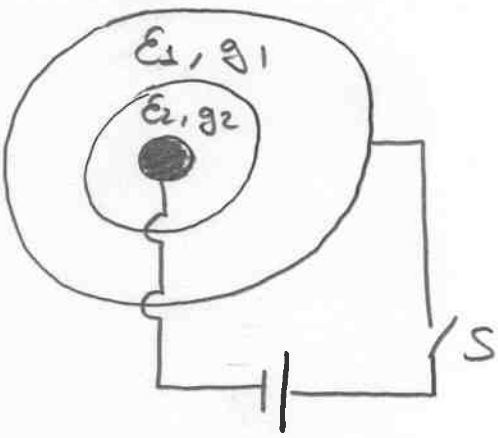
EXAMEN DICIEMBRE 2009

Problema 1:



$$\begin{cases} \varepsilon_2 = 2\varepsilon_1 \\ b = 2a \\ c = 4a \end{cases}$$

a) $\vec{E}_1 = \dfrac{K_1}{r^2} \cdot \hat{e_r}$; $\vec{E}_2 = \dfrac{K_2}{r^2} \cdot \hat{e_r}$

Por otro lado: $- \displaystyle\int_a^c \vec{E} \cdot d\vec{l} = V_1$

$$+K_2 \left(\frac{1}{b} - \frac{1}{a} \right) + K_1 \left(\frac{1}{c} - \frac{1}{b} \right) = V_1$$

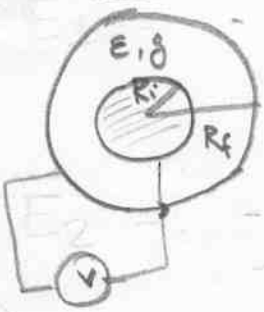
$$\frac{K_2}{2a} + \frac{K_1}{4a} = -V_1$$

Además: $\left(\vec{J}_1 - \vec{J}_2 \right)\Big|_{r=b} - \dot{\sigma} = \varnothing$

$$g_1 \vec{E}_1 = g_2 \vec{E}_2$$

$$\frac{g_1 K_1}{b^2} = \frac{g_2 K_2}{b^2} \Rightarrow K_2 = \frac{g_1}{g_2} K_1$$

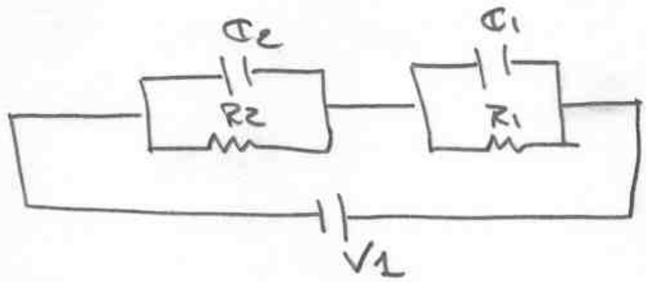
$\Rightarrow$ También es válido:



Capacitor esférico: $C = 4\pi\varepsilon \dfrac{R_i R_f}{R_f - R_i}$

Resistencia: $R = \dfrac{1}{4\pi\varepsilon g} \dfrac{R_f - R_i}{R_f R_i}$

Sistema equivalente:



Luego:

$$\begin{cases} \vec{D}_1 = \varepsilon_1 \vec{E}_1 \\ \vec{D}_2 = \varepsilon_2 \vec{E}_2 \end{cases}$$

$$\begin{cases} \vec{J}_1 = g_1 \vec{E}_1 \\ \vec{J}_2 = g_2 \vec{E}_2 \end{cases}$$

FUERA de LA ESFERA de Radio c:

$\vec{E}_3 = \emptyset$; $\vec{D}_3 = \emptyset$; $\vec{J}_3 = \emptyset$.

En el interior de la esf. de radio a los campos también son nulos.

