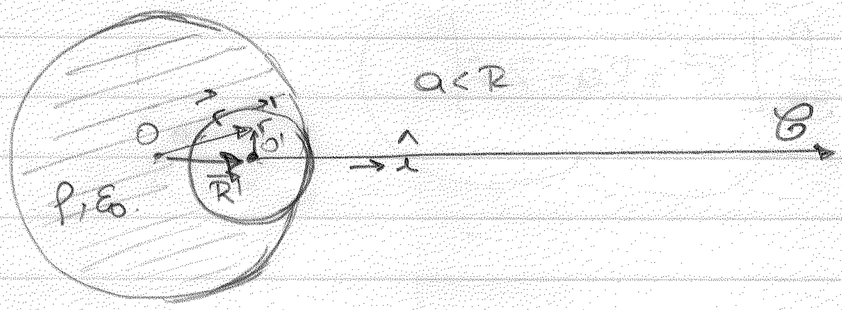
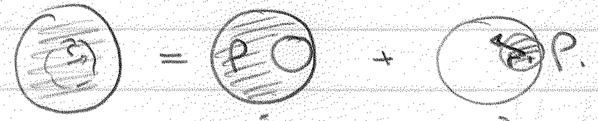
EXAMEN ELECTROMAGNETISMO - JULIO 2013.

1



$a < R$

a) $\vec{E}_{esfera}$?



SUPERPOSICIÓN

Gauss. $E\,\cancel{4\pi r^2} = \dfrac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$

$$\vec{E}(r) = \frac{\rho \vec{r}}{3\varepsilon_0} \qquad \vec{E}_2 = \frac{\rho \vec{r}\,'}{3\varepsilon_0} \quad \text{si } r' < a.$$

$$\Rightarrow \vec{E} = \frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{r}\,') = \frac{\rho(R-a)}{3\varepsilon_0}\hat{\imath}$$

$\vec{R}' + \vec{r}\,' = \vec{r}$

$\vec{R}' = (R-a)\,\hat{\imath}$

b) $\phi_{O'}$?

$$\Delta\phi_{O'\to\infty} = \overset{0}{\phi_\infty} - \phi_{O'} = -\int_{O'}^{\infty} \vec{E}\cdot d\vec{l} = \underbrace{-\int_{O'}^{R} \vec{E}\cdot d\vec{l}}_{\frac{\rho(R-a)}{3\varepsilon_0}\cdot a} - \underbrace{\int_{R}^{\infty} \vec{E}\cdot d\vec{l}}$$

a fuera de ambas esferas: $r' = r - R + a$

$$\int_R^\infty \Big(\overbrace{\frac{\rho R^3}{3r^2\varepsilon_0}}^{E_1} - \overbrace{\frac{\rho a^3}{3(r-R+a)^2\varepsilon_0}}^{E_2}\Big) dr$$

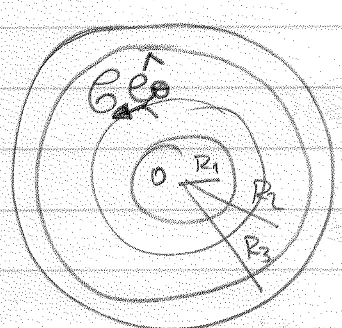
$$\frac{\rho R^3}{3\varepsilon_0}\cdot\left(-\frac{1}{r}\Big|_R^\infty\right) - \frac{\rho a^3}{3\varepsilon_0}\cdot\left(-\frac{1}{r-R+a}\right)\Big|_R^\infty$$

$$\frac{\rho R^2}{3\varepsilon_0} - \frac{\rho a^2}{3\varepsilon_0}$$

$$\phi_{O'} = \frac{\rho(R-a)a}{3\varepsilon_0} + \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho a^2}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\cdot\left[Ra - a^2 + R^2 - 2a^2\right]$$

$$\phi_{O'} = \frac{\rho}{3\varepsilon_0}\left[R^2 + Ra - 2a^2\right]$$

2



a) Energía disipada x efecto Joule → en las resistencias

$R_1 = \frac{\ell}{g\pi R_1^2}$ cilindro interior

$R_2 = \frac{\ell}{g\pi(R_3^2 - R_2^2)}$ cascarón

$\Rightarrow \boxed{P = (R_1 + R_2)\, I^2}$

b) $\oint_{\mathcal{C}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc.} \Rightarrow B\cdot 2\pi r = \mu_0 I \begin{cases} r^2/R_1^2 & r<R_1 \\ 1 & R_1<r<R_2 \\ 1 - \frac{r^2 - R_2^2}{R_3^2 - R_2^2} & R_2<r<R_3 \\ 0 & r>R_3 \end{cases}$

