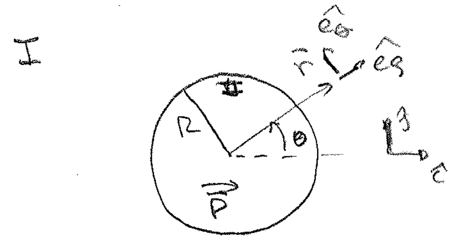
$\boxed{1}$



$\vec{P} = P_0 \hat{x}$

a) En I se cumple Laplace por ser vacío

En II tenemos una polarización uniforme $\vec{P} = P_0 \hat{x}$

no hay carga libre

$\nabla \cdot \vec{D} = 0$

$$\nabla \cdot \vec{D} = \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = 0$$

$$\varepsilon_0 \nabla \cdot \vec{E} + \nabla \cdot \vec{P} = 0 \qquad \varepsilon_0 \nabla \cdot \vec{E} = \nabla \cdot \vec{P}$$

en cilíndricas $\vec{P} = P_0 (\cos\theta \, \hat{e}_\rho - \operatorname{sen}\theta \, \hat{e}_\theta)$

$$\nabla \cdot \vec{P} = \frac{1}{r} \frac{\partial (r P_0 \cos\theta)}{\partial r} + \frac{1}{r} \frac{\partial (-P_0 \operatorname{sen}\theta)}{\partial \theta}$$

$$\nabla \cdot \vec{P} = \frac{1}{r} \left(P_0 \cos\theta - P_0 \cos\theta \right) = 0 \Rightarrow \nabla \cdot \vec{E} = 0 \Rightarrow \boxed{\nabla^2 \phi = 0}$$

b) $\rho_L = \sigma_L = 0$ no hay densidades de carga libre.

$\rho_P = -\nabla \cdot \vec{P} = 0$ (parte a)

$\sigma_P = \vec{P} \cdot \hat{n} = P_0 (\cos\theta \, \hat{e}_\rho - \operatorname{sen}\theta \, \hat{e}_\theta) \cdot \hat{e}_\rho = P_0 \cos\theta$

c) $$\phi(r,\theta) = \begin{cases} \phi_I & r \geq a \\ \phi_{II} & r \leq a \end{cases}$$

Tomo el desarrollo de la solución en cilíndricas hasta el orden 1 por unicidad de la solución si se verifican las C.B. es la solución del problema.

$$\phi_I(r,\theta) = A_I + B_I \ln r + \cos\theta \left(C_I r + D_I r^{-1} \right) + \operatorname{sen}\theta \left(E_I r + F_I r^{-1} \right)$$

$$\phi_{II}(r,\theta) = A_{II} + B_{II} \ln r + \cos\theta \left(C_{II} r + D_{II} r^{-1} \right) + \operatorname{sen}\theta \left(E_{II} r + F_{II} r^{-1} \right)$$

C. B. $\phi_{I}(R,\theta)=\phi_{II}(R,\theta)$ (1)

$\phi_{I}(r\to\infty,\theta)$ no diverge (2)

$\phi_{II}(r\to 0,\theta)$ no diverge (3)

$$\left(\vec{D}_{II}-\vec{D}_{I}\right)\cdot\hat{n}_{II}\Big|_{r=R}=0 \Rightarrow \left(\varepsilon_0\vec{E}_{II}+\vec{P}-\varepsilon_0 E_{I}\right)\cdot\hat{e}_{\rho}\Big|_{r=R}=0$$

$$\left[\varepsilon_0\nabla\phi_{I}-\varepsilon_0\nabla\phi_{II}+\vec{P}\right]\cdot\hat{e}_{\rho}\Big|_{r=R}=0 \quad (4)$$

de (2) $B_{I}=0$ $C_{I}=E_{I}=0$ tomo el valor cte $A_{I}=0$

de (3) $B_{II}=0$ $D_{II}=F_{II}=0$

(1) $$\frac{D_{I}}{R}\cos\theta+\frac{F_{I}}{R}\operatorname{sen}\theta=A_{II}+C_{II}R\cos\theta+E_{II}R\operatorname{sen}\theta$$

$$\Rightarrow A_{II}=0 \qquad \underset{(i)}{D_{I}=C_{II}R^2} \qquad F_{I}=E_{II}R^2 \ (ii)$$

$$\nabla\phi_{I}\cdot\hat{e}_{\rho}=-\frac{D_{I}}{r^2}\cos\theta-\frac{F_{I}}{r^2}\operatorname{sen}\theta$$

$$\nabla\phi_{II}\cdot\hat{e}_{\rho}=C_{II}\cos\theta+E_{II}\operatorname{sen}\theta$$

