Ejercicio 1:

a) $\rho_i(t) = \rho_{0_i} e^{-\frac{g_i}{\varepsilon_i} t}$, $\quad \rho_{0_i} = 0 \quad \Rightarrow \rho(t) = 0$

Por Gauss: $Q(t) = \int_S \vec{D}(\vec{r}) \cdot \hat{n} \, dS$

$\Rightarrow$ Por simetría: $Q(t) = D_1 2\pi r^2 + D_2 2\pi r^2$

Por otro lado: $E_1(r) = E_2(r) = E(r)$

$\Rightarrow Q(t) = E(r) 2\pi r^2 (\varepsilon_1 + \varepsilon_2) \quad (I)$

En el conductor interior:

$$\dot{\sigma}_1 + \dot{\sigma}_2 = -J_1(R_1) - J_2(R_1) = -E(g_1 + g_2)$$

$$\Rightarrow \dot{Q} = -E(R_1) 2\pi R_1^2 (g_1 + g_2) \quad (II)$$

De (I) y (II):

$$\frac{Q}{\dot{Q}} = -\frac{\varepsilon_1 + \varepsilon_2}{g_1 + g_2} \Rightarrow \boxed{Q_t = Q_0 e^{-\frac{g_1 + g_2}{\varepsilon_1 + \varepsilon_2} t}}$$

b) I) $Q(t) = E(r)\, 2\pi r^2 (\varepsilon_1 + \varepsilon_2)$

$$\Rightarrow E(r) = \frac{Q(t)}{2\pi r^2(\varepsilon_1+\varepsilon_2)}$$

$$\Rightarrow \boxed{\vec{E}(r) = \frac{Q_0}{2\pi r^2(\varepsilon_1+\varepsilon_2)} e^{-\frac{g_1+g_2}{\varepsilon_1+\varepsilon_2} t} \hat{e}_r}$$

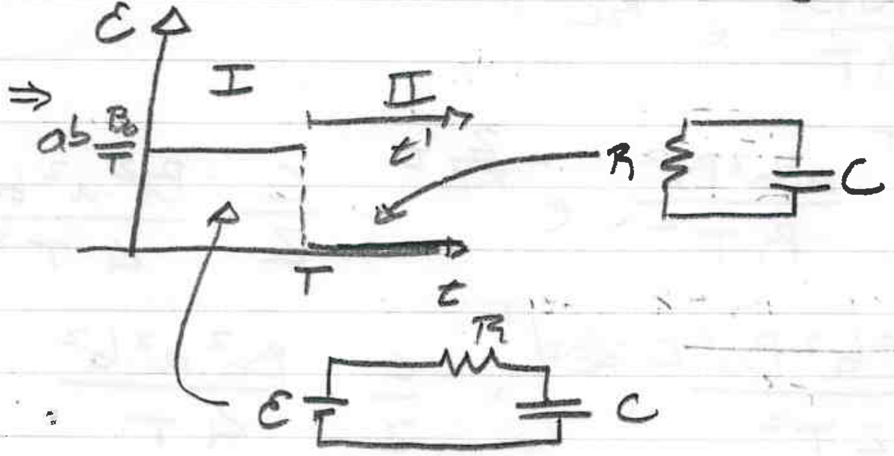
Condicion de borde en $r = R_1$

$$\boxed{\vec{E}_i(R_1)\cdot\hat{n} = \frac{\sigma_i}{\varepsilon_i}}$$

$$\Rightarrow \boxed{\sigma_i = \frac{Q_0\, \varepsilon_i}{2\pi R_1^2(\varepsilon_1+\varepsilon_2)} e^{-\frac{g_1+g_2}{\varepsilon_1+\varepsilon_2} t}}$$

