Instituto de Física – Facultad de Ingeniería

Soluciones del segundo parcial de Electromagnetismo - 2005

Problema 1

a1) Sea:

$$V(t) = \mathrm{Re}\left(V_0 e^{j\omega t}\right)$$
$$i(t) = \mathrm{Re}\left(I_0 e^{j\omega t}\right)$$

$$I_0 = \frac{V_0}{Z_{Eq.}} \qquad \text{con:} \quad \frac{1}{Z_{Eq.}} = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$Z_R = R$$
$$Z_L = j\omega L = jX_L$$
$$Z_C = \frac{1}{j\omega C} = -jX_C$$

$$\frac{1}{Z_{Eq.}} = \frac{1}{R} + \frac{1}{jX_L} - \frac{1}{jX_C} = \frac{1}{R} + \frac{X_L - X_C}{X_L X_C} j \quad \Rightarrow \quad I_0 = V_0\left(\frac{1}{R} + \frac{X_L - X_C}{X_L X_C} j\right)$$

- $$\left|I_0\right| = V_0 \sqrt{\frac{1}{R^2} + \left(\frac{X_L - X_C}{X_L X_C}\right)^2}$$

$$\boxed{i(t) = \left|I_0\right| \cos(\omega t + \phi)}$$

- $$\tan(\phi) = \frac{\mathrm{Im}(I_0)}{\mathrm{Re}(I_0)} = R\frac{X_L - X_C}{X_L X_C} = R\frac{\omega^2 LC - 1}{\omega L}$$

a2) En la resistencia:

$$P_R(t) = V(t)i(t) = \frac{V^2(t)}{R} \qquad \Rightarrow \qquad \boxed{\overline{P} = \frac{1}{R}\overline{V}^2(t) = \frac{V_0^2}{2R}}$$

a3)

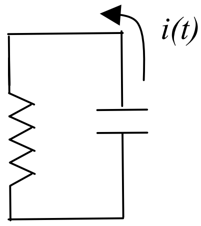
$$\phi = 0 \ \Rightarrow \ \tan(\phi) = R\frac{\omega_1^2 LC - 1}{\omega_1^2 L} = 0 \qquad \therefore$$

$$\boxed{\begin{array}{l} \omega_1 = \dfrac{1}{\sqrt{LC}} \\ f_1 = \dfrac{1}{2\pi\sqrt{LC}} \end{array}}$$

b)

Sea $t'=0$ cuando se abren L1 y L2, a partir de ese instante se tiene un circuito RC:

$Q_0 = CV_0$ (en $t'=0$ *V(t)* es máxima)



$$\left.\begin{array}{l}\dfrac{Q}{C} - Ri = 0 \\ i = -\dot{Q}\end{array}\right. \Rightarrow \dfrac{Q}{C} = -R\dot{Q}$$

$$\Rightarrow Q(t) = Q_0 e^{-t'/RC} = CV_0 e^{-t'/RC} \quad \therefore \quad \boxed{i(t) = \frac{V_0}{R} e^{-t'/RC}}$$

Problema 2

a) En régimen:

$$\begin{cases} V_0 = \mathrm{Re}\left(E_0 e^{j\omega t}\right) \\ v = \mathrm{Re}\left(v_0 e^{j\omega t}\right) \end{cases}$$

Ley de mallas: $V_0 - Ri - Blv = 0$

Ley de Newton: $m\dot{v} = Bli$ (según un versor hacia la derecha)

$$i = \frac{m}{Bl}\dot{v} \qquad \Rightarrow \qquad V_0 - \frac{mR}{Bl}\dot{v} - Blv = 0$$

