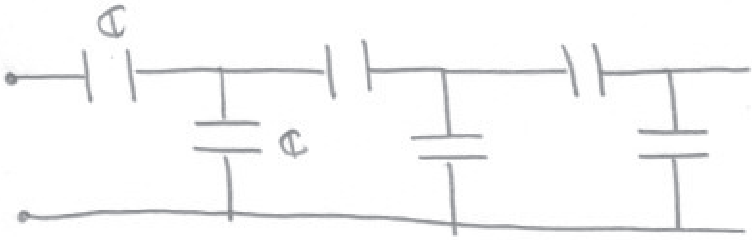
1er PARCIAL 2003

1) 

$$\begin{cases} \dfrac{1}{C_{eq}} = \dfrac{1}{C} + \dfrac{1}{C_{2eq}} \\ C_{2eq} = C + C_{eq} \quad \text{(impongo convergencia)} \end{cases}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C + C_{eq}}$$

$$\Rightarrow C^2 + C \cdot C_{eq} = 2C \cdot C_{eq} + C_{eq}^2$$

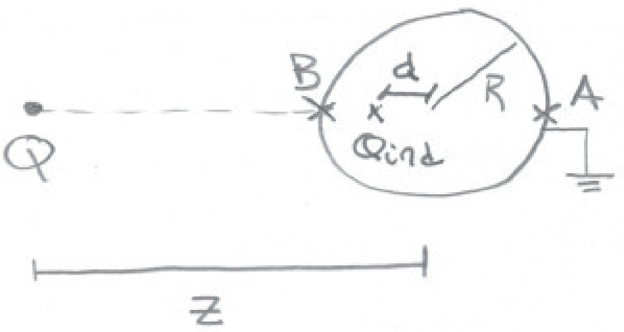
$$C_{eq}^2 + C\,C_{eq} - C^2 = 0$$

$$C_{eq} = \frac{-C \pm \sqrt{C^2 + 4C^2}}{2} = \frac{-C + \sqrt{5}\,C}{2}$$

solución positiva

$$\boxed{C_{eq} = (\sqrt{5} - 1)\,C/2}$$

2)



La "carga inducida" es aquella que se obtiene de aplicar el método de las imágenes de forma tal de sustituir el conductor por una carga imagen q_{ind} que anula el potencial sobre la sup. de la esfera.

El potencial en A deberá ser nulo:

$$\frac{Q}{z+R} + \frac{Q_{ind}}{d+R} = \emptyset$$

El potencial en B deberá ser nulo:

$$\frac{Q}{z-R} + \frac{Q_{ind}}{R-d} = \emptyset$$

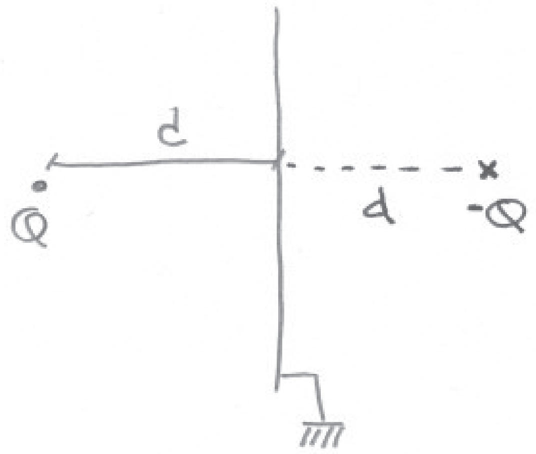
$$\Rightarrow \frac{Q}{z-R} = \frac{-Q_{ind}}{R-d} \Rightarrow Q(R-d) = -Q_{ind}(z-R)$$

$$d = R + \frac{Q_{ind}}{Q}(z-R)$$

$\Rightarrow$ Por otro lado: $d = -R - \frac{Q_{ind}}{Q}(z+R)$

$$\Rightarrow -R - \frac{Q_{ind}}{Q}(z+\cancel{R}) = R + \frac{Q_{ind}}{Q}(z-\cancel{R}) \Rightarrow \boxed{Q_{ind} = -\frac{R}{z} \cdot Q}$$

3)



Aplicando el método de las imágenes:

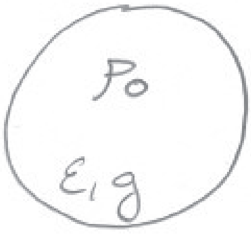
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-(Q)}{r^2}\,\hat{e_r}$$

⇒ Fuerza sobre Q:

$$|\vec{F}| = |Q\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{(2d)^2}$$

$$\boxed{|\vec{F}| = \frac{1}{16\pi\varepsilon_0} \cdot \frac{Q^2}{d^2}}$$

1)



Inicialmente hay un campo adentro de la esfera y otro afuera, cuando llega al régimen toda la carga se aloja en la superficie, por lo que el campo en el interior es nulo. Por lo tanto la energía disipada es la que había dentro de la esfera al comienzo.

