FIRST ESTIMATES OF INDUSTRIAL FURNACE PERFORMANCE - THE ONE-GAS-ZONE MODEL REEXAMINED

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1. INTRODUCTION

The design of a furnace, more specifically the prediction of the performance of a chosen design, can be carried out at several levels of sophistication. Although a determination of the distribution of heat-flux density over the surface of the stock is desirable - sometimes, in high flux-density systems, necessary - the attainment of the simpler objective of determining the total heat transfer rate as a function of firing rate and excess air is a proper orienting first step; and often it suffices. Even if that is the sole objective, knowledge of the detailed interaction of radiation and convection with mass transfer and combustion is in principle necessary. But integral formulations are tolerant of casual treatment of detail, especially in the presence of the leveling effect of radiation, responsive to a high power of temperature; and a surprisingly accurate overall performance is predictable from a relatively simple model. Even though the knowledge of flux distribution over the stock may be the ultimate objective, it is still good engineering practice to start with an almost-quantitative understanding of the overall process. In fact, it may be asserted that prospects of success with the zone method are poor if the simpler and less ambitious approach, which is after all a one-gas-zone example of the zone method, is not thoroughly understood.

It is the object here to set up a simple overall furnace performance model, in form as general as is consistent with the assumption of a single-gas-radiating temperature, a single equilibrium refractory temperature and, a single equilibrium refractory temperature and a single term characterizing the exchange area between the combustion gases and the sink, which allows for:

1. The effect of the adiabatic flame temperature T_F, dependent on the entering fuel and air enthalpies and the gas heat capacity

2. The effect of stock or sink temprature T_1, measured by the ratio of its mean value to T_F.

3. The effect of stock temperature variation, measured by the ratio r of stock temperatue rise to its arithmetic mean temperature.

4. The value of the characteristic or effective sink area A_S, that area which, multiplied by the difference of black-body emissive powers of the gas and stock temperatures, gives the flux from gas to stock. The major problem of making the model describe realistically the effects of gas composition, furnace shape, and disposition of heat sinks in the furnace clies in the evaluation of A_S, the total gas-sink exchange area.

5. The use of a single gas-radiating temperature T_g.
6. The use of a difference Δ between the gas-radiating and leaving-gas-enthalpy temperature, which varies with firing rate.
7. The loss of heat through the refractory walls.
8. The loss of heat by radiation through openings.
9. Other factors, contributing to the evaluaiton of A_S.

2. THE ONE-GAS-ZONE FURNACE MODEL

Although parts of the following development have appeared before [1,2a], parts are new; and for completeness the full derivation will be presented. The well stirred furnace gases are at temperature T_g in consequence of loss of heat (a) by radiative exchange with the stock or sink at T_1, (b) by convection to that part of the sink $A_{1,e}$ which does not include any curtain tubes across the gas exit from chamber, (c) by convection to and through the refractory walls, and (d) by radiation through furnace openings of area A_0.

a) The net radiative flux to the sink - direct as well as with the aid of refractory surfaces which reflect diffusely or absorb and reradiate - must, if the refractory is radiatively adiabatic and the gas is gray, be proportional to the difference in black-body emissive power of the gas and sink. The proportionality constant, having the dimensions of area, is called the gas-surface total-exchange area $(\overline{GS})_R$ the formulation of which will be discussed later. (The subscript indicates that allowance has been made for the aid given by refractory surfaces).The flux is then $((\overline{GS})_R\, \sigma(T_g-T_1))$.

b) Convective flux to those surfaces $A_{1,e}$ which affect the stirred-gas enthalpy is $hA_{1,e}(T_g-T_1)$. Because this term is quite small compared to (a), it is convenient to combine the two by forcing the convection into a fourth-power form:

$$[\, hA_{1,e}(T_g-T_1) \sim hA_{1,e}\sigma(T_g^4-T_1^4)/4\sigma T_{g1}^3\,]$$

where T_{g1} is the arithmetic mean of T_g and T_1. Then

$$(\overline{GS})_R\,\sigma(T_g^4-T_1^4)+hA_{1,e}(T_g-T_1)=[(\overline{GS})_R+hA_{1,e}/4\sigma T_{g1}]\;\sigma(T_g^4-T_1^4)$$

The bracket has in other contributions been called $(\overline{GS})_{R,c}$ to indicate that convection has been included. The simpler term A_s. the effective area of the sink, will be used here.

c) Convection to and through refractory walls. If the walls are in rediative equilibrium, convection - gas to wall - equals conduction through the wall. The flux is $U_rA_r(T_g-T_0)$, where T_0 is the ambient tempreature and U_r is given, conventionally, by $U_r=\dfrac{1}{\dfrac{1}{h_i}+\dfrac{W}{K}+\dfrac{1}{h_{c+r,\,0}}}$

The inside flux is not in fact equal to the flux through the wall, but the difference is so small compared to the radiative flux as hardly to negate the assumption of radiative equilibrium. Without that assumption one would need to introduce an additional unknown and an additional equation,and the slight improvement in final accuracy does not justify the complication.

d) Radiation through peep holes or other openings, of area A_0. Rigorous allowance for this usually small effect would introduce such comlexities as to prevent obtaining a solution capable of easy engineering use. Altough the view from the outside through furnace openings is a view of sink and refractory surfaces seen dimly through partly diathermanous gas, the assumption will be made that the effective furnace tempreature (the inside plane of the openings) is T_g. With $\overline{F}$ representing the exchange factor to allow for wall thickness [3], the loss through the openings becomes $A_0\overline{F}\sigma(T_g^4-T_0^4)$. Furnaces with openings large enough to make this casual treatment inadequate are rare.

The equation of transfer from the gas is, from the above,

$$\dot{Q}_G=A_S\sigma(T_g^4-T_1^4)+U_rA_r(T_g-T_0)+\overline{F}A_0\sigma(T_g^4-T_0^4) \qquad (1)$$

An energy balance on the gas is needed. Although a single gas radiating temperature has been postulated, it can be a space-mean value rather than the uniform gas temperature of a perfectly stirred chamber; and the gas temperature measuring the gas enthalpy leaving the chamber is usually lower. Let $T_g-\Delta$ represent the leaving-gas temperature, between which and the base tempreature T_o the mean heat capacity is $\overline{C}_{p,g}$. Then the energy balance is

$$\dot{Q}_G = \dot{H}_F - (T_g-\Delta-T_0)\,\dot{m}_g\,\overline{C}_{p,g} \qquad (2)$$

where $\dot{H}_F$ is the hourly entering enthalpy, chemical plus sensible, in the fuel and air and recirculated flue gas, if any, To is the enthalpy-base temperature, and $\dot{m}_g$ is the mass flow rate of gas/hour. Let the same mean heat capacity be used to define an adiabatic pseudo-flame tempreature T_F

$$(T_F-T_0)= \dot{H}_F\,/\,\dot{m}_g\,\overline{C}_{p,g}$$

(T_F will in general be much higher than the true adiabatic flame temperature which allows for a temperature-varying C_p and for dissociation). The energy balance may then be written in the form

$$\frac{\dot{H}_F-\dot{Q}_G}{\dot{H}_F}=\frac{T_g-\Delta-T_0}{T_F-T_0} \qquad (2a)$$