$$\frac{K_1}{4a} + \frac{g_1}{g_2} \cdot \frac{k_1}{2a} = -V_1$$

$$K_1 \left(\frac{1}{2} + \frac{g_1}{g_2} \right) = -V_1 \cdot 2a$$

$$k_1 \left(\frac{g_2 + 2g_1}{2g_2} \right) = -V_1 \cdot 2a$$

$$\begin{cases} k_1 = \dfrac{-4a\,g_2\,V_1}{2g_1 + g_2} \\[2ex] K_2 = \dfrac{-4a\,g_1\,V_1}{2g_1 + g_2} \end{cases}$$

$$\Rightarrow \begin{cases} \vec{E}_1 = \dfrac{-4a\,g_2\,V_1}{(2g_1 + g_2)\,r^2} \cdot \hat{e}_r \\[2ex] \vec{E}_2 = \dfrac{-4a\,g_1\,V_1}{(2g_1 + g_2)\,r^2} \cdot \hat{e}_r \end{cases}$$

b) Previamente debemos hallar $\vec{P}$

$$\vec{D}_2 = \varepsilon_0 \vec{E}_2 + \vec{P}_2$$

$$\varepsilon_2 \cdot \vec{E}_2 = \varepsilon_0 \vec{E}_2 + \vec{P}_2$$

$$\begin{cases} \vec{P}_2 = (\varepsilon_2 - \varepsilon_0)\vec{E}_2 \\ \vec{P}_1 = (\varepsilon_1 - \varepsilon_0)\vec{E}_1 \end{cases}$$

Sobre la superficie de $R=a$:

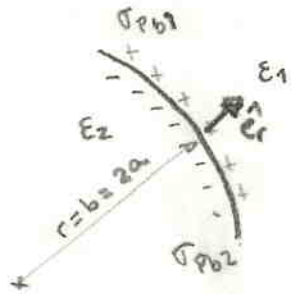
$$\sigma_{P_a} = \vec{P}_2 \cdot (-\hat{e}_r)\Big|_{r=a} = +\frac{(\varepsilon_2 - \varepsilon_0) \cdot 4 g_2 V_1}{(2g_1 + g_2)a}$$

$(-\hat{e}_r)$: normal saliente al dieléctrico.

Sobre la superficie de $R=b=2a$:

$$\sigma_{P_{b2}} = \vec{P}_2 \cdot \hat{e}_r\Big|_{r=b^-} = -\frac{(\varepsilon_2 - \varepsilon_0) \, g_2 V_1}{(2g_1 + g_2)a}$$

$$\sigma_{P_{b1}} = \vec{P}_1 \cdot (-\hat{e}_r)\Big|_{r=b^+} = \frac{(\varepsilon_1 - \varepsilon_0) \cdot g_1 V_1}{(2g_1 + g_2)a}$$



Sobre la superficie de $R=c=4a$:

$$\sigma_{P_c} = \vec{P}_1 \cdot (\hat{e}_r)\Big|_{r=c} = -\frac{(\varepsilon_1 - \varepsilon_0) \, g_1 V_1}{4(2g_1 + g_2)a}$$

c) Se abre la llave S:

Gauss: $\vec{E}_2(\vec{r},t) = \dfrac{Q_a(t)}{4\pi\varepsilon_2 r^2}\,\hat{e}_r$ $\left(\begin{array}{c}\text{al abrir la llave} \\ \rho(t) = \emptyset \quad \forall t \text{ pues} \\ \rho_0 = \emptyset\end{array}\right)$

Por otro lado: $\vec{J}_2 \cdot \hat{e}_r \Big|_{r=a} = -\dot{\sigma}_a(t)$

$$\vec{J}_2 = g_2 \vec{E}_2$$

$$\Rightarrow \frac{g_2 Q_a(t)}{4\pi \varepsilon_2 a^2} = \frac{\dot{Q}_a(t)}{4\pi a^2}$$

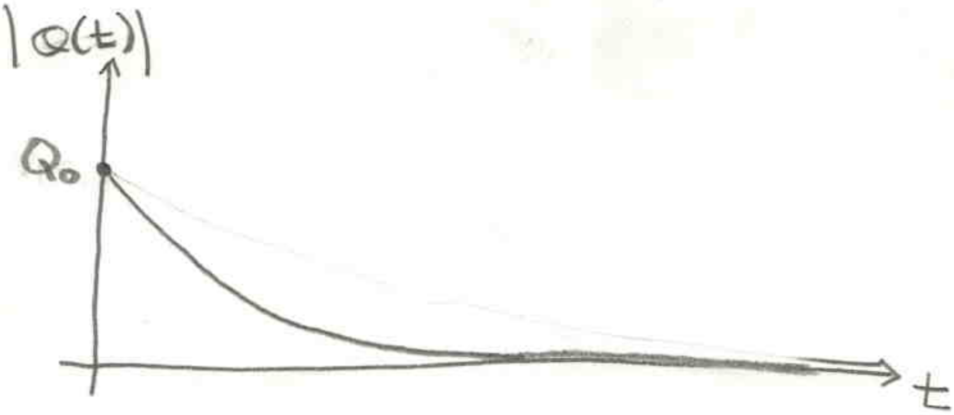
$$\dot{Q}_a(t) = -\frac{g_2}{\varepsilon_2} \cdot Q_a(t)$$