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon}\cdot\frac{\rho_0 \frac{4}{3}\pi r^3}{r^2} = \frac{\rho_0 r}{3\varepsilon}\hat{e}_r$$

$$\vec{D} = \frac{\rho_0 r}{3}\hat{e}_r$$

$$u = \frac{1}{2\varepsilon}\left(\frac{\rho_0 r}{3}\right)^2 \Rightarrow U = \int_V u\,dV$$

$$U = \frac{4\pi}{2\varepsilon}\cdot\frac{\rho_0^2}{9}\cdot\int_0^R r^4\,dr = \frac{2\pi}{\varepsilon}\cdot\frac{\rho_0^2}{9}\cdot\frac{R^5}{5}$$

$$\boxed{U = \frac{4\pi\rho_0^2 R^5}{90\varepsilon}}$$

5) $$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\nabla \cdot g\vec{E} + \frac{\partial \rho}{\partial t} = 0$$

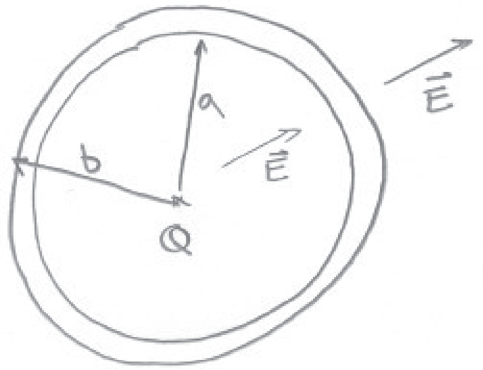
$$\frac{g}{\varepsilon}\rho + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \rho(t) = \rho_0 \, e^{-\frac{g}{\varepsilon}t}$$

$$\begin{cases} \nabla \cdot \vec{J} = +\rho_0 \cdot \dfrac{g}{\varepsilon} \cdot e^{-\frac{g}{\varepsilon}t} \\ \nabla \cdot \vec{J} = \dfrac{1}{r^2} \cdot \dfrac{\partial (r^2 J)}{\partial r} \end{cases}$$ (divergencia en esféricas)

$$\cancel{r^2} J = \frac{r^3}{3} \cdot \rho_0 \frac{g}{\varepsilon} \cdot e^{-\frac{g}{\varepsilon}t}$$

$$\boxed{\vec{J}(r,t) = \frac{g\rho_0 r}{3\varepsilon} \cdot e^{-\frac{g}{\varepsilon}t} \cdot \hat{e}_r}$$

6)



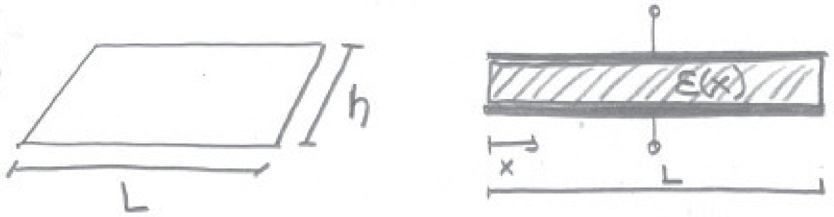
La diferencia de la energía electrostática de este sistema con la de una carga Q en el vacío es la energía contenida en el espesor del cascarón

$$u = \frac{1}{2}\vec{D}\vec{E} = \frac{1}{2}\varepsilon_0 E^2$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{\hat{e_r}}{r^2} \Rightarrow \Delta U = \frac{1}{2}4\pi\varepsilon_0\int_a^b \frac{Q^2}{(4\pi\varepsilon_0)^2}\cdot\frac{dr}{r^2} =$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\cdot\frac{-2}{r}\Big|_a^b = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

7)