The additional relation needed is that giving furnace efficiency η.

$$\eta=\frac{\dot{Q}_G \text{ -wall losses}}{\dot{H}_F}=\frac{\dot{Q}_G-[U_rA_r(T_g\text{-}T_O)+FA_0\sigma(T_g{}^4\text{-}T_O{}^4)]}{\dot{H}_F} \tag{3}$$

Equations (1) and (2a) contain as unknowns T_g an $\dot{Q}_G$ (assuming rules available for finding $(\overline{GS})_R$ and choosing Δ). Solution for these and insertion into (3) gives the furnace efficiency. But a much deeper understanding of the nature of furnaces can be obtained by further manipulation. Let (1) be made dimensionless by division through by $A_S\sigma T_F{}^4$ and let the dimensionless ratios of various tempreatures to T_F be denoted by their primes.

$$\frac{\dot{Q}_G}{A_S\sigma T_F{}^4}=T'_g{}^4-T'_1{}^4+\frac{U_rA_r}{\sigma A_ST_F{}^3}(T'_g-T'_0)+\frac{\overline{F}A_0}{A_S}(T'_g{}^4-T'_0{}^4) \tag{4}$$

Equation (2a) may be written

$$\frac{\dot{Q}_G}{\dot{H}_F}(1-T'_0)=1-T'_g+\Delta' \tag{5}$$

where $\Delta'=\Delta/T_F$. To complete the normalization, let

$$\frac{\dot{Q}_G}{\dot{H}_F}(1-T'_0)=Q', \text{ the reduced gas efficiency}$$

$$\frac{\dot{H}_F}{\sigma A_ST_F{}^4(1-T_O)}=Q', \text{ the reduced firing density}$$

$$\frac{U_rA_r}{\sigma A_ST_F{}^3}=L'_r, \text{ the refractory loss factor}$$

$$\frac{\overline{F}A_0}{A_S}=L'_0, \text{ the furnace-opening loss factor}$$

Equations (4) and (5) then become

$$Q'D'=T'_g{}^4-T'_1{}^4+L'_r(T'_g-T'_0)+L'_0(T'_g{}^4-T'_0{}^4) \tag{6}$$

and

$$Q'=1-T'_g+\Delta' \tag{7}$$

When loss factors L'_r and L'_0 are small enough to be neglected and Δ' is assumed zero, Q' becomes the reduced furnace efficiency - the furnace efficiency times $(1-T'_0)$ - given by the solution of the equation

$$Q'D' = (1-Q')^4 - T'^4_1 \tag{8}$$

This extraordinarily simple relation, giving furnace efficiency as a function of two dimensionless parameters, D', proportional to the firing rate per unit of effective sink area, and T'_1, the ratio of sink temperature to adiabatic flame temperature, is the basic relationship governing the efficiency of furnaces of almost any class. Performance data on furnaces of a wide variety of types, from gas turbine combustors to openhearth furnaces, can be put on a diagram of efficiency versus firing rate, with sink temperature as a parameter [1,2b] (efficiency here refers to transfer of heat from the combustion gases rather than to the stock).

Better agreement between prediction and experiment, however, may be expected if allowance is made for some difference between mean-radiating temperature and leaving enthalpy temperature. Experience with furnaces of various types as well as with computations based on the multi-zone model indicate that Δ varies inversely with H_F, and the assumption that Δ' is proportional to Q' is much more realistic than treating Δ as constant. For a reason which will emerge later let the proportionality constant be $(1-1/d)$.(There is some evidence that d is about 4/3, putting Δ in the range of 150 to 240C for heavily fired cracking coils or marine boilers). The energy balance, Equation (7) then becomes

$$T'_g = 1 - \frac{Q'}{d} \tag{9}$$

An additional modification is desirable. In many modern high-output furnaces - e.g., reformers and ethylene furnaces - the stock is often heated through a significant temperature interval: and the stock temperature leaving the combustion chamber may come to within a few hundred degrees of the leaving-gas temperature. It may readily be shown that, if the stock has a constant specific heat and T_1 rises from $T_{1,i}$ to $T_{1,o}$ within the combustion chamber, the term $(T'^4_g - T'^4_1)$ in Eq. (6) should be replaced by

$$\frac{8T'^3_g T'_1 r}{\ln\dfrac{T'_g - T'_1(1-r)}{T'_g + T'_1(1-r)} \cdot \dfrac{T'_g + T'_1(1+r)}{T'_g - T'_1(1+r)} + 2\left(\tan^{-1}\dfrac{T'_1(1+r)}{T'_g} - \tan^{-1}\dfrac{T'_1(1-r)}{T'_g}\right)} \tag{10}$$

where T_1 is now $(T_{1,i}+T_{1,0})/2$ and $r = (T_{1,o} - T_{1,i})/(T_{1,0}+T_{1,i})$. As before, $T'_1 = T_1/T_F$. In the limit as $r \to 0$, (10) approaches $(T'^4_g - T'^4_1)$.*

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* Although chemical or phase change within the stock invalidates the assumption of constant specific heat, it is always possible to find an equivalent r to make (10) numerically correct. With $H(T_1)$ representing the stock enthalpy as a function of T_1 varying from $H_{1,i}$ at entry to $H_{1,0}$ at exit, the true value of $\dot{Q}_{stock}/\sigma A_S$ is

$$(H_{1,0}-H_{1,i})\ \backslash i(H_{1,i},H_{1,0},\backslash f(dH(T_1),\ T_g^4-T_1^4)) \quad ,$$

readily evaluated graphically. The result must equal T_F^4 times (10) or (10) with the primes missing. For the T_g and T_1 of interest, r can be found by trial and error. Since the ratio of r defined as $(T_0-T_i)/(T_0+T_i)$ to r from above equality depends on the $H-T_1$ relation and on T_G/T_1 and the latter is substantially constant for a particluar furnace type, the ratio of the two r's requires but infrequent determination. It is to be remembered that if the stock is a fluid in a tube, T_1 in the $H-T_1$ relation is the outer tube surface