Ejercicio 2

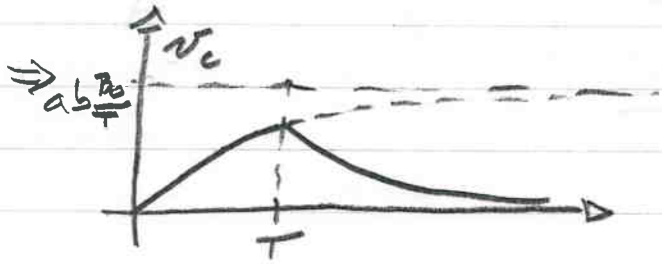
$$\phi = ab\,B \Rightarrow \left|\frac{d\phi}{dt}\right| = \begin{cases} ab\,\frac{B_0}{T}, & 0<t<T \\ 0, & \text{en otro caso} \end{cases}$$



$$\text{I}: \quad v_c^{(I)}(t) = ab\,\frac{B_0}{T}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\text{II}: \quad v_c^{II}(t') = v_c^{(I)}(T)\, e^{-\frac{t'}{RC}}$$

$$\hookrightarrow t' = t - T$$



b) Máximo en $t = T$

$$\left.\begin{aligned} \Rightarrow V_{c\max} &= \frac{B_0 ab}{2T} \\ V_{c\max} &= ab\,\frac{B_0}{T}\left(1 - e^{-\frac{T}{RC}}\right) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \frac{1}{2} = \left(1 - e^{-\frac{T}{RC}}\right) \Rightarrow T = RC\ln(2)$$

c) $U = \int_0^\infty P_R(t)\,dt = \int_0^T R\, i_I^2\, dt + \frac{C V_{c\,max}^2}{2}$

$$i_I = \frac{a b B_0}{R T} e^{-\frac{t}{RC}}$$

$$\Rightarrow U = \int_0^T \frac{a^2 b^2 B_0^2}{R\, T^2} e^{-\frac{2}{RC} t} + \frac{C}{2} \frac{B_0^2 a^2 b^2}{4 T^2}$$

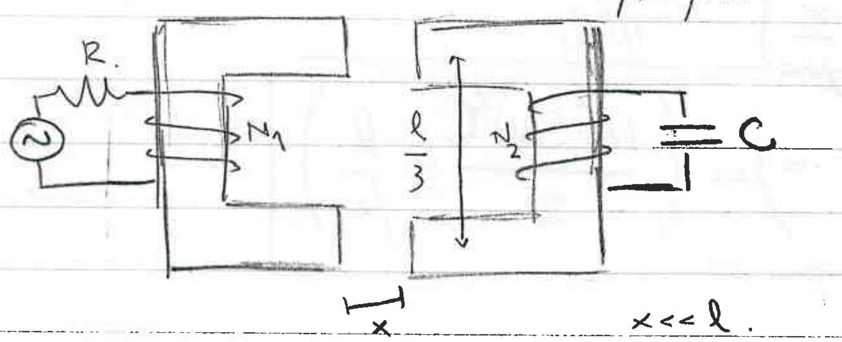
$$= -\frac{a^2 b^2 B_0^2 C}{2 T^2} e^{-\frac{2}{RC} t}\Big|_0^T + \frac{c}{2} \frac{B_0^2 a^2 b^2}{4 T}$$

$$= \frac{a^2 b^2 B_0^2 C}{T^2} \left(-\frac{1}{2} e^{-\frac{2}{RC} T} + \frac{5}{8} \right)$$

$$e^{-\frac{2}{RC} T} = e^{-\frac{2}{RC} RC \ln(2)} = \frac{1}{4}$$

$$= \frac{1}{2} \frac{a^2 b^2 B_0^2 C}{T^2}$$

Ejercicio 3:



$\mathcal{E}(t) = \mathcal{E}_0 \cdot \cos(\omega t)$.

a) ¿L_1, L_2, M?

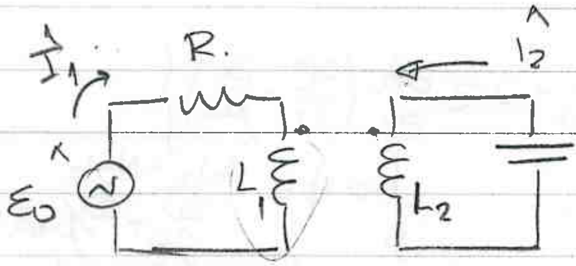
$$L_1 = \frac{N_1^2}{R_0} = \frac{N_1^2}{\frac{2l}{\mu S} + \frac{2x}{\mu_0 S}} = \boxed{\frac{N_1^2 S}{2\left(\frac{l}{\mu} + x/\mu_0\right)}}$$

x simetría, $\boxed{L_2 = \frac{N_2^2 S}{2\left(\frac{l}{\mu} + x/\mu_0\right)}}$

$$M = \sqrt{L_1 L_2} = \boxed{\frac{N_1 N_2 S}{2\left(\frac{l}{\mu} + x/\mu_0\right)}}$$

no hay pérdida de flujo.