Sustituyendo por las expresiones de régimen:

$$E_0 - \frac{mR}{Bl} v_0 j\omega - Blv_0 = 0$$

$$\Rightarrow \quad v_0 = \frac{BlE_0}{B^2 l^2 + mR\omega j} = \frac{BlE_0}{B^4 l^4 + m^2 R^2 \omega^2}(B^2 l^2 - mR\omega j)$$

$$\therefore \quad \boxed{\begin{cases} |v_0| = \dfrac{BlE_0}{\sqrt{B^4 l^4 + m^2 R^2 \omega^2}} \\ \tan(\phi) = -\dfrac{mR\omega}{B^2 l^2} \end{cases}}$$

b)

$$-1 = \tan(-\frac{\pi}{4}) = -\frac{mR\omega_1}{B^2 l^2} \qquad \Rightarrow \qquad \boxed{\omega_1 = \frac{B^2 l^2}{mR} = 2\pi f_1}$$

c) Con $\omega = \omega_1$ es: $|v_0| = \frac{E_0}{\sqrt{2}Bl} = v(t'=0)$

Las ecuaciones de mallas y de Newton son:

$$\begin{cases} -Ri(t) - Blv = 0 \\ m\dot{v} = Bli(t) \end{cases} \Rightarrow \quad \dot{v} = -\frac{B^2l^2}{mR}v$$

$$v(t') = \frac{E_0}{\sqrt{2}Bl}\exp(-\frac{B^2l^2}{mR}t') \quad \therefore \quad \boxed{i(t) = -\frac{Bl}{R}v = \frac{E_0}{\sqrt{2}Bl}\exp(-\frac{B^2l^2}{mR}t')}$$

d)

$$E_d = \frac{1}{2}mv^2(t'=0) \quad \Rightarrow \quad \boxed{E_d = \frac{mE_0^2}{4B^2l^2}}$$

Problema 3

a)

$$\left.\begin{array}{l} 4aH = NI \\ H < H_{sat} \Rightarrow H = \frac{B}{\mu} \end{array}\right\} \Rightarrow \Phi = \frac{B}{A} = \frac{\mu AN}{4a}I$$

$$\boxed{L = \frac{d}{dt}(N\Phi) = \frac{\mu AN^2}{4a}}$$

b)

$$\left.\begin{array}{l} \frac{Q}{C} - L\frac{dI}{dt} = 0 \\ I = -\dot{Q} \end{array}\right\} \Rightarrow \ddot{Q} = -\frac{1}{LC}Q = -\omega^2 Q \qquad \left(\omega = \frac{1}{\sqrt{LC}}\right)$$

$$Q(t) = A\sin(\omega t) + B\cos(\omega t) \qquad \begin{array}{l} A = Q(0) = CV_0 \\ B = \dot{Q}(0) = 0 \end{array}$$

$$Q(t) = CV_0\cos(\omega t) \Rightarrow I(t) = \omega CV_0\sin(\omega t)$$

Ahora:

$$H = \frac{N}{4a}I = \frac{\omega NCV_0}{4a}\sin(\omega t) \le H_{sat} \quad \Leftrightarrow \quad \frac{\omega NCV_0}{4a} \le H_{sat}$$

$$\boxed{V_0^{máx} = \frac{4aH_{sat}}{N}\sqrt{\frac{L}{C}} = H_{sat}\sqrt{\frac{4a\mu A}{C}} = B_{sat}\sqrt{\frac{4aA}{\mu C}}}$$

c) Es:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} C V_0^2 \cos^2(\omega\, t)$$

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2} L \omega^2 C^2 V_0^2 \sin^2(\omega\, t) = \frac{1}{2} C V_0^2 \sin^2(\omega\, t)$$

$\Rightarrow$

$$U_E\left(t = \frac{\pi}{4}\sqrt{LC}\right) = \frac{1}{2} C V_0^2 \cos^2\left(\omega \frac{\pi}{4}\sqrt{LC}\right) = \frac{1}{4} C V_0^2$$

$$U_B\left(t = \frac{\pi}{4}\sqrt{LC}\right) = \frac{1}{2} C V_0^2 \sin^2\left(\omega \frac{\pi}{4}\sqrt{LC}\right) = \frac{1}{4} C V_0^2$$