With (10) replacing $(T'^{4}_{g} - T'^{4}_{1})$ in (6) and with T'_g replaced by its value from (8), Equation (6) becomes

$$\left(\frac{Q'}{d}\right)(dD') = \frac{8\left(1-\frac{Q'}{d}\right)^3 T'_1 r}{\ln\dfrac{1-\frac{Q'}{d}-T'_1(1-r)}{\frac{Q'}{d}+T'_1(1-r)}\cdot\dfrac{1-\frac{Q'}{d}+T'_1(1+r)}{1-\frac{Q'}{d}-T'_1(1+r)}+2\left(\tan^{-1}\dfrac{T'_1(1+r)}{1-\frac{Q'}{d}}-\tan^{-1}\dfrac{T'_1(1-r)}{1-\frac{Q'}{d}}\right)}$$

$$+L'_r\left(1-\frac{Q'}{d}-T'_0\right)+L'_0\left[\left(1-\frac{Q'}{d}\right)^4-T'^{4}_0\right] \tag{11a}$$

with $r = 0$, this reduces to

$$\left(\frac{Q'}{d}\right)(dD') = \left(1-\frac{Q'}{d}\right)^4 - T'^{4}_1 + L'_r\left(1-\frac{Q'}{d}-T'_0\right)+L'_0\left[\left(1-\frac{Q'}{d}\right)^4-T'^{4}_0\right] \tag{11b}$$

The error in the use of (11b) rather than (11a) is less then one percent when $T_1/T_g = 0.8$ and $r < 0.05$ or $T_1/T_g = 0.9$ and $r < 0.02$. r is often many times these values and the error mounts rapidly.

If loss coefficients L'_r and L'_0 are both zero, (11b) has the same structure as (8), with (Q'/d) replacing Q' and $(D'd)$ replacing D'. Thus the form of relation chosen for Δ has permitted inclusion of allowance for it without increasing the number of dimensionless groups.

The furnace efficiency η, when there are wall losses, may be shown from (3) to take the form

$$\frac{\eta}{d} = \frac{Q'/d}{1-T'_0} - \frac{L'_r}{(D'd)}\left(1-\frac{Q'/d}{1-T'_0}\right) - \frac{L'_0}{(D'd)(1-T'_0)}\left[\left(1-\frac{Q'}{d}\right)^4 - T'^{4}_0\right] \tag{12}$$

The desired relation between efficiency η and reduced firing density D' - or rather between η/d and $D'd$ - is obtianable from (11a or b) and (12) considered as parametric equations in Q'/d. They express the relation

$$\eta/d = f(dD', T_1' \text{ (or } T'_1 \text{ and } r), L'_r \text{ and } L'_0) \tag{13}$$

which is in a form involving no commitment as to what value will be assigned to d, the measure of the difference between radiating and enthalpy temperature of the combustion gases.

3. OVERALL FURNACE PERFORMANCE - GRAPHICAL PRESENTATION

To indicate the character and use of relation (13), three graphs have been presented. Figure 1 shows, on logarithmic coordinates, the consequences of allowing for wall losses, for the case of L'_0 and r both zero, and for $T'_0 = 1/8$, $T'_1 = 0.5$, and $L'_r = 0.02$ (a realistic wall-loss coefficient). The lower curve is the furnace efficiency,

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temperature, not the bulk temperature of the fluid. Enough is generally known in advance to construct an H–T_1 diagram allowing for temperature drop through the fluid film and tube wall.

efficiency, going to 0 when the normalized firing density drops to 0.015, * passing through a

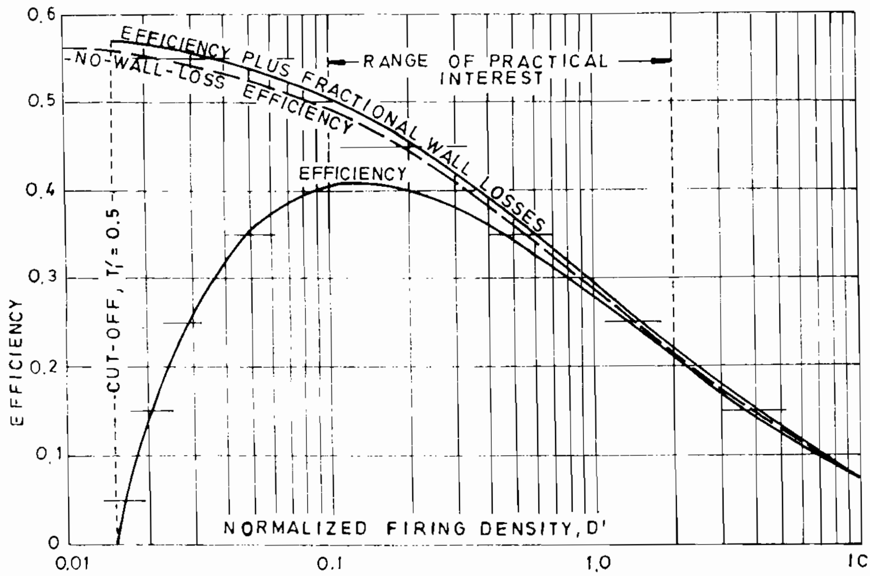

Fig. 1. Effect of wall-loss factor L'_r on combustion chamber performance. $L'_r = 0.02$; $T'_1 = 0.5$; $T'_0 = 1/8$

maximum, and dropping at high firing rate. The top curve is the sum of efficiency and fractional wall loss. The middle curve is the no-wall-loss efficiency, which increases continuously as the firing rate drops, and at $D' \to 0$ is asymptotic to $(1-T'_1)/(1-T'_0)$.

Figure 2 shows, on cartesion coordinates, the (η/d) versus $D'd$ relation, and gives the effect of sink temperature ($T'_1 = 0.2$ to 0.7) and L'_r (0.02 and 0.04).

The same picture on logarithmic coordinates is much more useful, Figure 3. Its importance is that in this form the relationship may be used to correlate and extrapolate furnace data without concern for the sometimes difficult technique of accurately evaluating the quantity A_S. Let us assume that furnace performance data are available in the form of effeciency η versus firing rate H_F. The quantitaties T_0, T_1, T_F, U_rA_r, A_0, $\overline{F}$ are known or readily calculable, and a vlaue of A_S sufficiently accurate for the determining the not too important quantity L'_r and L'_0 should also be calculable. Let the efficiency η be plotted vs $H_F/\sigma T'^4_F(1-T'_0)$ on transparent logarithmic paper which matches Fig. 3, and let the plot be superimposed on an equivalent of Fig. 3 which has been constructed for the values of r and L'_0 which characterize the furnace. Let the plot be displaced vertically and horizontally until tha data fit the proper T'_1 and L'_r curves. The relative vertical displacement of the two plots yields the value of d; their relative horizontal displacement yields the value $D'd/[H_F/\sigma T'^4_F(1-T'_0)]$ which is d/A_S. The two quantities d and A_S are characteristic of the furnace; one is an

* $(dD')_{\eta=0} = [L'_r(T'_1 - T'_0) + L'_0(T'^4_1 - T'^4_0)]/(1-T'_1)$; $(Q'/d)_{\eta=0} = 1 - T'_1$