(4) $$-\varepsilon_0\frac{D_{I}}{R^2}\cos\theta-\frac{\varepsilon_0 F_{I}}{R^2}\operatorname{sen}\theta-\varepsilon_0 C_{II}\cos\theta-\varepsilon_0 E_{II}\operatorname{sen}\theta+P_0\cos\theta=0$$

$$-\varepsilon_0\left(\frac{D_{I}}{R^2}+C_{II}\right)=-P_0 \quad (iii)$$

$$-\varepsilon_0\left(\frac{F_{I}}{R^2}+E_{II}\right)=0 \rightarrow R^2E_{II}=-F_{I} \overset{ii}{\Rightarrow} F_{I}=E_{II}=0$$

i y iii $$2C_{II}=\frac{P_0}{\varepsilon_0} \qquad \boxed{C_{II}=\frac{P_0}{2\varepsilon_0}} \qquad \boxed{D_{I}=\frac{P_0R^2}{2\varepsilon_0}}$$

$$\phi(r,\theta)=\begin{cases}\dfrac{P_0R^2}{2\varepsilon_0}\dfrac{1}{r}\cos\theta & r\geq R\\[2ex] \dfrac{P_0}{2\varepsilon_0}r\cos\theta & r\leq R\end{cases}$$

d) $\vec{E}_I = -\nabla\Phi_I = \frac{P_0}{2\varepsilon_0} R^2 \left(\frac{\cos\theta}{r^2}\,\hat{e}_r + \frac{\operatorname{sen}\theta}{r^2}\,\hat{e}_\theta \right)$

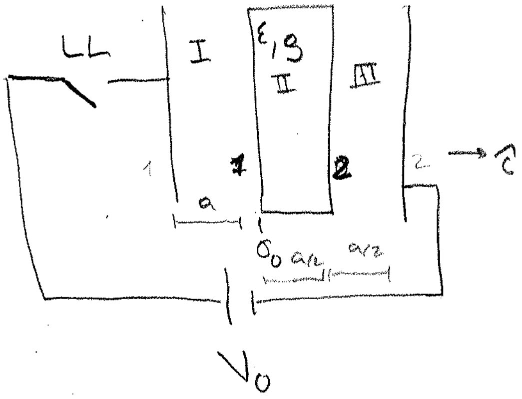
$\vec{E}_{II} = -\nabla\Phi_{II} = \frac{P_0}{2\varepsilon_0}\left(-\cos\theta\,\hat{e}_r + \operatorname{sen}\theta\,\hat{e}_\theta \right) = -\frac{P_0}{2\varepsilon_0}\,\hat{z}$

$\vec{D}_{II} = \varepsilon_0 \vec{E}_{II} + \vec{P} = \frac{P_0}{2}\left(-\cos\theta\,\hat{e}_r + \operatorname{sen}\theta\,\hat{e}_\theta + 2\cos\theta\,\hat{e}_r - 2\operatorname{sen}\theta\,\hat{e}_\theta \right)$

$= \frac{P_0}{2}\left(\cos\theta\,\hat{e}_r - \operatorname{sen}\theta\,\hat{e}_\theta \right) = \frac{P_0}{2}\,\hat{z}$

$$\boxed{\begin{aligned} \vec{E}_{II} &= -\frac{P_0}{2\varepsilon_0}\,\hat{z} \\ \vec{D}_{II} &= \frac{P_0}{2}\,\hat{z} \end{aligned}}$$

2



$$\vec{E}_I = E_I \hat{\imath}$$
$$\vec{E}_{II} = E_{II} \hat{\imath}$$
$$\vec{E}_{III} = E_{III} \hat{\imath}$$

a)

condiciones de borde

en 1) $\vec{D}_I \cdot \hat{n}_I + \vec{D}_{II} \cdot \hat{n}_{II} = \sigma_1 \to \varepsilon . E_{II} - \varepsilon_0 E_I = \sigma_1 \to \varepsilon_0 E_I = \varepsilon E_{II} - \sigma_1$ (1)

en 2) $\vec{D}_{II} \cdot \hat{n}_2 + D_{III} \cdot \hat{n}_{III} = \sigma_2 \to \varepsilon_0 E_{III} - \varepsilon E_{II} = \sigma_2 \to \varepsilon_0 E_{III} = \sigma_2 + \varepsilon E_{II}$ (2)

además: $-\sigma_1 + \sigma_2 = \sigma_0$ (3) $\forall t$

$-\dot{\sigma}_1 = g E_{II}$ (4)

por la fuente V_0.

$$V_0 = \int_0^a E(x)\, dx = E_I a + E_{II} \frac{a}{2} + E_{III} \frac{a}{2}$$

$$\frac{2 V_0 \varepsilon_0}{a} = 2 E_I \varepsilon_0 + E_{II} \varepsilon_0 + E_{III} \varepsilon_0 \quad (5)$$