$\varepsilon(x) = \varepsilon_0 + \varepsilon_1 x$

$$\vec{E} = \frac{\sigma}{\varepsilon(x)}\hat{j} \quad ; \quad \Delta\phi = \int_0^d \frac{\sigma}{\varepsilon(x)}\cdot dr = \frac{d\cdot\sigma}{\varepsilon(x)}$$

$$Q = \int \sigma\cdot dA = \frac{\Delta\phi}{d}\cdot h\int_0^L(\varepsilon_0+\varepsilon_1 x)\,dx =$$

$$Q = \frac{\Delta\phi h}{d}\left(\varepsilon_0 + \frac{\varepsilon_1}{2}L\right)L \quad ; \quad Q = C\,\Delta\phi$$

$$\Rightarrow \boxed{C = \frac{(\varepsilon_0 + \varepsilon_1 L/2)hL}{d}}$$

8)

$$\nabla^2\phi = 0 \quad \text{(simetria cilíndrica)}$$

$$\frac{d\phi}{dr} = \frac{A}{r} \Rightarrow \phi = A\ln(r) + B$$

$$\phi(R_2) - \phi(R_1) = V \Rightarrow A = \frac{-V}{\ln(R_2/R_1)}$$

$$\Rightarrow \vec{E} = \frac{V}{\ln(R_2/R_1)} \cdot \frac{\hat{e}_r}{r} \Rightarrow \begin{cases} \vec{D}_1 = \dfrac{\varepsilon_0 V}{\ln(R_2/R_1)} \cdot \dfrac{\hat{e}_r}{r} \\ \vec{D}_2 = \dfrac{\varepsilon V}{\ln(R_2/R_1)} \cdot \dfrac{\hat{e}_r}{r} \end{cases}$$

$$\Rightarrow U = \frac{1}{2} \cdot \frac{2\pi\varepsilon V^2}{\ln^2(R_2/R_1)} \cdot h \int_{R_1}^{R_2} \frac{dr}{r} + \frac{1}{2} \cdot \frac{2\pi\varepsilon_0 V^2}{\ln^2(R_2/R_1)} \cdot (L-h) \int_{R_1}^{R_2} \frac{dr}{r}$$

$$U = \frac{\pi V^2}{\ln(R_2/R_1)} \left[\varepsilon h + \varepsilon_0 (L-h)\right]$$

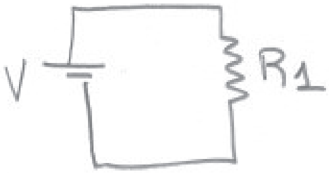
$$\frac{dU}{dh} = \frac{\pi V^2}{\ln(R_2/R_1)} \overbrace{\left[\varepsilon - \varepsilon_0\right]}^{\chi} = |\vec{F}_{elect}|$$

$$|m\vec{g}| = \rho g (R_2^2 - R_1^2) \pi h$$

$$\frac{\cancel{\pi} V^2 \chi}{\ln(R_2/R_1)} = \rho g (R_2^2 - R_1^2) \cancel{\pi} h$$

$$\boxed{\chi = \rho g (R_1^2 - R_2^2) \ln(R_1/R_2) / V^2}$$

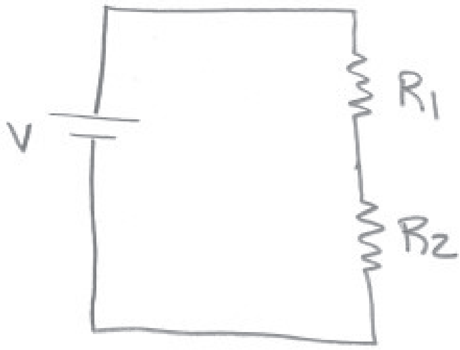
9) Modelo la lamparilla como una resistencia:



$$\Rightarrow P = VI \underset{\uparrow \text{ohm}}{=} \frac{V^2}{R_1} = W_1$$

$$\Rightarrow W_2 = \frac{V^2}{R_2}$$

$$\Rightarrow \quad R_1 = \frac{V^2}{W_1} \qquad R_2 = \frac{V^2}{W_2}$$



$$W_3 = \frac{V^2}{R_1 + R_2} = \frac{V^2}{\frac{V^2}{W_1} + \frac{V^2}{W_2}}$$

$$W_3 = \frac{1}{\frac{1}{W_1} + \frac{1}{W_2}} \Rightarrow \boxed{W_3 = \frac{W_1 W_2}{W_1 + W_2}}$$

10)

$$I = n I_i$$

$$\varepsilon = R_i \cdot I_i + R I = (R_i + nR) I_i$$

$$I_i = \frac{\varepsilon}{R_i + nR} \Rightarrow \boxed{I = \frac{n\varepsilon}{R_i + nR}}$$