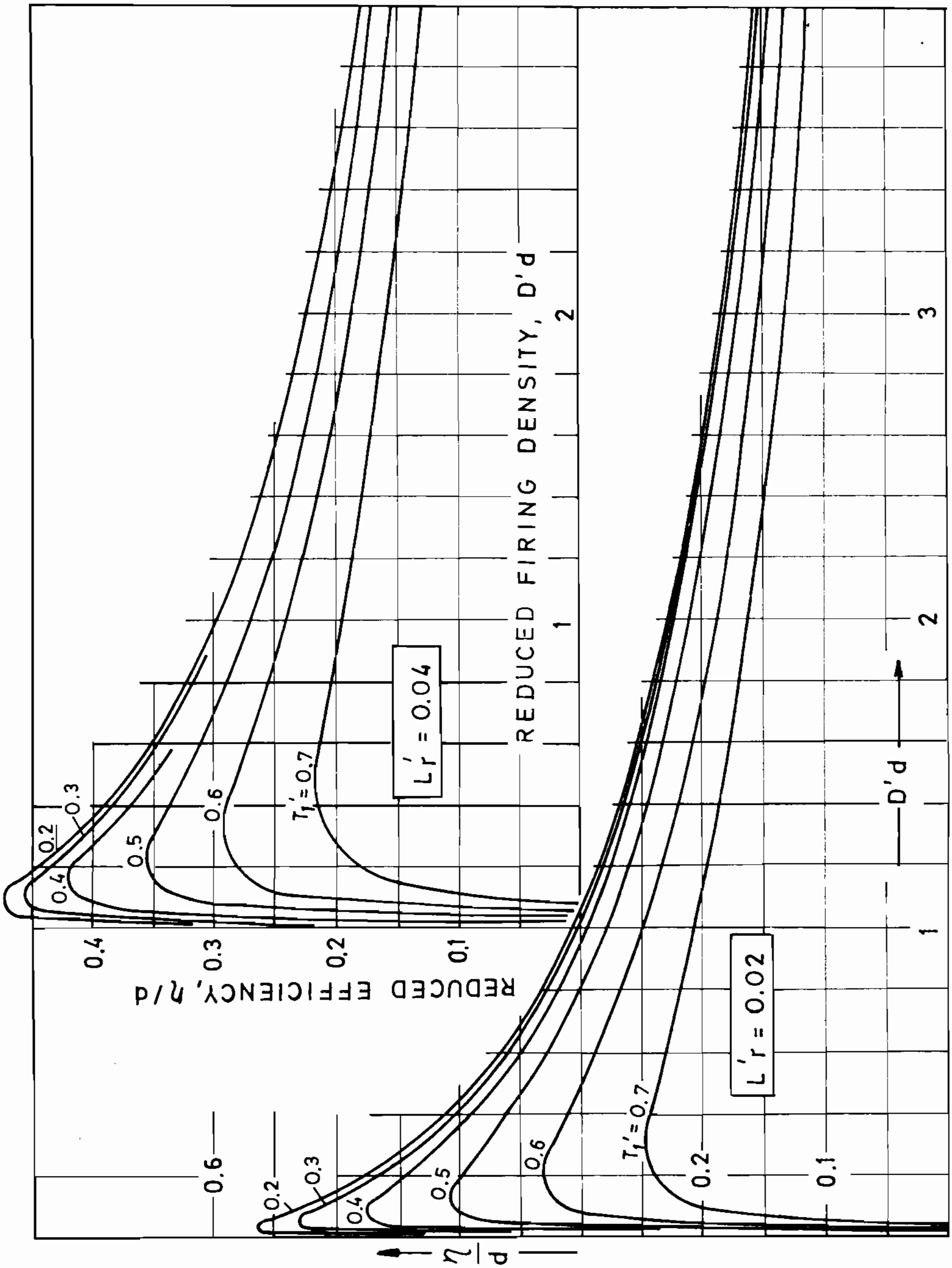

Figure 2. Efficiency vs firing density for 2 values of the wall-loss factor L'_r. T'_1 = reduced sink temperature, T_1/T_F. $L'_0 = 0$, $r = 0$.

