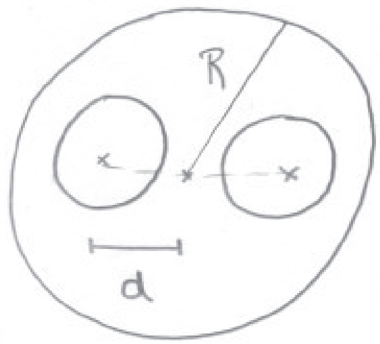
1er Parcial 26/9/09

1)



Esfera uniformemenete cargada con densidad ρ_0

Si estuviera llena:

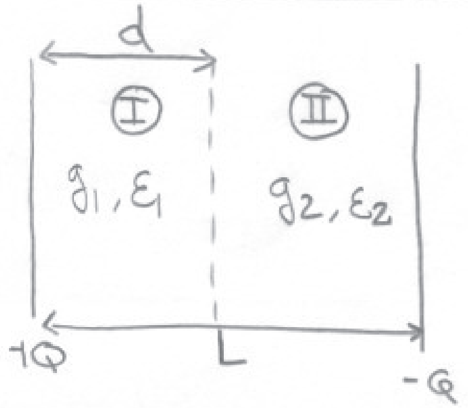
$$\begin{cases} \vec{E}_{in} = \dfrac{\rho_0 r}{3\varepsilon_0} \cdot \hat{e_r} \\ \vec{E}_{out} = \dfrac{\rho_0 R^3}{3\varepsilon_0 r^2} \cdot \hat{e_r} \end{cases}$$

Aplicando superposición:

$$\vec{E} = \frac{\rho_0 \cdot d}{3\varepsilon_0} - \frac{\rho_0 \cdot a^3}{3\varepsilon_0 (2d)^2}$$

$$\boxed{\vec{E} = \frac{\rho_0}{3\varepsilon_0}\left(d - \frac{a^3}{4d^2}\right)}$$

2)



Calculamos la energía inicial y la misma se va a disiper TODA en las resistencias, ya que es un sistema aislado

Aplicando superposición:

$$\vec{E}_1 = \frac{Q}{\varepsilon_1 A}\hat{i} \Rightarrow \vec{D}_1 = \frac{Q}{A}\hat{i}$$

$$\vec{E}_2 = \frac{Q}{\varepsilon_2 A}\hat{i} \Rightarrow \vec{D}_2 = \frac{Q}{A}\hat{i}$$

Energía inicial:

$$U = \frac{1}{2} \int_0^d \frac{Q^2}{\varepsilon_1 A^2} dV + \frac{1}{2} \int_d^L \frac{Q^2}{\varepsilon_2 A^2} dV$$

$$U = \frac{1}{2} \frac{Q^2}{\varepsilon_1 A} d + \frac{1}{2} \frac{L-d}{\varepsilon_2 A} Q^2$$

$$\boxed{U = \frac{Q^2}{2A} \left[\frac{d}{\varepsilon_1} + \frac{L-d}{\varepsilon_2} \right]}$$

3) Para que no se acumule carga libre en la interfaz $\dot{\sigma}_L = \emptyset$ (Ya que $\sigma_L(\emptyset) = 0$)

Planteando la condición de borde:

$$\vec{J}_1 - \vec{J}_2 = \dot{\sigma}$$

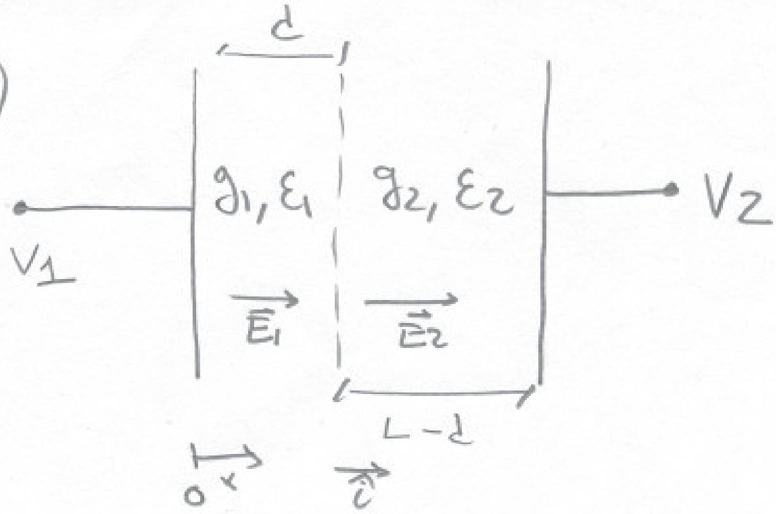
$$\Rightarrow \vec{J}_1 = \vec{J}_2 \underset{\substack{\text{medio} \\ \text{óhmico}}}{\Longrightarrow} g_1 \vec{E}_1 = g \vec{E}_2$$

$$\Rightarrow \frac{g_1 \vec{D}_1}{\varepsilon_1} = \frac{g_2 \vec{D}_2}{\varepsilon_2}$$

Pero si no se acumula carga libre: $\vec{D}_1 = \vec{D}_2$

$$\Rightarrow \boxed{\frac{\varepsilon_2}{\varepsilon_1} = \frac{g_2}{g_1}}$$

4)



$$\Delta\phi = -\int_0^L \vec{E}\, d\vec{r}$$

$\Rightarrow V_1 - V_2 = E_2 (L-d) + E_1 d$

En estado estacionario:

$\vec{J_1} - \vec{J_2} = -\dot{\sigma} = 0$ (en la interfaz)

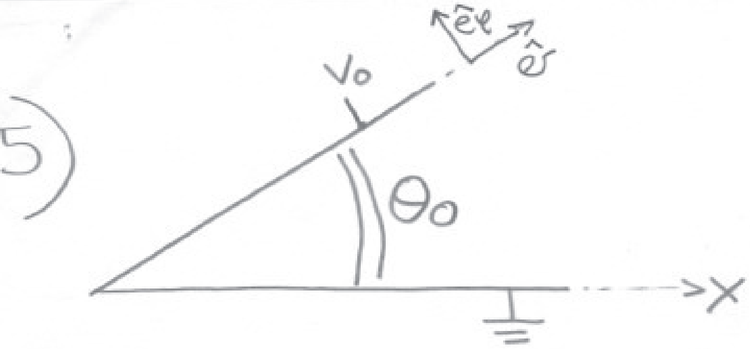
$\Rightarrow \vec{J_1} = \vec{J_2} = \vec{J}$

$$\Rightarrow V_1 - V_2 = \frac{J(L-d)}{g_2} + \frac{Jd}{g_1}$$

$$\Rightarrow J\left(\frac{g_1}{g_2}(L-d) + d\right) = g_1 (V_1 - V_2)$$

$$\Rightarrow \boxed{\vec{J} = \frac{(V_1 - V_2)\, g_1\, \hat{i}}{d + \frac{g_1}{g_2}(L-d)}}$$

5)



. Simetría cilíndrica

Aplicamos Laplace entre las placas:

$$\nabla^2\phi = 0$$

Campos según $-\hat{e}_\varphi$: $\phi = \phi(\varphi)$

$$\nabla^2\phi = \frac{1}{r^2} \cdot \frac{\partial^2\phi}{\partial\varphi^2} = 0 \Rightarrow \phi(\varphi) = A\varphi + B$$

Condiciones de borde:

$$\phi(0) - \phi(\theta_0) = -V_0 \Rightarrow A = \frac{V_0}{\theta_0}$$

$$\Rightarrow \vec{E} = \frac{-A}{r} \cdot \hat{e}_\varphi \Rightarrow \vec{E} = \frac{-V_0}{\theta_0 r} \cdot \hat{e}_\varphi$$

Gauss sobre el plano a ángulo 0:

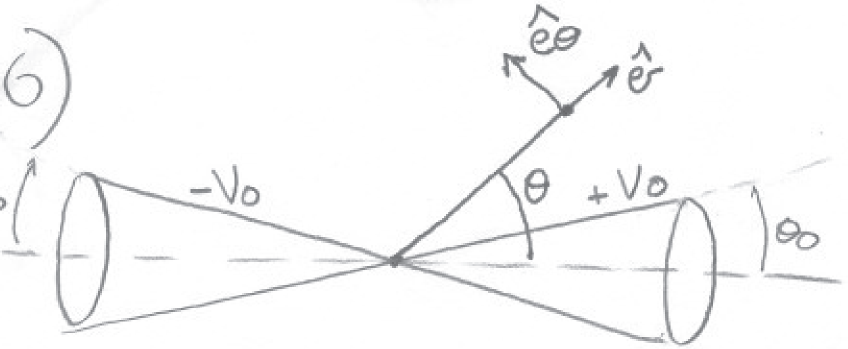
$$\oint_S \vec{E} \cdot \vec{n}\, dA = \frac{-V_0}{\theta_0} \cdot \frac{S}{r} = \frac{\sigma_0 \cdot S}{\varepsilon_0}$$

$$\Rightarrow \sigma_0 = \frac{-V_0 \varepsilon_0}{\theta_0 r}$$

Pero sobre dicho plano $r = x$

$$\Rightarrow \boxed{\sigma_0 = \frac{-V_0 \varepsilon_0}{\theta_0 x}}$$

6)



- Problema con simetría esférica.
- Campos según $\hat{e}_\theta$

$\phi = \phi(\theta)$

$$\nabla^2\phi = 0 \underset{\text{Laplace en esféricas}}{\Longrightarrow} \nabla^2\phi = \frac{1}{r^2 \operatorname{sen}\theta} \cdot \frac{\partial}{\partial\theta}\left[\operatorname{sen}\theta \cdot \frac{\partial\phi}{\partial\theta}\right] = 0$$

$$\Rightarrow \frac{d\phi}{d\theta} = \frac{A}{\operatorname{sen}\theta} \quad ; \quad \phi(\theta) = \int d\phi = \int \frac{A\,d\theta}{\operatorname{sen}\theta} \underset{\operatorname{sen}\theta = 2\operatorname{sen}\frac{\theta}{2}\cos\frac{\theta}{2}}{=}$$

$$= \int \frac{A \cos(\theta/2)\, d\theta}{2 \operatorname{sen}(\theta/2) \cos(\theta/2)} = \left[\begin{array}{l} u = tg\,\theta/2 \\ du = \frac{1}{2\cos^2\theta/2}\, d\theta \end{array}\right] = A \int \frac{du}{u} = A \ln\left(tg\frac{\theta}{2}\right) + B$$

$$\frac{\cos(\theta/2)}{\operatorname{sen}(\theta/2)} = \frac{1}{tg\,\theta/2}$$

Impongo CB:
$$\begin{cases} \phi(\theta_0) = V_0 = A \cdot \ln\left(tg\frac{\theta_0}{2}\right) + B \\ \phi(\pi - \theta_0) = -V_0 = A \ln\left(tg\frac{\pi - \theta_0}{2}\right) + B \end{cases}$$

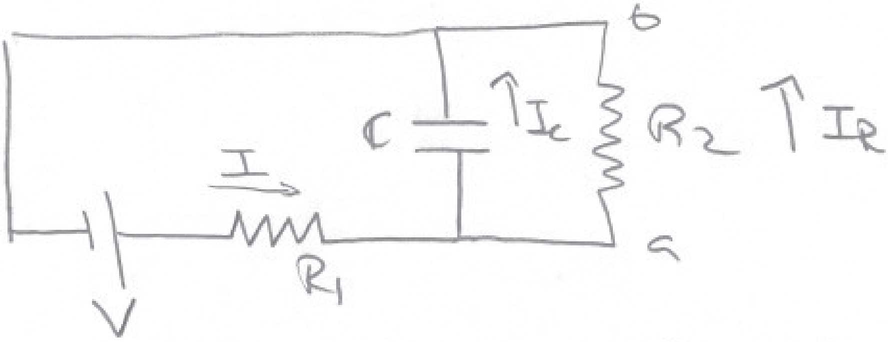
Prop:
$$\left[tg\left(\frac{\theta_0}{2}\right) = \frac{1}{tg\left(\frac{\pi-\theta_0}{2}\right)} \Rightarrow \ln\left(tg\frac{\pi-\theta_0}{2}\right) = -\ln\left(tg\frac{\theta_0}{2}\right) \right]$$

