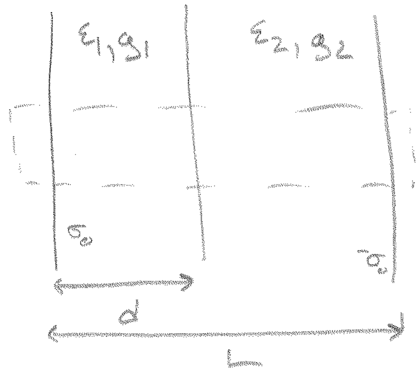
$\boxed{1}$



Inizialmente $0 = \sigma_I + \sigma_1(0) + \sigma_2(0)$

$\sigma_I(0) = -(\sigma_1(0) + \sigma_2(0))$

$\sigma_I(0) = 0$

a) $\nabla \cdot \vec{J} = -\dot{\rho}$
$\vec{J} = g\vec{E}$ $\Big\| \Rightarrow g \nabla \cdot \vec{E} = -\dot{\rho}$

$\Rightarrow g_1 (\vec{E}_1 \cdot \hat{n}) = -\dot{\sigma}_1 \qquad \vec{E}_1 = \frac{\sigma_1}{\varepsilon_1} \hat{e}$

$\frac{g_1}{\varepsilon_1} \sigma_1 = -\dot{\sigma}_1$

$\Rightarrow \sigma_1(t) = \sigma_1(0) e^{-\frac{g_1}{\varepsilon_1} t} \qquad \sigma_1(0) = \sigma_0$

analogamente $\frac{g_2}{\varepsilon_2} \sigma_2 = -\dot{\sigma}_2$

$\sigma_2(t) = \sigma_2(0) e^{-\frac{g_2}{\varepsilon_2} t} \qquad \sigma_2(0) = -\sigma_0$

$$\boxed{\begin{aligned} \sigma_1(t) &= \sigma_0 e^{-\frac{g_1}{\varepsilon_1} t} \\ \sigma_2(t) &= -\sigma_0 e^{-\frac{g_2}{\varepsilon_2} t} \end{aligned}}$$

$$\boxed{\sigma_I(t) = \sigma_0 \left(e^{-\frac{g_2}{\varepsilon_2} t} - e^{-\frac{g_1}{\varepsilon_1} t} \right)}$$

$E_{2n} - E_{1n} = \frac{\sigma_T}{\varepsilon_0} = \frac{\sigma_L + \sigma_P}{\varepsilon_0}$

$\sigma_L = D_{2n} - D_{1n} = \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n}$

$\Rightarrow (\varepsilon_1 - \varepsilon_2) E_{2n} - (\varepsilon_0 - \varepsilon_1) E_{1n} = \sigma_P$

$$\boxed{\sigma_P = \sigma_0 \left\{ \left(\frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1} \right) e^{-\frac{g_1}{\varepsilon_1} t} - \left(\frac{\varepsilon_2 - \varepsilon_0}{\varepsilon_2} \right) e^{-\frac{g_2}{\varepsilon_2} t} \right\}}$$

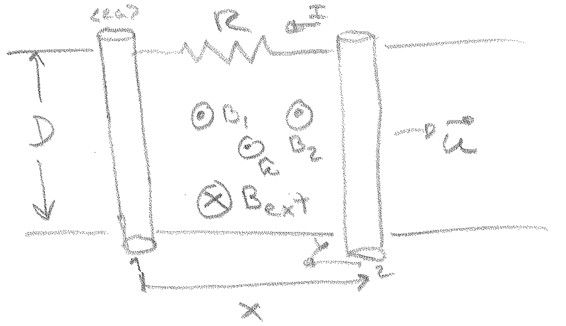
b) $\phi(L) - \phi(d) = -\int_d^L \vec{E}_2 \cdot d\vec{l}$