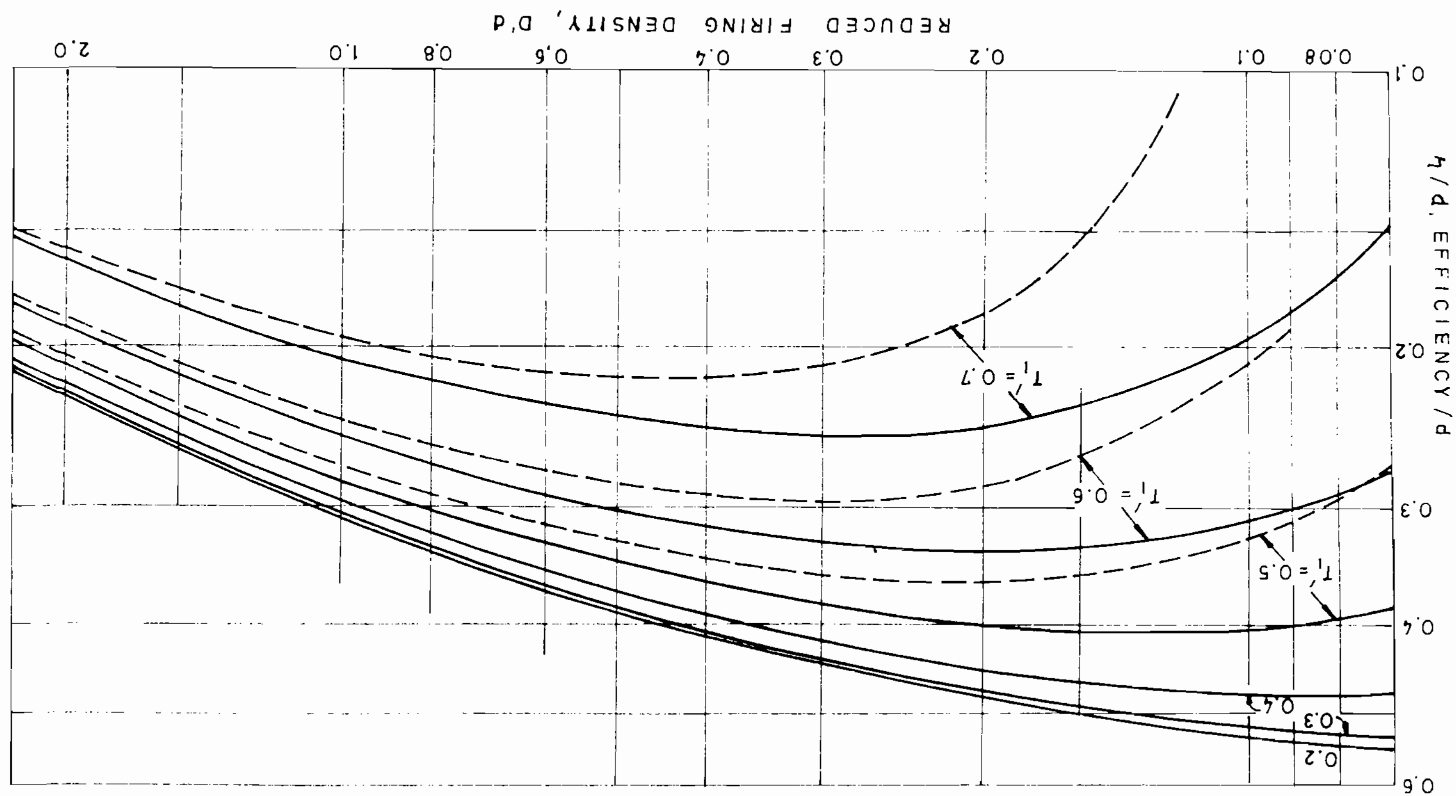

Figure 3. (η/d) vs ($D'd$) for furnace chambers. For comparison with data on efficiency vs. firing rate, to determine furnace performance characteristics d and A_e by curve fitting. Conditions: $T'_0 = 1/8$, $L'_0 = 0$, $r = 0$, $L'_r = 0.02$ (solid lines) and 0.04 (dotted lines).

empirical constant taking account of Δ, the other a measure of the many factors affecting radiative exchange.They permit a computation of what will happen under other furnace operating conditions (provided these don't change the general character of the flow pattern or the system's radiation geometry); and when determined for each of several furnaces in a particular class they lead to an empirical determination of the effects of design variables on performance.For predictions requiring no prior data, however, it is necessary to be able to calculate A_S from first principles, and this depends primarily on the total-exchange area the evaluation of which will now be considered.

4. THE TOTAL-EXCHANGE AREA

Given: a gray isothermal gas volume enclosed by an isothermal sink surface A_1 and by a refractory surface A_r in radiative equilibrium. The net flux from gas to sink is $\overline{GS_1}\,\sigma(T'^{4}_{g}-T'^{4}_{1})$ where $\overline{GS_1}$ is the total-exchange area. The restriction must be imposed that the gas and two surfaces are each treated as a single zone, implying not that all of surface A_1, for example, is segregated into a single area but that a single mean view that A_1 has of A_r can be used in evaluating all of the radiation emitted or reflected from A_1 toward A_r. The total-exchange area $\overline{GS}$ allows for multiple reflections off all surfaces and for assistance given by the refractory in absorbing gas radiation and reradiating a part of it to the sink. In the model under discussion it is clear that $\overline{GS}$ carries a major burden of making the model agree with reality. Many degrees of complexity exist in evaluating $\overline{GS}$, and the engineer has the choice of advancing as far along the path as the importance of his problem warrants.

It may be shown that the total-exchange area depends on sink emissivity and on the inter-zone direct-exchange areas (lower case letter pairs)

$$\overline{GS}_1=\cfrac{1}{\cfrac{1-\varepsilon_1}{A_1-\varepsilon_1}+\cfrac{1}{\overline{gs_1}+\cfrac{1}{\overline{gs_r}+\cfrac{1}{\overline{s_r s_1}}}}} \qquad (14)$$

with $\overline{gs_1}\equiv A_1[\varepsilon_g(L_{m,1})]$ and $\overline{gs_r}\equiv A_r[\varepsilon_g(L_{m,r})]$ and $\overline{g_r s_1}\equiv A_r F_{r1}\{1-[\varepsilon_g(L_{m,m,r1})]\}$. Here the gas emissivity ε_g is written to indicate its dependence on mean beam length L_m, and the latter in turn to indicate that its value depends on the source and sink of radiation. If the three ε_g*'s* are assumed representable by a single gas emissivity applicable to the whole enclosure, (14) becomes

$$\overline{GS_1}=\cfrac{A_1}{\cfrac{1}{\varepsilon_1}-\cfrac{1}{\varepsilon_g\left(1+\cfrac{A_r/A_1}{1+\varepsilon_g/(1-\varepsilon_g)F_{r1}}\right)}} \qquad (15)$$

This ancient relation* was replaced early by a simplified version [5] based on what later became known as the speckled-furnace approximation. If A_1 and A_r are

* Its black – sink equivalent appeared in 1928 [4], perhaps earlier.

intimately mixed over the surface of the enclosure rather than more or less segregated the view factor either surface has of A_1 is $A_1/(A_1+A_r)$. Subsituttion of this value for F_{r1} in (15) together with the replacement of A_1 by CA_T and A_r by $(1\text{-}C)A_T$, where C is the fraction of the total envelope area A_T which is "cold" (i.e., which is A_1), yields the very simple relation

$$\overline{GS}_1 = \frac{A_T}{\dfrac{1}{C\varepsilon_1}+\left(\dfrac{1}{\varepsilon_1}-1\right)} \tag{16}$$

If instead of A_1 and A_r being intimately mixed A_1 lies in a single plane, F_{1r} becomes 1 and, since $A_1F_{1r}=A_r/F_{r1}$, F_{r1} is A_1/A_r or $C(1-C)$. Substitution of this into (15) yields a result which may be shown to have the structure of (16) except that the parenthesis in the denominator is now multiplied by

$$\left(\frac{1-\varepsilon_g}{1-C\varepsilon_g}\right) \tag{16a}$$

The object of converting (15) to (16) or to (16) modified has of course been to dodge the often tedious formulation of F_{r1} (or F_{1r}). From the two results which appear to be limiting cases it is tempting to seek an interpolation procedure, i.e., to find a function by which to multiply the parenthesis in (16) which varies from 1 for speckled furnaces to (16a) for the case of A_1 in a single plane.

It may be shown that Eq. (15), with A_1 and A_r replaced by CA_T and $(1\text{-}C)A_T$, becomes

$$\frac{GS_1}{A_1} \equiv \frac{GS_1}{CA_T} = \frac{1}{\dfrac{1}{\varepsilon_1}+\left(\dfrac{1}{\varepsilon_1}-1\right)C\left[\dfrac{\varepsilon_g+F_{r1}(1-\varepsilon_g/C)}{C\varepsilon_g+F_{r1}(1-\varepsilon_g)}\right]} \tag{17}$$

The bracketed term can be written in a form more nearly symmetrical with respect to C and $(1\text{-}C)$ by substitution of the exchange area $\overline{1r}$ for A_1F_{1r} $(\equiv A_rF_{r1})$ to give

$$\left[\frac{\varepsilon_g/C+(1-\varepsilon_g/C)\,\overline{1r}\,/A_TC(1-C)}{\varepsilon_g+(1-\varepsilon_g)\,\overline{1r}\,/A_TC(1-C)}\right] \tag{18}$$

The term takes the three limiting (?) forms

(a)

If A_1 lies in a single plane, $(C \leq 1/2)$, substitution of $F_{1r}=1$ yields

$$[\,]=\frac{1-\varepsilon_g}{1-C\varepsilon_g}$$

(b)