$\Rightarrow$ Sumando $\Rightarrow 2B = 0 \Rightarrow B = 0$

$\Rightarrow$ Restando $\Rightarrow 2V_0 = 2A \ln(tg\,\theta_0/2) \Rightarrow A = \dfrac{V_0}{\ln\left(tg\frac{\theta_0}{2}\right)}$

$$\Rightarrow \boxed{\phi(\theta) = V_0 \ln(tg\,\theta/2) / \ln(tg\,\theta_0/2)}$$

7)



Tenemos que imponer que la diferencia de Potencial en el capacitor llegue a V/2

$$I = I_C + I_R$$

$$V = R_1 I + V_c(t)$$

$$\begin{cases} C \cdot V(t) = Q(t) \\ V(t) = R_2 I_R \\ \text{(Paralelo).} \end{cases}$$

$$V = R_1 (I_C + I_R) + V_c(t)$$

$$V = R_1 I_C + R_1 \frac{V_c(t)}{R_2} + V_c(t)$$

$$\begin{cases} V_c(t) = \frac{Q(t)}{C} \\ \dot{V}_c(t) = \frac{I_c(t)}{C} \end{cases}$$

$$\boxed{V = R_1 C \dot{V}_c(t) + \left(\frac{R_1 + R_2}{R_2}\right) V_c(t)}$$

$\underline{\underline{V_{CH}(t)}}$: $\dot{V}_c = -\left(\frac{R_1 + R_2}{R_1 R_2 C}\right) V_c \Rightarrow V_{CH}(t) = A e^{-\left(\frac{R_1+R_2}{R_1 R_2 C}\right)t}$

$\underline{\underline{V_{CP}}}$: $V_{CP} = \frac{R_2}{R_1 + R_2} \cdot V$ $\qquad V(t) = V_{CH}(t) + V_{CP}$

Aplicando condiciones iniciales $(V_c(0) = 0)$

$$A + \frac{R_2}{R_1 + R_2} \cdot V = 0 \quad \Rightarrow \quad A = \frac{-R_2}{R_1 + R_2} \cdot V$$

$$\Rightarrow V_c(t) = \frac{R_2}{R_1+R_2} \cdot V \left(1 - e^{-\left(\frac{R_1+R_2}{R_1 R_2 C}\right)t}\right)$$

* Para que $V_c(t) = \frac{V}{2} \Rightarrow \frac{R_2}{R_1+R_2} > \frac{1}{2}$

$$2R_2 > R_1 + R_2$$

$$\Rightarrow \boxed{R_2 > R_1}$$

* Tiempo que demore en quemarse:

$$\frac{\cancel{V}}{2} = \frac{R_2}{R_1+R_2} \cdot \cancel{V} \left(1 - e^{-\frac{R_1+R_2}{R_1 R_2 C} t'}\right)$$

