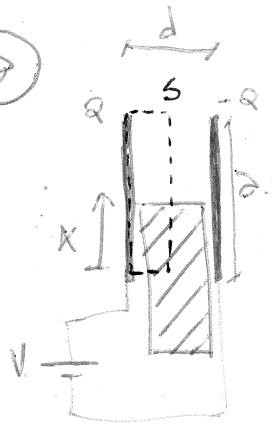
Ejercicio 1

a)



$$\oint_S \vec{D}\cdot\hat{n}\,ds = Q \Rightarrow$$

$$\Rightarrow E\varepsilon_1 x b + E\varepsilon_0 (a-x) b = Q \Rightarrow$$

$$\Rightarrow E = \frac{Q}{\varepsilon_1 x b + \varepsilon_0 (a-x) b}$$

$$V = E\cdot d$$

$$\Rightarrow V = \frac{Q d}{[\varepsilon_0 a + (-\varepsilon_0+\varepsilon_1)x]b}$$

$$V\cdot C = Q$$

$$\Rightarrow \boxed{C = \frac{[\varepsilon_0 a + (\varepsilon_1-\varepsilon_0)x]\,b}{d}}$$

b)

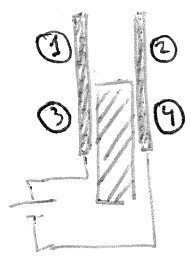
$$K x_{eq} = \left.\frac{\partial U}{\partial x}\right|_{V=cte}$$

$$U = \frac{1}{2} C V^2 = \frac{V^2}{2}\left(\frac{[\varepsilon_0 a + (\varepsilon_1-\varepsilon_0)x]\,b}{d}\right)$$

$$\Rightarrow K x_{eq} = \frac{V^2}{2}(\varepsilon_1-\varepsilon_0)\frac{b}{d} \Rightarrow$$

$$\Rightarrow \boxed{x_{eq} = \frac{V^2 b(\varepsilon_1-\varepsilon_0)}{2dK}}$$

c)



$V = E\cdot d$

1) $\sigma_{L1} = \varepsilon_0 E = \frac{\varepsilon_0 V}{d}$ 2) $\sigma_{L2} = -\frac{\varepsilon_0 V}{d}$

3) $\sigma_{L3} = \varepsilon_1 E = \frac{\varepsilon_1 V}{d}$ 4) $\sigma_{L4} = -\frac{\varepsilon_1 V}{d}$

$$\sigma_{p3} = \vec{P}\cdot\hat{n} = -(\varepsilon_1-\varepsilon_0)\frac{V}{d} \qquad \sigma_{p4} = (\varepsilon_1-\varepsilon_0)\frac{V}{d}$$

$$Q_L = V\cdot C\Big|_{x=x_{eq}} = V\cdot\left[\varepsilon_0 a + \frac{V^2 b(\varepsilon_1-\varepsilon_0)^2}{2dK}\right]\frac{b}{d}$$

$$Q_p = \sigma_{p3}\cdot x_{eq}\cdot b = -(\varepsilon_1-\varepsilon_0)\frac{V}{d}\cdot\frac{V^2 b(\varepsilon_1-\varepsilon_0)}{2dK}\cdot b = -\frac{(\varepsilon_1-\varepsilon_0)^2 V^3 b^2}{2d^2 K}$$