(1) y (2) en (5) $\quad \dfrac{2 V_0 \varepsilon_0}{a} = 2 \varepsilon E_{II} - 2\sigma_1 + \varepsilon_0 E_{II} + \sigma_2 + \varepsilon E_{II}$

$$\frac{2 V_0 \varepsilon_0}{a} = E_{II} (3\varepsilon + \varepsilon_0) - 2\sigma_1 + \underbrace{\sigma_2}_{\sigma_0 - \sigma_1}$$

$$(3\varepsilon + \varepsilon_0) E_{II} = \frac{2 V_0 \varepsilon_0}{a} - \sigma_0 + 3\sigma_1$$

$$-\dot{\sigma}_1 (3\varepsilon + \varepsilon_0) = g \left(\frac{2 V_0 \varepsilon_0}{a} - \sigma_0 \right) + 3 g \sigma_1$$

homogenea $\quad \dot{\sigma}_1 = -\dfrac{3g}{3\varepsilon + \varepsilon_0} \sigma_1 \to \sigma_{1H}(t) = A e^{-\alpha t} \quad \alpha = \dfrac{3g}{3\varepsilon + \varepsilon_0}$

particular $\quad \sigma_p = \dfrac{1}{3} \left(\sigma_0 - \dfrac{2 V_0 \varepsilon_0}{a} \right)$

$$\sigma_1(t) = A e^{-\alpha t} + \frac{1}{3}\left(\sigma_0 - \frac{2V_0\varepsilon_0}{a}\right)$$

$$\sigma(0) = \sigma_0 = A + \frac{1}{3}\left(\sigma_0 - \frac{2V_0\varepsilon_0}{a}\right) \rightarrow A = \frac{1}{3}\left(2\sigma_0 + 2\frac{V_0\varepsilon_0}{a}\right)$$

$$\sigma_1(t) = \frac{\sigma_0}{3}\left(1+2e^{-\alpha t}\right) + \frac{2V_0\varepsilon_0}{3a}\left(e^{-\alpha t} - 1\right) \; ; \quad \alpha = \frac{3g}{3\varepsilon + \varepsilon_0}$$

$$\sigma_2(t) = \sigma_0 - \sigma_1(t) = \frac{2\sigma_0}{3}\left(1 - e^{-\alpha t}\right) - \frac{2V_0\varepsilon_0}{3a}\left(e^{-\alpha t} - 1\right)$$

$$E_{II}(t) = \frac{1}{3\varepsilon+\varepsilon_0}\left[\frac{2V_0\varepsilon_0}{a} - \sigma_0 + \sigma_0\left(1+2e^{-\alpha t}\right) + 2\frac{V_0\varepsilon_0}{a}\left(e^{-\alpha t} - 1\right)\right]$$

$$E_{II}(t) = \frac{1}{3\varepsilon+\varepsilon_0}\left[2\sigma_0 e^{-\alpha t} + \frac{2V_0\varepsilon_0}{a} e^{-\alpha t}\right]$$

b) para $t \gg 1$

$$\sigma_1(t) = \frac{\sigma_0}{3} - \frac{2}{3}\frac{V_0\varepsilon_0}{a} \qquad \sigma_2(t) = \frac{2\sigma_0}{3} + \frac{2V_0\varepsilon_0}{3a}$$

$$E_{II} = 0$$

$$\varepsilon_0 E_I = \frac{2}{3}\frac{V_0\varepsilon_0}{a} - \frac{\sigma_0}{3} \; ; \quad \varepsilon_0 E_{II} = \frac{2\sigma_0}{3} + \frac{2V_0\varepsilon_0}{3a}$$

Se cumple que: $V_0 = a E_I + \frac{a}{2} E_{II} + \frac{a}{2} E_{III}$

Problema 3

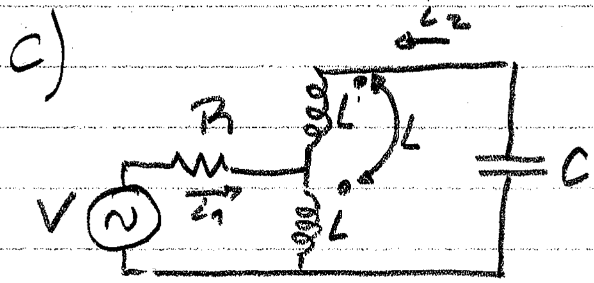
a) $L_1 = \dfrac{N_1^2}{\Re} = \dfrac{N_1^2 \mu S}{4 l}$

$L_2 = \dfrac{N_2^2}{\Re} = \dfrac{N_2^2 \mu S}{4 l}$

$M = \sqrt{L_1 L_2} = \dfrac{N_1 N_2 \mu S}{4 l}$

b) $L_1 = L_2 = M \Rightarrow N_1 = N_2$

c)



$$\left.\begin{array}{l} \Rightarrow -j\omega L(i_1+i_2) - j\omega L(i_2) - \dfrac{1}{j\omega C}(i_2) - \\ \qquad - j\omega L(i_2) - j\omega L(i_1+i_2) = 0 \\ \\ P_R = 0 \Rightarrow i_1 = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow -j\omega 4L = \frac{1}{j\omega C} \quad \Rightarrow \quad C = \frac{1}{4\omega^2 L}$$