$$Q_a(t) = Q_0 \cdot e^{-\frac{g_2}{\varepsilon_2} \cdot t}$$

DONDE

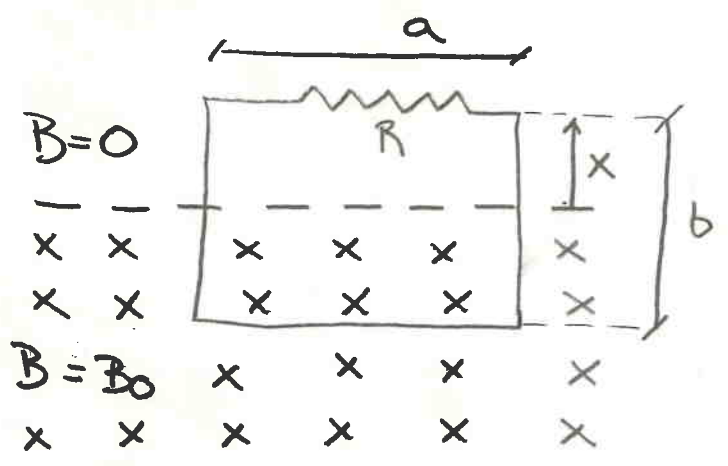
$$Q_0 = 4\pi a^2 \cdot D_2(a) = -\left[\frac{16\pi\varepsilon_2 g_2}{(2g_1+g_2)a}\right] V_1$$

de la parte anterior

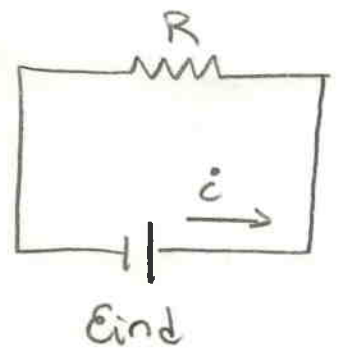


Nota: $Q(t) < \emptyset$

Problema 2:



a)



$$\phi = Bo \cdot a(b - x)$$

$$\varepsilon ind = |Bo \cdot a \cdot \dot{x}|$$

$$\varepsilon ind = R\, i(t)$$

$$Bo \cdot a \cdot v = R \cdot i(t)$$

Newton:
$$Mg - F_{mag} = M\dot{v}$$

$$Mg - Bo \cdot a \cdot i = M\dot{v}$$

$$Mg - \frac{Bo^2 \cdot a^2 \cdot v}{R} = M\dot{v}$$

Solución particular: $vp = \dfrac{Mg \cdot R}{Bo^2 \cdot a^2}$

Homogenea: $V_{HOM} = K e^{-\frac{Bo^2 a^2}{MR} \cdot t}$

$$v(t) = \frac{MgR}{B_0^2 . a^2} + K e^{-\frac{B_0^2 . a^2}{MR} . t}$$

$$v(t=\emptyset) = \emptyset \Rightarrow K = \frac{-MgR}{B_0^2 . a^2}$$

$$\boxed{v(t) = \frac{MgR}{B_0^2 a^2} \left(1 - e^{-\frac{B_0^2 a^2}{MR} . t} \right)}$$

$$i(t) = \frac{B_0 . a}{R} . v(t)$$

$$\boxed{i(t) = \frac{Mg}{B_0 . a} \left(1 - e^{-\frac{B_0^2 . a^2}{MR} . t} \right)}$$

b) $B_0 . a . \dot{x} = L . \frac{di}{dt}$

$$Mg - B_0 . a . i = M\dot{v}$$

$$B_0 . a . i = Mg - M\dot{v}$$

$$B_0 . a . \frac{di}{dt} = -M\ddot{v}$$

$$\Rightarrow B_0 . a . v = \frac{-LM}{B_0 . a} . \ddot{v}$$

$$\ddot{V} + \frac{Bo^2 . a^2}{L M} . V = \emptyset \qquad \omega = \frac{Bo . a}{\sqrt{L . M}}$$

$$v(t) = Vo . sen(\omega t)$$

$$\left.\begin{array}{l} a(t) = Vo . \omega . \cos(\omega t) \\ a(t=\emptyset) = g \end{array}\right] \Rightarrow \quad Vo = \frac{g}{\omega}$$