If the walls are speckled, $(0<C<1)$ substitution of $F_{r1}=C$ or $F_{1r}=1-C$ yields

$$[\,]=1$$

(c)

If A_r lies in a single plane, $(C>1/2)$ substitution of $F_{r1}=1$ yields

$$[\,]=\frac{1-\varepsilon_g(1-C)/C}{1-\varepsilon_g(1-C)}$$

The first two of these three results have already been given. Since F_{1r} changes from (1-C) for case (b) above - the speckled system - to 1 for A_1 segregated in a plane, let us make the heuristic assumption that a term S - called the speckledness - measures the shift from complete speckledness to A_1 in a plane, according to the relation

$$\left.\begin{aligned} F_{1r} &= 1 - SC \\ \text{or} \qquad \overline{1r}/A_T &= C(1-SC) \end{aligned}\right\} \quad C \leq 1/2 \tag{19}$$

with $S = 1$ for a speckled enclosure and 0 for A_1-segregation. Substitution of this into the bracket of (17) gives

$$[\,] = \left[\frac{1-\varepsilon_g - S(C-\varepsilon_g)}{1-C\varepsilon_g - SC(1-\varepsilon_g)}\right] \tag{20}$$

Values of $S = 1$ and 0 substituted into (20) yield the results of cases (b) and (a) above respectively.

Similarly, since F_{r1} changes from C for the speckled system to 1 for A_r in a single plane, we make the parallel assumption that S' measures the shift from speckledness to A_r-segregation according to the relation

$$\left.\begin{aligned} F_{r1} &= 1 - S'(1-C) \\ \text{or} \qquad \overline{r1}/A_T &= (1-C)(1-S'(1-C)) \end{aligned}\right\} \quad C \geq 1/2 \tag{21}$$

with $S' = 1$ for a speckled enclosure and 0 for A_r in a plane. Substitution of this into the bracket of (17) give

$$[\,] = \left[\frac{\varepsilon_g + (1-\varepsilon_g/C)(1-S'(1-C))}{\varepsilon_g/C + (1-\varepsilon_g)(1-S'(1-C))}\right] \tag{22}$$

Values of $S' = 1$ and 0 substituted into (22) yield the results of cases (b) and (c) above, respectively.

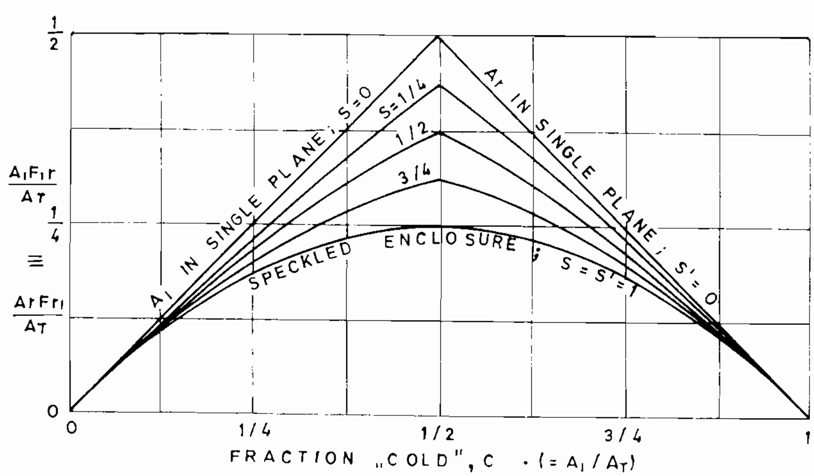

Figure 4. Evaluation of ratio of Sink-refractory direct-exchange area to total furnace envelope area, in terms of fraction cold C, and speckledness S.

In Fig. 4 the value $\overline{Ir}/A_T$ appears as a function of C for the three limiting values:

speckled-wall enclosure, lower parabola ($S = S' = 1$)
A_1 in a plane, upper left straight line ($S = 0$)
A_r in a plane, upper right straight line ($S' = 0$)

The values of $\overline{Ir}/A_T$ acoording to (19) and (21) are given in Fig. 4 at S and S' of 1/4, 1/2, 3/4. The question arises as to the meaning of these curves intermediate between S(or S') = 1 and 0 where the meanings are quantitatively identifiable. Or if the questions is rephrased, does the S-concept have utility in quick identification of $\overline{Ir}$ for use in (17) to evaluate $\overline{GS_1}$? How significant is an error in $\overline{Ir}/A_T$?

An examination of a few geometrical shapes is illuminating:

1. Spheres. The view that A_1 has of A_r depends on areas only, not on location. ($A_1F_{1r} = A_1A_r/A_T$). No matter what the disposition of surfaces or their degree of segregation, $S = 1 = S'$, and all spherical enclosures are in the "speckled" category.

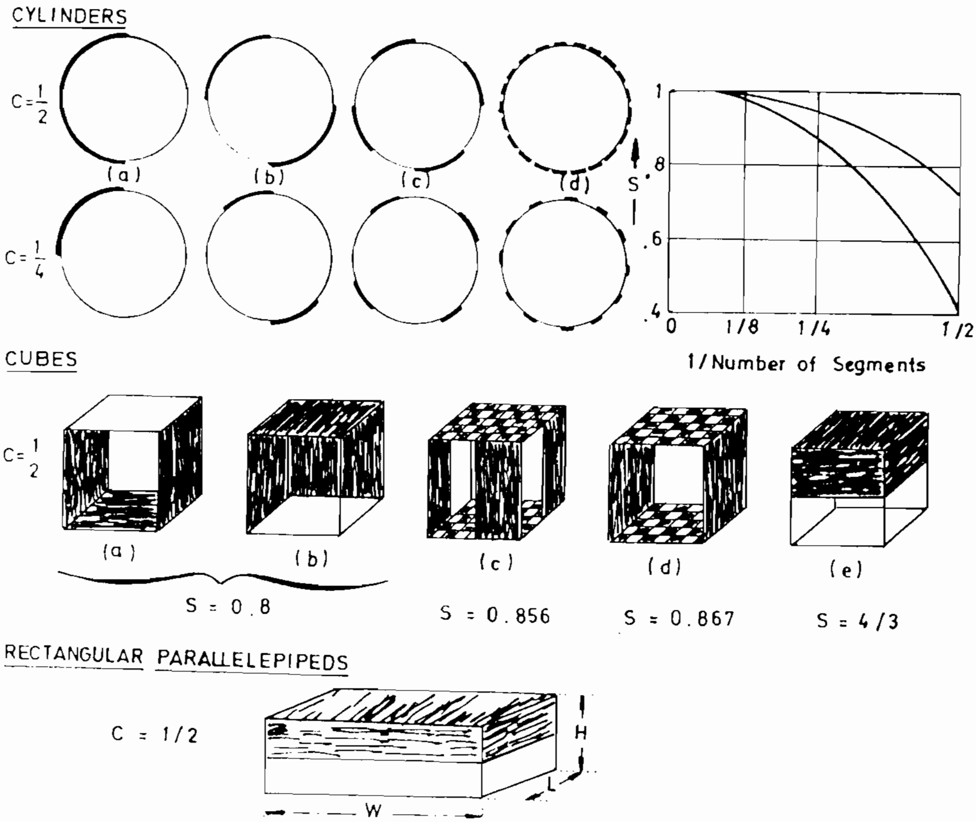

Figure 5. Effect of division of the surface of various enclosures into A_1 (sink) and A_r (refractory) on the speckledness necessary to predict the exchange area $A_1F_{1r}(\equiv A_rF_{r1})$.