cfa de radio r
centro en O

$$\Rightarrow \vec{B} = \begin{cases} \frac{\mu_0 I r}{2\pi R_1^2}\,\hat{e}_\theta & r<R_1 \\ \frac{\mu_0 I}{2\pi r}\,\hat{e}_\theta & R_1<r<R_2 \\ \frac{\mu_0 I\,(R_3^2 - r^2)}{2\pi r\,(R_3^2 - R_2^2)}\,\hat{e}_\theta & R_2<r<R_3 \\ 0\,\hat{e}_\theta & r>R_3 \end{cases}$$

c) $R_2 = R_3$

$$U = \int_V \frac{B^2}{\cancel{2}\mu_0} \underbrace{dV}_{\cancel{2}\pi r\,dr\,h} = \frac{\pi}{\mu_0} l \left(\frac{\mu_0 I}{2\pi}\right)^2 \left(\int_0^{R_1} \left(\frac{r}{R_1^2}\right)^2 r\,dr + \int_{R_1}^{R_2} \frac{1}{r^{\cancel{2}}} \cdot \cancel{r}\,dr \right)$$

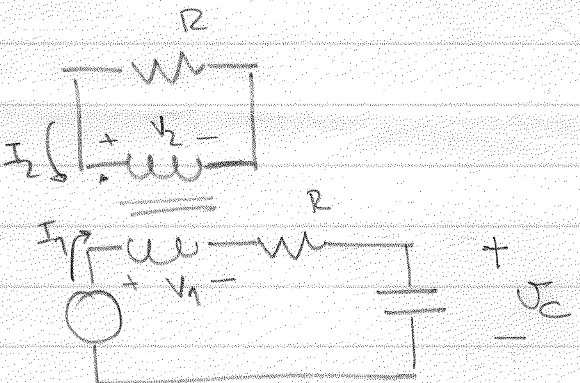
$$\underbrace{\left.\frac{r^4}{4R_1^4}\right|_0^{R_1}}_{1/4} \qquad \left. \ln r \right|_{R_1}^{R_2}$$

$$1/4 + \ln(R_2/R_1).$$

$$\Rightarrow U = \frac{\mu_0 I^2}{4\pi} l \left[1/4 + \ln(R_2/R_1) \right] = \frac{1}{2} L I^2$$

$$\Rightarrow \boxed{L = \frac{\mu_0 l}{2\pi} \cdot \left[1/4 + \ln(R_2/R_1) \right]}$$

3



$v(t) = V_0 \cdot e^{j\omega t}$

$|\omega L_2| \gg R$

$M = \sqrt{L_1 L_2}$

a) $v_C(t) = ?$

$$\begin{cases} \hat{V}_2 = j\omega L_2 \hat{I}_2 + j\omega M \hat{I}_1 = -R\hat{I}_2 \Rightarrow \hat{I}_1 = -\frac{(j\omega L_2 + \cancel{R})}{j\omega M} \hat{I}_2 \cong -\frac{L_2}{M} \hat{I}_2 \\ \hat{V}_1 = j\omega L_1 \hat{I}_1 + j\omega M \hat{I}_2 \end{cases}$$

mallas: $\hat{V} - \hat{V}_1 - R\hat{I}_1 - \frac{\hat{I}_1}{j\omega C} = 0.$

$$V_0 = \Big(\underbrace{j\omega L_1 - j\omega \frac{\overset{L_1 \cancel{L_2}}{M^2}}{\cancel{L_2}}}_{0} + R + \frac{1}{j\omega C} \Big) \hat{I}_1 \Rightarrow \boxed{\hat{I}_1 = \frac{V_0}{R - j/\omega C}}$$

$$\hat{V}_C = \frac{\hat{I}}{j\omega C} = \frac{U_0}{j\omega RC + 1}$$

$$u_C(t) = \frac{U_0}{\sqrt{1+(\omega RC)^2}} \cos(\omega t - \phi)$$

$\text{con } \phi = \arctan(\omega RC)$

b) $R_1 \rightarrow \bar{P}_1 = \frac{1}{2} R I_1^2 = \frac{1}{2} R \frac{U_0^2 \omega^2 C^2}{(R\omega C)^2 + 1}$

$R_2 \rightarrow \bar{P}_2 = \frac{1}{2} R I_2^2 = \frac{1}{2} R \cdot \frac{M^2}{L_2^2} \cdot \frac{V_0^2 \omega^2 C^2}{(R\omega C)^2 + 1} = \frac{1}{2} R \frac{L_1}{L_2} \frac{V_0^2 \omega^2 C^2}{1 + (R\omega C)^2}$

$I_2 = -\frac{M}{L_2} I_1$

$L_1 L_2 = M^2$

$$\bar{P}_{perte} = \bar{P}_1 + \bar{P}_2$$