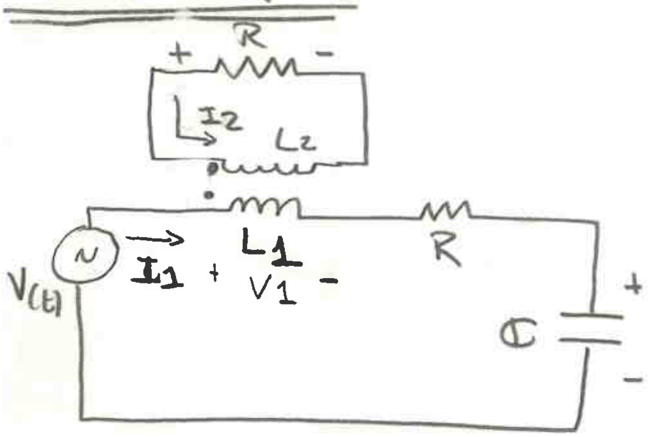
$$\boxed{v(t) = \frac{\sqrt{L . M} . g}{Bo . a} . sen(\omega t)}$$

$$Bo . a . i = Mg - M\dot{v}$$

$$Bo . a . i = Mg - M . \frac{g}{\cancel{\omega}} . \cancel{\omega} . \cos(\omega t)$$

$$\boxed{i(t) = \frac{Mg}{Boa}\left(1 - \cos(\omega t)\right)}$$

Problema 3



$$V(t) = V_0\, e^{j\omega t}$$

$$|\omega L_2| \gg R$$

a) $$\bar{V} = \bar{V_1} + R\,\bar{I_1} + \frac{\bar{I_1}}{C j\omega}$$

$$\bar{V_2} = -R\,\bar{I_2}$$

$$\begin{cases} \bar{V_1} = L_1 . j\omega\, \bar{I_1} + M j\omega\, \bar{I_2} \\ \bar{V_2} = L_2\, j\omega\, \bar{I_2} + M j\omega\, \bar{I_1} \end{cases}$$

$$\Rightarrow \underbrace{(L_2\, j\omega + R)}_{\simeq\, L_2 j\omega} \bar{I_2} + M j\omega\, \bar{I_1} = \varnothing$$

$$\Rightarrow \bar{I_2} = -\frac{M}{L_2} . \bar{I_1}$$

$$\bar{V_1} = \left(L_1\, j\omega - \frac{M^2}{L_2} . j\omega\right) \bar{I_1}$$

Por ambos bobinados pasa el mismo flujo magnético $\Rightarrow$ Transformador perfecto

$$\Rightarrow M = \sqrt{L_1 L_2}$$

$$\overline{V_1} = \left(L_1 j\omega - \frac{L_1 \cancel{L_2}}{\cancel{L_2}} j\omega \right) \overline{I_1} = \emptyset.$$

$$\Rightarrow \overline{V} = \left(R + \frac{1}{Cj\omega} \right) \overline{I_1}$$

$\Rightarrow$ Divisor de tensiones:

$$\overline{V_c} = \frac{\overline{V} \cdot 1/Cj\omega}{R + 1/Cj\omega} = \frac{\overline{V}}{RCj\omega + 1}$$

$$|V_c| = \frac{V_0}{\sqrt{1 + (RC\omega)^2}}$$

$$\angle V_c = - \operatorname{Arctg}(RC\omega)$$

$$\boxed{v_c(t) = \frac{V_0}{\sqrt{1 + (RC\omega)^2}} \cdot \cos(\omega t - \operatorname{Arctg}(RC\omega)).}$$

b) $$\bar{I}_1 = \frac{\bar{V} \cdot Cj\omega}{1 + RCj\omega}$$

$$\bar{I}_1 = \frac{\bar{V} \cdot Cj\omega\,(1 - RCj\omega)}{1 + (RC\omega)^2}$$

$$\bar{I}_1 = \frac{V_0\,(RC^2\omega^2 + Cj\omega)}{1 + (RC\omega)^2}$$

$$P = \frac{1}{2} \operatorname{Re}\left(\bar{V} \cdot \bar{I}_1^{*}\right).$$

$$P = \frac{1}{2} \operatorname{Re}\left\{ \frac{V_0^2\,(R \cdot C^2 \cdot \omega^2 - Cj\omega)}{1 + (RC\omega)^2} \right\}$$

$$\boxed{P_{fuente} = \frac{V_0^2 R C^2 \omega^2}{2\left(1 + (RC\omega)^2\right)}}$$

$$\boxed{P_{Resist} = P_{Fuente}}$$