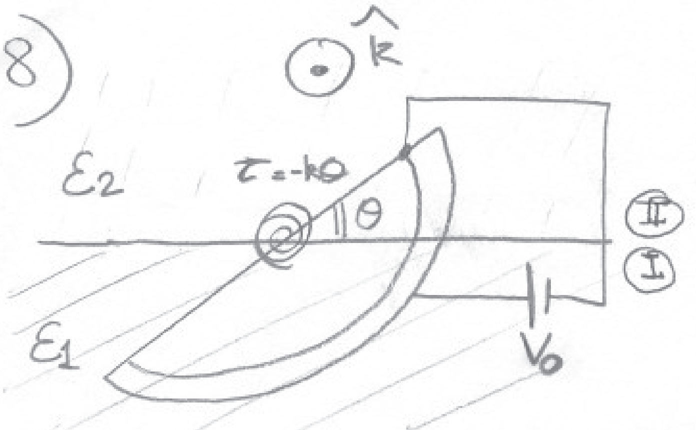
$$\frac{R_1+R_2}{2R_2} = 1 - e^{-\frac{R_1+R_2}{R_1 R_2 C} t'}$$

$$e^{-(\ )t'} = 1 - \frac{R_1+R_2}{2R_2}$$; Tomo logaritmos.

$$\frac{-R_1+R_2}{R_1 R_2 C} \cdot t' = \ln\left(\frac{R_2-R_1}{2R_2}\right)$$

$$\boxed{t' = \frac{C \cdot R_1 R_2}{R_1+R_2} \cdot \ln\left(\frac{2R_2}{R_2-R_1}\right)}$$

8)



$\varepsilon_2 > \varepsilon_1$

* Campos radiales:

$\Rightarrow \vec{E}_I = \vec{E}_2 = \vec{E}$

(imponiendo CB en la interfaz)

Dieléctricos lineales $\Rightarrow$ vale Laplace:

$$\nabla^2\phi = \frac{1}{r}\cdot\frac{d}{dr}\left[r\frac{d\phi}{dr}\right] = \emptyset \Rightarrow \frac{d\phi}{dr} = \frac{A}{r}$$

$$\Rightarrow \phi = A\ln(r) + B$$

$$\phi(R_2) - \phi(R_1) = A.\ln\left(\frac{R_2}{R_1}\right) = V_0$$

$$A = V_0/\ln(R_2/R_1) \Rightarrow \vec{E} = \frac{-V_0}{r\ln(R_2/R_1)}\cdot\hat{e_r}$$

<u>Energía:</u>

$$u = \begin{cases} \dfrac{\varepsilon_1 E^2}{2} = \dfrac{\varepsilon_1 V_0^2}{\ln^2(R_2/R_1)\, r^2} & (\text{zona I}) \\[2ex] \dfrac{\varepsilon_2 E^2}{2} = \dfrac{\varepsilon_2 V_0^2}{\ln^2(R_2/R_1)\, r^2} & (\text{zona II}) \end{cases}$$

$$U = \int u\,dV = \frac{\varepsilon_2 V_0^2}{\ln^2(R_2/R_1)}\int_{R_1}^{R_2}\frac{\theta\, \cancel{r} L}{r^{\cancel{2}}}dr + \frac{\varepsilon_1 V_0^2}{\ln^2\left(\frac{R_2}{R_1}\right)}\cdot\int_{R_1}^{R_2}\frac{(\pi-\theta)\cancel{r}L}{r^{\cancel{2}}}dr$$

$$U = \frac{-V_0^2 L(\varepsilon_1 - \varepsilon_2)\theta}{2\ln(R_2/R_1)} + \frac{\varepsilon_1 V_0^2 \pi L}{2\ln(R_2/R_1)}$$

$\Rightarrow$ A tensión constante:

$$\vec{\tau} = \left(\frac{dU}{d\theta}\right)\hat{k}\Bigg|_{V_0} = \frac{V_0^2 L (\varepsilon_2 - \varepsilon_1)}{2 \ln (R_2/R_1)}\hat{k}$$

2da cardinal en θ_0:

$$\frac{V_0^2 L (\varepsilon_2 - \varepsilon_1)}{2 \ln (R_2/R_1)} - k\,\theta_0 = \emptyset$$

$$\boxed{\theta_0 = \frac{L V_0^2 (\varepsilon_2 - \varepsilon_1)}{2 k \ln (R_2/R_1)}}$$