$\cancel{\phi(L)}^{\,0} - \phi(d) = \frac{\sigma_0}{\varepsilon_2} e^{-\frac{g_2}{\varepsilon_2} t} (L - d)(-1)$

$$\boxed{\phi(d) = \frac{\sigma_0 (L - d)}{\varepsilon_2} e^{-\frac{g_2}{\varepsilon_2} t}}$$

c) $\sigma_I(t) = \sigma_0 \left(e^{-\frac{g_2}{\varepsilon_2} t} - e^{-\frac{g_1}{\varepsilon_1} t} \right) = 0 \quad \forall t \Rightarrow \boxed{\frac{g_2}{\varepsilon_2} = \frac{g_1}{\varepsilon_1}}$

2



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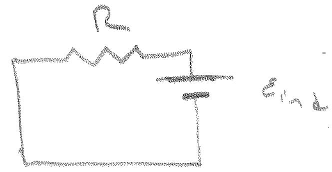

$$\varepsilon_{ind} = -\frac{d\phi}{dt}$$

$$\phi = -B_{ext} D (x-2a) + \phi_{ai}$$

$$\phi_{ai} = \int_a^{x-a} B_1 D\, dx + \int_a^{x-a} B_2 D\, dy$$

$$\vec{B}_1(x) = \frac{\mu_0 I}{2\pi}\frac{1}{x}\hat{k} \quad ; \quad \vec{B}_2 = \frac{\mu_0 I}{2\pi}\frac{1}{y}\hat{k}$$

$$\Rightarrow \phi_{ai} = 2 \cdot \frac{\mu_0 I}{2\pi} D \int_a^{x-a} \frac{1}{x}\, dx = \frac{\mu_0 I D}{\pi} \ln\left(\frac{x-a}{a}\right)$$

$I = \frac{\varepsilon_{ind}}{R}$ busco $I(t)$ cuando $\frac{dI}{dt} = 0$

$$\Rightarrow R\, I(t) = B_{ext} D \dot{x} - \frac{\mu_0 I D}{\pi}\frac{\dot{x}}{x-a} \qquad \dot{x} = u$$

$$\boxed{I(t) = \frac{B_{ext} D u}{R + \frac{\mu_0 D u}{x-a}}}$$

b) $P_u - P_r$ con P_r y P_u tomadas positivas

$$P_r = R I^2$$

$$P_u = \vec{F}_{mag} \cdot \vec{u}$$

$$\vec{F}_{mag} = -B_{ext} D I \hat{u} + \frac{\mu_0 I^2 D}{2\pi}\frac{1}{x}\hat{u}$$

$$P_u = -B_{ext} D I u + \frac{\mu_0 I^2 D}{2\pi}\frac{1}{x} u$$

$$P_u - P_r = -R I^2 + B_{ext} D I v_0 - \frac{\mu_0 D}{2\pi x} v_0 I^2$$

$$\boxed{P_u - P_r = B D v_0 I - \left(\frac{\mu_0 D}{2\pi x} v_0 + R\right) I^2}$$

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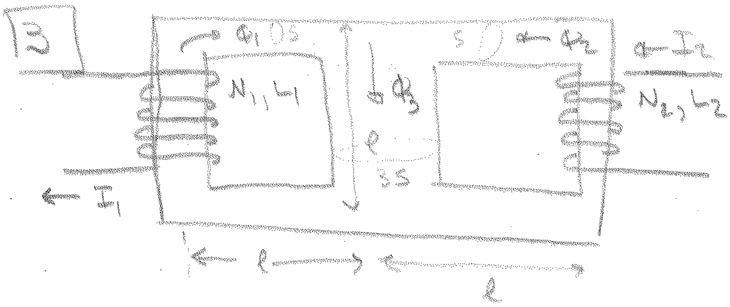


