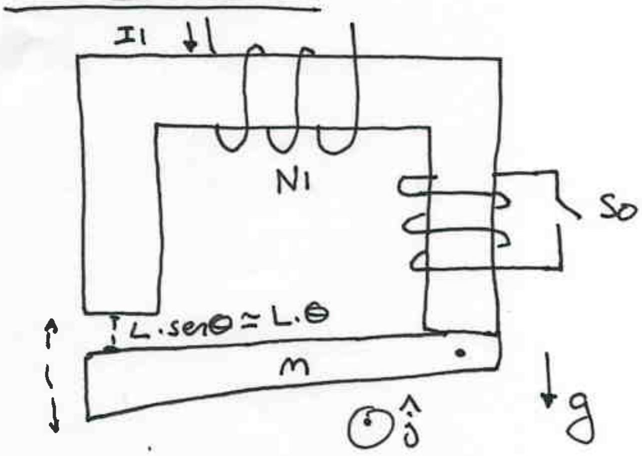
SOLUCIÓN 2° PARCIAL 2009

Problema 1:



Al cerrar la llave congelo el Flujo: $\mathcal{E} = \frac{d\phi}{dt} = \emptyset$

$\Rightarrow \underline{\underline{\phi \text{ cte}}}$

$N_1 I_1 = \mathcal{R}\phi$

$\phi = B \cdot S \Rightarrow H_i = \frac{\phi}{\mu S} \; ; \; H_e = \frac{\phi}{\mu_0 S}$

$$\Rightarrow U = \frac{\mu \cdot \phi^2 \cdot L \cdot \theta \cdot S}{2 \mu_0 S^2 \mu} + \frac{\mu_0 \cdot \phi^2 \cdot 4 L \cdot S}{\mu_0 2 \mu S^2}$$

$$= \frac{\phi^2 (L\theta\mu + 4\mu_0 L)}{2\mu_0 \mu \cdot S}$$

$$\tau = -\left.\frac{dU}{d\theta}\right|_{\phi = cte} = -\frac{L\phi^2}{2\mu_0 \cancel{S}} = \frac{-L\left(\frac{N_1 I_1}{\mathcal{R}}\right)^2}{2\mu_0 S}$$

$|\vec{\tau}_{peso}| = mg\frac{L}{2}$

$$\Rightarrow \frac{(N_1 I_1)^2 \mu^2 \cancel{S}}{32 \mu_0 L} = \frac{mgL}{2}$$

$$\Rightarrow \boxed{I_1 = \sqrt{\cancel{16}\, \mu_0 L^2 mg / \mu^2 S \cdot N_1^2}}$$

Problema 2:

Ampere : $N_1 . I_1 = 3L H_i + L\theta . H_e + L . \left(\frac{B}{\mu_0} + M \right)$

$$\Rightarrow N_1 I_1 = \frac{3L . \phi}{\mu S} + \frac{L\theta . \phi}{\mu_0 S} + L \left(\frac{\phi}{\mu_0 S} + M \right)$$

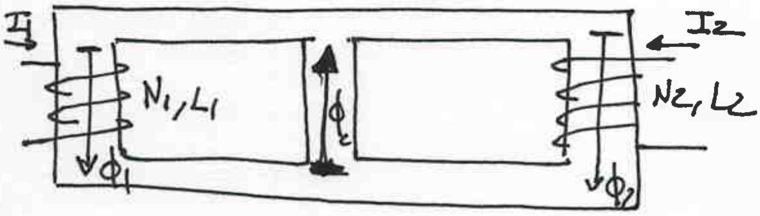
$$\Rightarrow \boxed{\phi = \frac{(N_1 I_1 - L M) . \mu_0 \mu S}{3L\mu_0 + \mu L\theta + \mu L}}$$

$$\tau = \frac{\phi^2 L}{2\mu_0 S} = \frac{(N_1 I_1 - LM)^2 (\mu_0 \mu S)^2 L}{(3L\mu_0 + \mu L) . \cancel{2} \mu_0 S} = \frac{mg L}{\cancel{2}}$$

$$(N_1 I_1 - LM)^2 = \frac{(3L\mu_0 + \mu L)^2 mg}{\mu_0 S \mu^2}$$

$$\boxed{I_1 = \frac{L |M|}{N_1} + \frac{(3\mu_0 + \mu) . L}{\mu \mu_0 N_1} . \sqrt{\frac{mg \mu_0}{S}}}$$

