_2 - V_1)\; g_1}{d g_2 + g_1 (L-d)}\,(\varepsilon_2 - \varepsilon_0) \qquad \text{derecha}$$

$$\sigma_p = \sigma_p^{Iz} + \sigma_p^{d}$$

$$\boxed{\sigma_p = \frac{(V_2 - V_1)}{d g_2 + g_1 (L-d)} \left\{ g_1 (\varepsilon_2 - \varepsilon_0) - g_2 (\varepsilon_1 - \varepsilon_0) \right\}}$$

2) $N_2 = n \cdot N_1$

i)

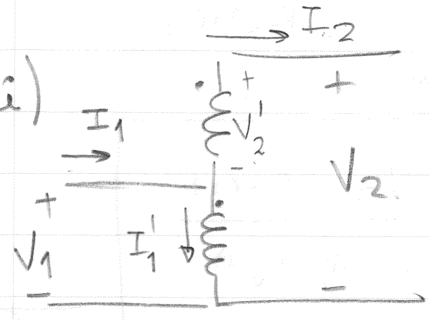


trafo ideal:

$$V_1/N_1 = V_2/N_2 \Rightarrow \boxed{V_2 = nV_1}$$

$$I_1 N_1 = I_2 N_2 \Rightarrow \boxed{I_2 = \frac{I_1}{n}}$$

ii)



AUTOTRANSFORMADOR.

<u>trafo ideal</u> $\begin{cases} V_2' = nV_1 \\ I_2 = I_1'/n \end{cases}$

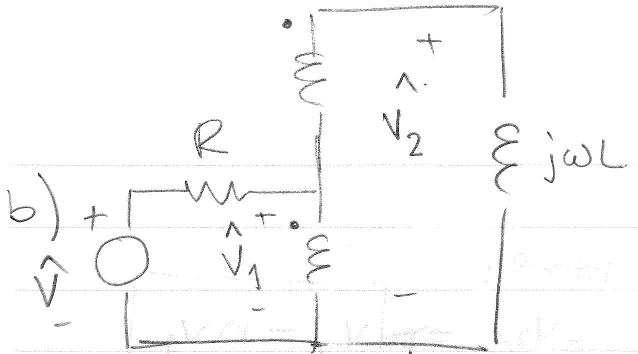
$$V_2 = V_1 + V_2' \rightarrow \boxed{V_2 = (1+n)V_1}$$

$$I_1 = I_1' + I_2 = (1+n)I_2 \Rightarrow \boxed{I_2 = \frac{I_1}{1+n}}$$

[obs. → En ambos se conserva la potencia].

- La conexión ii impone un voltaje y entrega menor corriente.
- En la conexión ii se pierde la aislación entre primario y secundario.

b)



$$\hat{I}_2 \underset{a)}{=} \frac{\hat{I}_1}{1+n} \underset{\text{ohm}}{=} \frac{\overbrace{\hat{V}}^{V_0} - \hat{V}_1}{R(1+n)} \underset{a)}{=} \frac{V_0 - \hat{V}_2/1+n}{R(1+n)} \underset{\text{ohm}}{=} \frac{V_0 - j\omega L\,\hat{I}_2/1+n}{R(1+n)}$$

$$\hat{I}_2\left[1+\frac{j\omega L}{R(1+n)^2}\right] = \frac{V_0}{R(1+n)} \Rightarrow \boxed{\hat{I}_2 = \frac{V_0(1+n)}{R(1+n)^2 + j\omega L}}$$

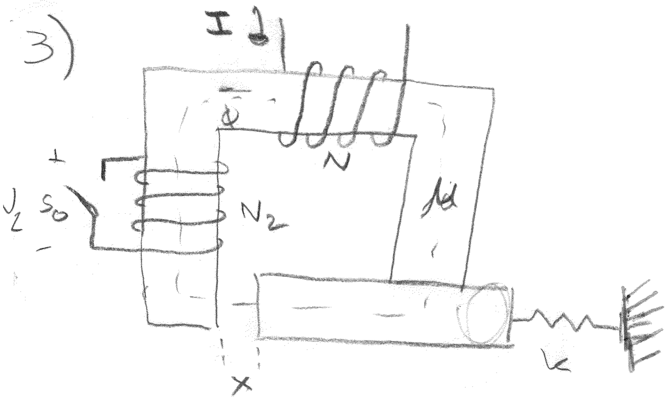
$$i_2(t) = \mathrm{Re}\left\{\hat{I}_2 e^{j\omega t}\right\} = \boxed{\frac{V_0(1+n)}{\sqrt{R(1+n)^2 + (\omega L)^2}}\cos(\omega t - \varphi) = i_2(t) \quad \text{con } \varphi = \operatorname{atan}\left(\frac{\omega L}{R(1+n)^2}\right)}$$