$$\mathcal{R} = \frac{l}{\mu S}$$

$$\Phi_1 + \Phi_2 = \Phi_3$$

$$3\mathcal{R}\Phi_1 + \frac{\mathcal{R}}{3}\Phi_3 = N_1 I_1$$

$$3\mathcal{R}\Phi_2 + \frac{\mathcal{R}}{3}\Phi_3 = N_2 I_2$$

$$9\mathcal{R}\Phi_1 + \mathcal{R}(\Phi_1+\Phi_2) = 3N_1 I_1 \rightarrow \Phi_2 = \frac{3N_1 I_1 - 10\mathcal{R}\Phi_1}{\mathcal{R}}$$

$$9\mathcal{R}\Phi_2 + \mathcal{R}(\Phi_1+\Phi_2) = 3N_2 I_2$$

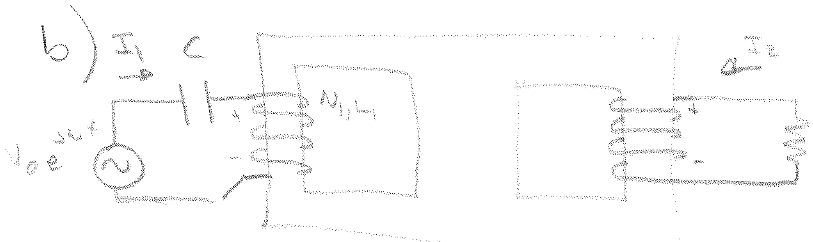
$$\rightarrow 30 N_1 I_1 - 100\mathcal{R}\Phi_1 + \Phi_1\mathcal{R} = 3N_2 I_2$$

$$\boxed{\Phi_1 = \frac{10 N_1 I_1 - N_2 I_2}{33\mathcal{R}}}$$

$$\boxed{\Phi_2 = \frac{10 N_2 I_2 - N_1 I_1}{33\mathcal{R}}}$$

$$L_1 = \frac{\partial \Phi_1}{\partial I_1} = \frac{10 N_1^2}{33\mathcal{R}} \; ; L_2 = \frac{10 N_2^2}{33\mathcal{R}} \; ; |M_{12}| = |M_{21}| = \frac{N_2 N_1}{33\mathcal{R}}$$

b)



$$V_0 - \frac{I_1}{j\omega C} = \frac{d\Phi_1}{dt} \rightarrow V_0 - \frac{I_1}{j\omega C} = \frac{10 N_1^2}{33\mathcal{R}} I_1 j\omega - \frac{N_1 N_2}{33\mathcal{R}} j\omega I_2$$

$$V_0 - \frac{I_1}{j\omega C} = L_1 I_1 j\omega - M j\omega I_2$$

$$-RI_2 = \frac{d\Phi_2}{dt} \rightarrow -RI_2 = j\omega L_2 I_2 - M I_1 j\omega \rightarrow$$

$$V_0 = \left(\frac{1}{j\omega C} + Lj\omega\right) I_1 - j\omega M I_2 \qquad \boxed{I_1 = \left(\frac{R + j\omega L_2}{j\omega M}\right) I_2}$$

$$V_0 = \left(\frac{(1-\omega^2 L_1 C)}{j\omega C}\frac{(j\omega L_2 + R)}{j\omega M} - j\omega M^2\right) I_2$$

$$V_0 = \frac{\left((\omega^2 LC - 1)(j\omega L_2 + R) + j\omega^3 M^2 C\right)}{\omega^2 MC} I_2$$

$$I_2 = \frac{V_0 \omega^2 MC}{R(1-\omega^2 L_1 C) + j(-\omega^3 L_1 L_2 C + \omega L_2 + \omega^3 M^2 C)}$$

$$|I_2| = \frac{V_0 \omega^2 MC}{\left[R^2(1-\omega^2 L_1 C)^2 + (\omega^3 M^2 C - \omega^3 L_1 L_2 C + \omega L_2)^2\right]^{1/2}}$$

$$|V_R| = R|I_2|$$

$$\boxed{|V_R| = \frac{V_0 \omega^2 RMC}{\left[R^2(1-\omega^2 L_1 C)^2 + \left(\omega^3(M^2 C - L_1 L_2 C) + \omega L_2\right)^2\right]^{1/2}}}$$