Problema 3:



$$\Rightarrow \begin{cases} \phi_c = \phi_1 + \phi_2 \\ N_1 I_1 = 3\mathcal{R}\phi_1 + \mathcal{R}\phi_c \\ N_2 I_2 = 3\mathcal{R}\phi_2 + \mathcal{R}\phi_c \end{cases}$$

$$N_1 I_1 + N_2 I_2 = 2\mathcal{R}\phi_c + 3\mathcal{R}(\phi_1 + \phi_2) = 5\mathcal{R}\phi_c$$

$$\Rightarrow \phi_c = \frac{N_1 I_1 + N_2 I_2}{5\mathcal{R}}$$

$$\Rightarrow 3\mathcal{R}\phi_L = \frac{4}{5} \cdot N_1 I_1 - \frac{1}{5} \cdot N_2 I_2$$

$$3\mathcal{R}\phi_R = \frac{4}{5} N_2 I_2 - \frac{1}{5} \cdot N_1 I_1$$

$$\begin{cases} \phi_L = \dfrac{4/5 \cdot N_1 I_1 - 1/5 \cdot N_2 I_2}{3\mathcal{R}} \\ \\ \phi_R = \dfrac{4/5 \cdot N_2 I_2 - 1/5 \cdot N_1 I_1}{3\mathcal{R}} \end{cases}$$

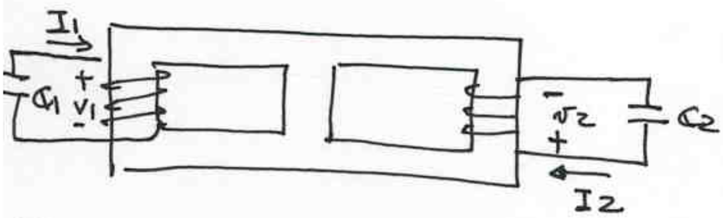
$$\boxed{L_1 = \frac{4 N_1^2 \mu S}{15 l}} \; ; \; \boxed{L_2 = \frac{4 N_2^2 \mu S}{15 l}} \; ; \; \boxed{|M_{12}| = \frac{N_1 N_2 \mu S}{15 l}}$$

Problema 4:

$N_1 = N_2$; $C_1 = C_2$.

$\hookrightarrow L_1 = L_2$

Recordar que $|M| \leq \sqrt{L_1 L_2} = L$.



Ecuaciones del transformador:

$$\begin{cases} v_1(t) = L \cdot \frac{di_1}{dt} + M \frac{di_2}{dt} = L \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} \\ v_2 = L \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2}. \end{cases}$$

Por otro lado:

$v_1(t) = -\frac{q_1(t)}{C}$

$v_2(t) = -\frac{q_2(t)}{C}$

(Por la convención de signos utilizada).

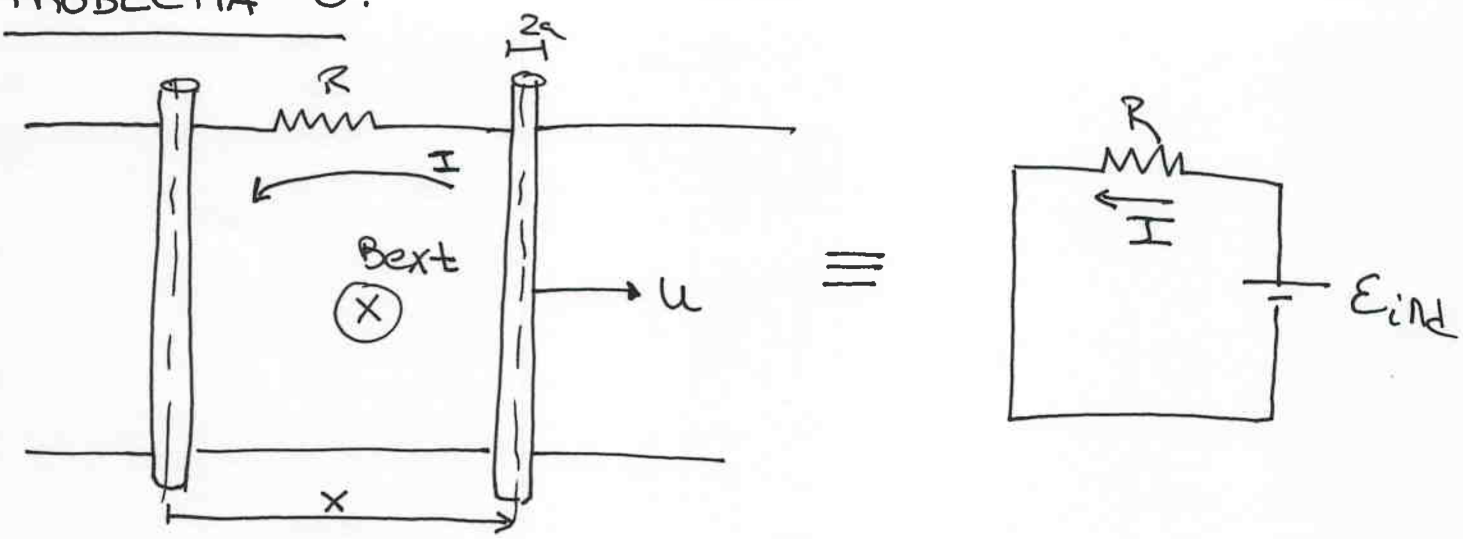
A su vez de la parte anterior sabemos que $M<0$.