b)



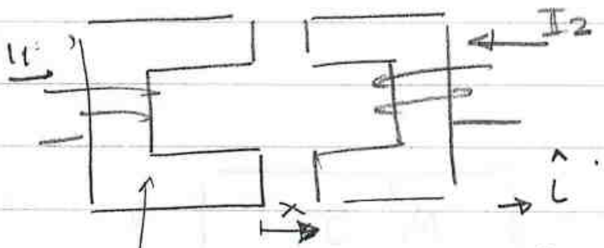
$\hat{I}_1 = 0 \iff \begin{cases} \hat{\mathcal{E}}_0 = j\omega M \hat{I}_2 & (1) \\ -j\omega L_2 \hat{I}_2 - \frac{1}{j\omega C}\hat{I}_2 = 0 \end{cases}$ $\rightarrow I_2 = 0$ Absurdo x (1)

$$\frac{1}{\omega C} = \omega L_2$$

$$L_2 = \frac{N_2^2 S}{2\left(\frac{l}{\mu}+\frac{x}{\mu_0}\right)} = \frac{1}{\omega^2 C}$$

$$\boxed{x = \mu_0\left(\frac{N_2^2 S\omega^2 C}{2} - \frac{l}{\mu}\right)}$$

c)



supongo herradura fija.

$$U = \frac{1}{2}\int_{VOL}\frac{B^2}{\mu}\,dV = \frac{1}{2}B^2 S\cdot\left(\frac{2l}{\mu}+\frac{2x}{\mu_0}\right)$$

@ $\phi = cte \Rightarrow B = cte.$

$$\vec{F} = -\frac{\partial U}{\partial x}\Big|_{\phi=cte}\hat{i} = -\frac{1}{\cancel{2}}B^2 S\cdot\frac{\cancel{2}}{\mu_0}\hat{i} \Rightarrow \boxed{\vec{F} = -\frac{B^2 S}{\mu_0}\hat{i}}$$

@ $I = cte.$

$$\vec{F} = \frac{\partial U}{\partial x}\Big|_{I=cte}\hat{i} = \frac{1}{2}\cdot S\cdot\left\{B^2\cdot\frac{2}{\mu_0} + 2\cdot B\cdot\frac{\partial B}{\partial x}\left(\frac{2l}{\mu}+\frac{2x}{\mu_0}\right)\right\}$$

$$\hookrightarrow \phi = \frac{N_1 I_1 + N_2 I_2}{\frac{2l}{\mu S}+\frac{2x}{\mu_0 S}}$$

$$B = \frac{\phi}{S} = \frac{N_1 I_1 + N_2 I_2}{\frac{2l}{\mu}+\frac{2x}{\mu_0}}$$

$$\frac{\partial B}{\partial x} = \frac{N_1 I_1 + N_2 I_2}{\left(\frac{2l}{\mu}+\frac{2x}{\mu_0}\right)^2}\cdot\left(\frac{-2}{\mu_0}\right)$$

$$= \frac{1}{2} \cdot S \cdot \left\{ B^2 \frac{2}{\mu_0} + 2 \cdot B \overbrace{\boxed{\cdot \frac{N_1 I_1 + N_2 I_2}{\left(\frac{2l}{\mu} + \frac{2x}{\mu_0}\right)^{\cancel{2}}}}}^{B.} \cdot \left(-\frac{2}{\mu_0}\right) \cdot \cancel{\left(\frac{2l}{\mu} + \frac{2x}{\mu_0}\right)} \right\} \hat{i}$$

$$= \left(\frac{SB^2}{\mu_0} - \frac{2 \cdot B^2 S}{\mu_0} \right) \hat{i} \quad = -\frac{B^2 S}{\mu_0} \hat{i}$$