2. Long cylinders. Figure 5 shows two sets of cylinders, $C = 1/2$ and 1/4, with A_1 successively divided into smaller segments symmetrically disposed. The value of S needed for the known value of $\overline{1r}/A_T$ is plotted versus the reciprocal of the number of segments into which the surface is divided. Note that division of A_1 and A_r into only two segments each is almost sufficient to put S above 0.9.

3. Cubes, $C = 1/2$. When three of the six faces are A_1, the only two possible arrangements will yield an S of 0.8, cases (a) and (b). Case (c), the four sidewalls symmetrically banded, with A_1 occupying one-half the area, and the roof and floor speckled. The overall S is 0.856. When this case is modified to (d), with A_1 occupying all opposite sides and a speckled half of the roof and floor, $S = 0.867$. But when a value of $C = 1/2$ is reached (e) by a plane parallel to one of the faces, with all A_1 on one side of it, $S = 4/3$. It becomes obvious that S values of 0 and 1 are not bounding values (see next case).

4. Rectangular parallelepipeds, $C = 1/2$, A_1 and A_r on either side of a plane parallel to one face. It may be shown that in this case $S = 2\text{–}2/[1 + H(1/W + 1/L)]$. The larger S the poorer the performance. When $H/(W + L)$ is small, S is small, zero in the limits as $H \rightarrow 0$(the case of infinite parallel planes). Under these conditions A_r makes a maximum contribution to the transfer of heat to A_1. As H increases to $1/2W$, with $L = W$, S becomes 1. With H great compared to W or L, S is increasingly greater than 1. Actual furnaces of such geometry as to make $S > 1$ are rare, and should be.

A study of the cases presented justifies the conclusion that for most furnace chambers S, which is near 1 unless C is small (when A_1 may be in a single plane), may generally be estimated within 0.2 on the basis of a qualitative examination of the chamber. To examine the accuracy to which S need be established, $\overline{GS_1}/A_1$ was evaluated for the case of $\varepsilon_1 = 0.9$ and $\varepsilon_g = 0.3$, realistic values for cracking coils and reformers. Figure 6 shows curves for $S = S' = 1$ and for $S = 0$ ($C < 1/2$) and $S' = 0$ ($C > 1/2$). Since the maximum difference between the two curves is only about 10% at $C = 0.4$, it is clear that estimation of S to within 0.2 should be adequate for most design purpose, and that in consequence it should rarely be necessary to evaluate F_{r1} rigorously.

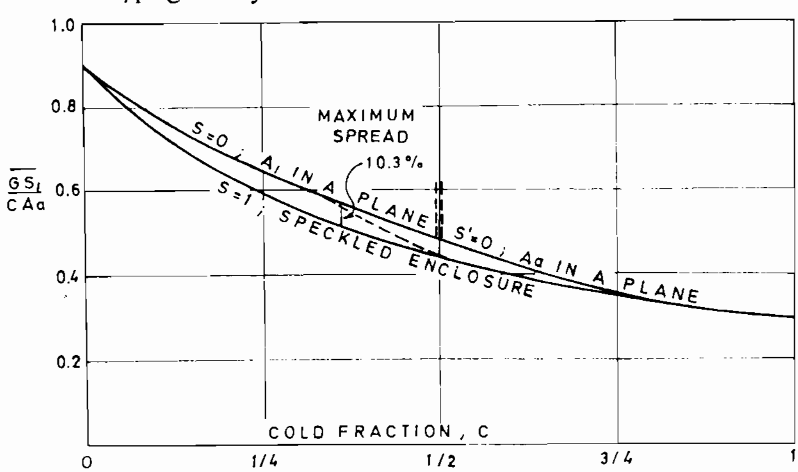

Figure 6. Example of the effect of the variation of speckledness from 0 to 1 on the relation between total gas-sink exchange area and "cold" fraction of a furnace chamber. ($\varepsilon_1 = 0.9$; $\varepsilon_g = 0.3$).

It is interesting to note that Lobo and Evans [6], in an early application of the equivalent of Eq. (15) to cracking-coil furnaces, recommended a procedure which can be shown to be the equivalent, in Fig. 6, of assuming that $\overline{GS_1}/CA_T$ is uniquely determined by C. The relation recommended was the equivalent of assuming that $S = 0$ for C up to 1/3, that S undergoes transition from 0 to 1 as C varies from 1/3 to 1/2, and that $S = 1$ from $C = 1/2$ to 1. The transition is shown as a dotted line in Fig. 6. In Fig. 4, the Lobo and Evans recommendation is equivalent to following the top line from the left at $C = 0$ to $C = 1/3$, dropping from there to the $S = 1$ curve at $C = 1/2$.

5. EFFECT OF NON-GRAY GAS ON $\overline{GS}$

Although refractory surfaces are in overall radiative equlibrium, they are capable of absorbing radiaton from a gas and then reradiating it with a different spectral distribution, some of the radiation passing through the spectral windows of the gas directly to the sink A_1. The mixed-gray-gas assumption [7,2c] is consequently more realistic than the gray-gas assumption on which Eqs. (14) and (15) are based.

The simple mixed-gray gas is a gray-plus-clear system, with a_g and $(1-a_g)$ representing the energy-fractions of the black-body spectrum in which the gas is gray (or the absorption coefficient is constant) and clear, respectively. The quantity a_g is obtained from a determination of gas emissivity ε_{g,L_m} at the mean length L_m which characterizes the enclosure, and of $\varepsilon_{g,2L_m}$ at twice that length.

$$a_g = \frac{(\varepsilon_{g,L_m})^2}{2\varepsilon_{g,L_m} - \varepsilon_{g,2L_m}} \tag{23}$$

The rather involved derivation [2d] leads to a reasonably simple expression for $\overline{GS_1}$.

$$\overline{GS}_1 = \frac{A_T}{\frac{1}{C\varepsilon_1} + \frac{1}{\varepsilon_{g,e}} - \frac{1}{a_g} + \left(\frac{1}{a_g} - 1\right)\frac{1}{C\varepsilon_1 + (1-C)\varepsilon_r}} \tag{24}$$

Note that for a gray gas $a_g = 1$, and (24) reduces to (16).