c) la potencia se disipa en R

$$\Rightarrow P = \frac{1}{2} R I_R^2 = \frac{1}{2} R I_1^2 = \frac{1}{2} R \cdot \left[I_2(1+n)\right]^2$$

$$\boxed{P = \frac{1}{2} R \cdot \frac{V_0^2 (1+n)^4}{R^2(1+n)^2 + (\omega L)^2}}$$

3)



a) $\mathcal{R} = \frac{4L}{\mu S}$

$\mathcal{R}_2 = \frac{x}{\mu_0 S}$

$$\Phi = \frac{NI}{\frac{4L}{\mu S} + \frac{x}{\mu_0 S}}$$

$B = \frac{\Phi}{S}$ $\qquad u = \frac{1}{2}\frac{B^2}{\mu_0}$

$$U = \frac{xS}{2}\frac{\Phi^2}{\mu_0 S^2} = \frac{x}{2\mu_0 S}\Phi^2$$

La bobina N_2 tiene resistencia nula $\Rightarrow v_2 = 0$ y $|v_2| = \left|N_2 \frac{d\Phi}{dt}\right| \Rightarrow \Phi$ cte

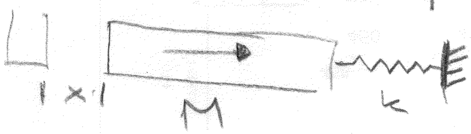
$$|\bar{F}| = \left|\frac{\partial U}{\partial x}\right| = \frac{\Phi^2}{2\mu_0 S} = \frac{(NI)^2}{2\mu_0 S}\frac{1}{\left(\frac{4L}{\mu S} + \frac{x}{\mu_0 S}\right)^2} = kx$$

inicialmente $x = x_0$

$$\Rightarrow \frac{(NI)^2}{2\mu_0 S}\frac{1}{\left(\frac{4L}{\mu S} + \frac{x_0}{\mu_0 S}\right)^2} = k x_0$$

$$\Rightarrow \boxed{I = \begin{cases} \left(\frac{4L}{\mu S} + \frac{x_0}{\mu_0 S}\right)\frac{\sqrt{k x_0 2\mu_0 S}}{N} & t=0 \\ \text{Puede Tomar cualquier valor} & t>0 \end{cases}}$$

b)



$B = \mu_0(\bar{H} + \bar{M})$

$$U = \frac{1}{2}\frac{B^2}{\mu_0} S x$$

$$x H_x + 3L H_\mu + L\left(\frac{B}{\mu_0} - M\right) = NI$$

$$\frac{x}{\mu_0}B + \frac{3L}{\mu}B + \frac{LB}{\mu_0} - LM = NI$$

$$U = \frac{1}{2}\frac{S}{\mu_0}\frac{(NI+LM)^2}{\left(\frac{3L}{\mu} + \frac{(L+x)}{\mu_0}\right)^2} x$$

$$\boxed{B = \frac{NI+LM}{\frac{3L}{\mu} + \frac{(L+x)}{\mu_0}}}$$

$$|\vec{F}| = \frac{1}{2}\frac{S}{\mu_0}\frac{(NI+LM)^2}{\left(\frac{3L}{\mu}+\frac{(L+x)}{\mu_0}\right)^2} = kx$$

$$I = \begin{cases} \frac{1}{N}\left\{\left(\frac{3L}{\mu}+\frac{(L+x)}{\mu_0}\right)\sqrt{\frac{2\mu_0 k x}{S}} - LM\right\} & t=0 \\ \text{puede tomar cualquier valor} & t>0 \end{cases}$$