$$\Rightarrow (L - |M|) \cdot \frac{d^2}{dt^2}(q_1 + q_2) = -\left(\frac{q_1 + q_2}{C}\right)$$

$$\Rightarrow q_1 = -q_2 - \underbrace{C(L - |M|)}_{\geq 0} \cdot \frac{d^2}{dt^2}(q_1 + q_2)$$

$$\Rightarrow \boxed{q_1(t) = -a \cdot \frac{d^2(q_1 + q_2)}{dt^2} - b \cdot q_2(t)}$$

PROBLEMA 5:



$$\phi = B_{ext} \cdot D \cdot (x(t) - 2a) + \phi \text{ autoinducido}$$

$$|\phi_{autoinducido}| = 2 * \cdot \int_{a}^{x-a} \frac{\mu_0 I}{2\pi \cdot x'} \cdot D \, dx' = \frac{\mu_0 D}{\pi} \cdot I \cdot Ln\left(\frac{x-a}{a}\right)$$

$$\phi = B_{ext} \cdot D\,(x(t) - 2a) - \frac{\mu_0 D}{\pi} \cdot I \cdot Ln\left(\frac{x-a}{a}\right).$$

$$\frac{d\phi}{dt} = B_{ext} \cdot D \cdot u - \frac{\mu_0 D I}{\pi} \cdot \frac{a}{x-a} \cdot \frac{u}{a} - \frac{\mu_0 D}{\pi} \cdot \underbrace{\cancel{Ln\left(\frac{x-a}{a}\right)}}_{\to 0} \cdot \underbrace{\frac{dI}{dt}}_{=0}$$

$$\mathcal{E}_{ind} = \frac{d\phi}{dt} \quad ; \quad \text{y además} \quad \mathcal{E}_{ind} = R\,I$$

$$\Rightarrow B_{ext} \cdot D \cdot u - \frac{\mu_0 D I}{\pi} \cdot \frac{u}{x-a} = R \cdot I$$

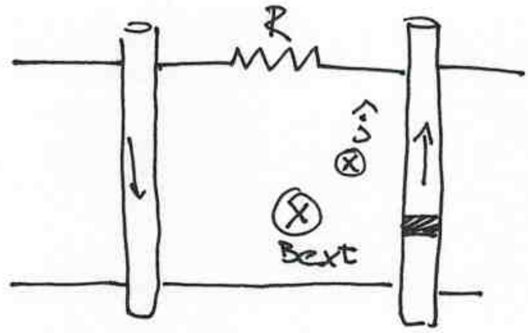
$$\Rightarrow \boxed{I(t) = \frac{B_{ext} \cdot D \cdot u}{R + \dfrac{\mu_0 D \cdot u}{\pi (x-a)}}}$$

Problema 6:

$\mu = cte.$

$P_R = R\, i^2$

$P_u = F_{ext}(t) \cdot u$



$dF = I \cdot d\vec{l} \wedge \vec{B}$

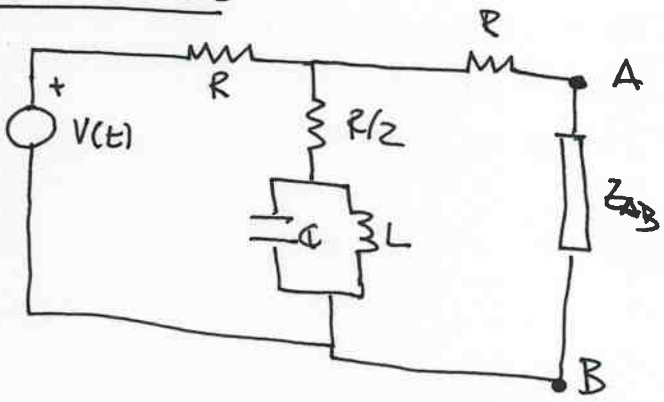
$\vec{B} = \vec{B}ext - \frac{\mu_0 I}{2\pi \cdot x} \cdot \hat{j}$

$$F_{ext}(t) = \int_0^D I \cdot dl \left(-\frac{\mu_0 I}{2\pi x} + Bext \right) = D \cdot I \cdot \left(Bext - \frac{\mu_0 I}{2\pi x} \right).$$

$$P_u = Bext \cdot D I \cdot u - \frac{D \cdot \mu_0 I^2}{2\pi x} \cdot u$$

$$\boxed{P_u - P_R = Bext \cdot D \cdot u \cdot i(t) - \left(\frac{D \cdot \mu_0 \cdot u}{2\pi \cdot x} + R \right) i^2(t)}$$

PROBLEMA 7:



$$\overline{V}(t) = V_0 e^{j\omega t}$$

$$\omega = R/L = 1/\sqrt{LC}$$

$$V_A - V_B = \frac{(1+j)\,\overline{V}(t)}{2}$$

Veamos la impedancia de la rama central:

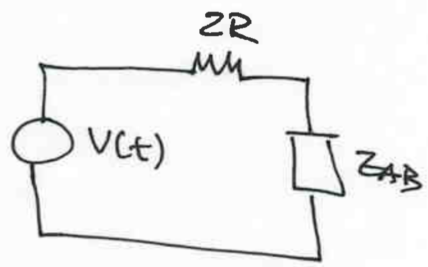
$$\overline{Z}_C = \frac{R}{2} + C /\!/ L = \frac{R}{2} + \frac{Lj\omega \cdot 1/Cj\omega}{Lj\omega + \frac{1}{Cj\omega}}$$

$$\overline{Z}_C = \frac{R}{2} + \frac{Lj\omega}{-LC\omega^2 + 1}$$

evaluando Z_C en $\omega = 1/\sqrt{LC}$

$Z_C = \infty \implies$ NO pasa corriente por la rama central.

$\Rightarrow$



$\Rightarrow$ Divisor resistivo:

$$V_{AB} = \frac{V(t) \cdot Z_{AB}}{2R + Z_{AB}} = \left(\frac{1+j}{2}\right) \cdot V(t)$$

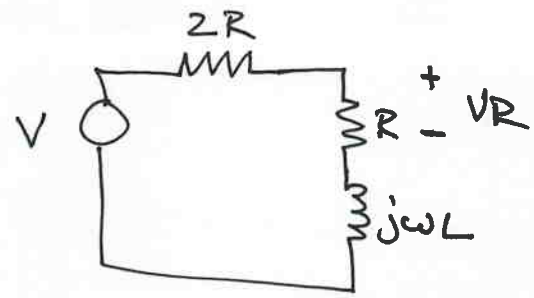
$$2 Z_{AB} = (1+j) \cdot [2R + Z_{AB}]$$

$$Z_{AB}(1-j) = 2R \cdot (1+j) \implies Z_{AB} = \frac{2R(1+j)}{(1-j)} = \frac{2R(1+j)^2}{2}$$

$$Z_{AB} = \frac{\cancel{2}R(\cancel{1} - \cancel{1} + 2j)}{\cancel{2}} \implies \boxed{Z_{AB} = 2jR}$$

Problema 8:

$$Z_{AB} = R + j\omega L$$



Divisor resistivo:

$$V_R = \frac{V \cdot R}{3R + j\omega L} = \frac{V R}{3R + j \frac{R}{\cancel{\omega}} \cdot \cancel{\omega}} = \frac{V}{3 + j}$$

$$|V_R| = \frac{V_0}{\sqrt{9 + 1}} = \frac{V_0}{\sqrt{10}}$$

$$\Rightarrow \text{POT} = \frac{1}{2} \cdot \frac{V_R^2}{R}$$

$$\boxed{\text{Pot} = \frac{V_0^2}{20R}}$$