The new subscript e on gas emissivity requires explanation. For a gray gas, emissivity and absorptivity are the same; and the net gas-sink flux is the black-body emissive-power difference $(E_g - E_1)$ multiplied by a $\overline{GS_1}$ dependent on ε_g. For a non-gray gas E_g and E_1 are separately multiplied by different values of $\overline{GS_1}$, based on gas emissivity and gas absorptivity, respectively. This complication may be avoided, however, by using a modified gas emissivity in (24), equal to the value at the arithmetic mean of T_g and T_1, then multiplied by the factor $[1 + (a' + b - c)/4]$ which allows primarily for the way emissivity varies with absorption strength and tempreature [6.2c].

$$a' = \partial \ln \varepsilon_g / \partial \ln pL$$

$$b = \partial \ln \varepsilon_g / \partial \ln T_g$$

$c = 0.65$ for CO_2, 0.45 for H_2O, 0.5 for average flue gas.

When $T_1 < T_g/2$, the simpler ε_g evaluated at gas temperature replaces $\varepsilon_{g,e}$ in (24).(Note that $\overline{GS_1}$ now depends on refractory emissivity, about 0.5).

6. SPECIAL PROBLEMS ASSOCIATED WITH TUBULAR HEATERS

The heat sinks of many furnaces are in the form of a row or rows of tubes, often mounted in a plane parallel to and near a refractory backing wall. When so mounted each tube row - backwall combination acts, with respect to radiative interchange with the remainder of the chamber - gas and walls - like a plane gray surface of area equal to the continuous tube plane A_p and of effectove emissivity given [7,2f] by

$$\varepsilon_1 = \frac{1}{\dfrac{1}{F+(1-F)F} + \dfrac{B}{\pi}\left(\dfrac{1}{\varepsilon'_1} - 1\right)} \tag{25}$$

Here ε_1' is the true emissivity of the tube metal (often taken as 0.9, lower for high-quality alloys), B is the ratio of tube center-to-center distance to diameter, and F is the fraction of radiation incident on the plane through 2π steradians which is intercepted by the tubes. The fraction $(1-F)$ impinges on the backwall and is reradiated or reflected.

The fraction F, the view-factor for radiation incident on a row of tubes, is conventionally evaluated for incident radiation which is isotropic, of which black-body radiation is an example. In that case

$$F_{iso} = 1 - \frac{1}{B}[(B^2-1)^{1/2} - \cos^{-1}\frac{1}{B}] \tag{26}$$

(The numerical equivalent of (26), clumsily obtained, first appeared in 1930[8]). The question unavoidably arises as to how much error is involved in using F from (26) when the incident radiation on a tube row is non-isotropic, as from a gray gas (the result for a real gas is then readily evaluated by use of the mixed gray gas concept).

Consider a two-dimensionally infinite tube row, Fig. 7, irradiated by a slab of

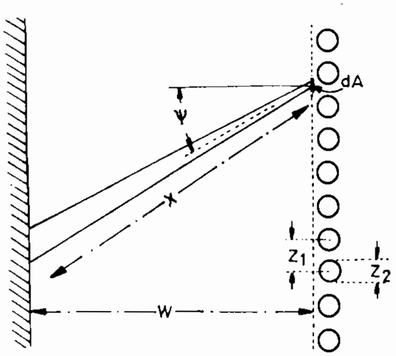

Figure 7. Section through a gas slab adjacent to a row of tubes $z_1/z_2 \equiv B$

gas of width W and absorption coefficient K. The interchange-area $\overline{ss}$ between surface elements bounding the ends of the wedge-shaped space [2g] is

$$\frac{d^2\,\overline{ss}}{dA\cos\psi\, d\psi}=\frac{2}{\pi}\int_0^{\pi/2} e^{-Kx/\cos\theta}\cos^2\theta d\theta \tag{27}$$

Call the definite integral $f_3(Kx)$; its numerical value is given in [2g]. The value of $\overline{ss}$ if the gas is non-absorbing is, from (27) with $K=0$, $(2/\pi)f_3(0)=1/2$. Since

$$\overline{ss}_{\,clear} - \overline{ss}_{\,absorbing\ gas\ present} = \overline{gs}$$

one may express $\overline{gs}$, with x replaced by $W/\cos\psi$, as

$$\frac{d^2\,\overline{gs}}{dA\cos\psi\, d\psi}=\frac{1}{2}-\frac{2}{\pi}f_3(KW/\cos\psi) \tag{28}$$

As dA is moved along the plane of the tube row without changing ψ, the interception of the beam varies intermittently from 0 to 1. Let the mean fractional interception for ψ-oriented radiation be F_ψ, given by

$$F_\psi=\begin{cases}\dfrac{\sec\psi}{B} \text{ when } \sec\psi\le B\\ 1 \text{ when } \sec\psi\ge B\end{cases}$$

The interchange area ratio $\overline{gs}/A$ - the flux from gas to tubes per unit area of tube plane and per unit black emissive power difference of gas and tube surface -- is then given by multiplying the r.h.s. of (28) by F_ψ and integrating.

$$\frac{\overline{gs}}{A}=2\int_0^1\left[\frac{1}{2}-\frac{2}{\pi}f_3(KW/\cos\psi)\right]F_\psi d\sin\psi \tag{29}$$

The same interchange-area ratio for a continuous plane receiver would be (29) evaluated with $F_\psi=1$; this may be shown to be $1-2\varepsilon_3(KW)$, where ε_3 is the third exponential integral. The ratio of these two values of $\overline{GS}$ is the desired mean fractional interception F_{gas} of radiation from a gas slab to a bounding tube row.

$$F_{gas}=\frac{2\int_0^1\left[\frac{1}{2}-\frac{2}{\pi}f_3\ (KW/\cos\psi)\right][F\psi(B)]d\ \sin\psi}{0.5-\varepsilon_3(KW)} \tag{30}$$

The ratio $F_{gas}/F_{isotropic}$, from (26)/(30), is the correction factor to the conventionally used graphs giving F_{iso} as a function of B. This ratio appears in Fig. 8. Since, for the gray-plus-clear gas model, tube furnaces have a KW of the order of 1, the correction for non-isotropic incidence on the tube row is generally small.

If the tube-row is mounted on a backwall, F is required in (25) in order to obtain the effective plane emissivity ε_1. The term $[F+(1-F)F]$, representing interception of the incoming beam plus interception of the beam returning from the refractory, is called F. Because the returning beam is either almost-isotropic emission or almost-isotropic diffuse reflection,

$$\overline{F_{gas}} = F_{gas} + (1-F_{gas})F_{iso} \tag{31}$$

Clearly, the ratio $\overline{F_{gas}} / \overline{F_{iso}}$ is even nearer 1 then the direct-radiation ratio F_{gas}/F_{iso}; and the correction can almost always be ignored.

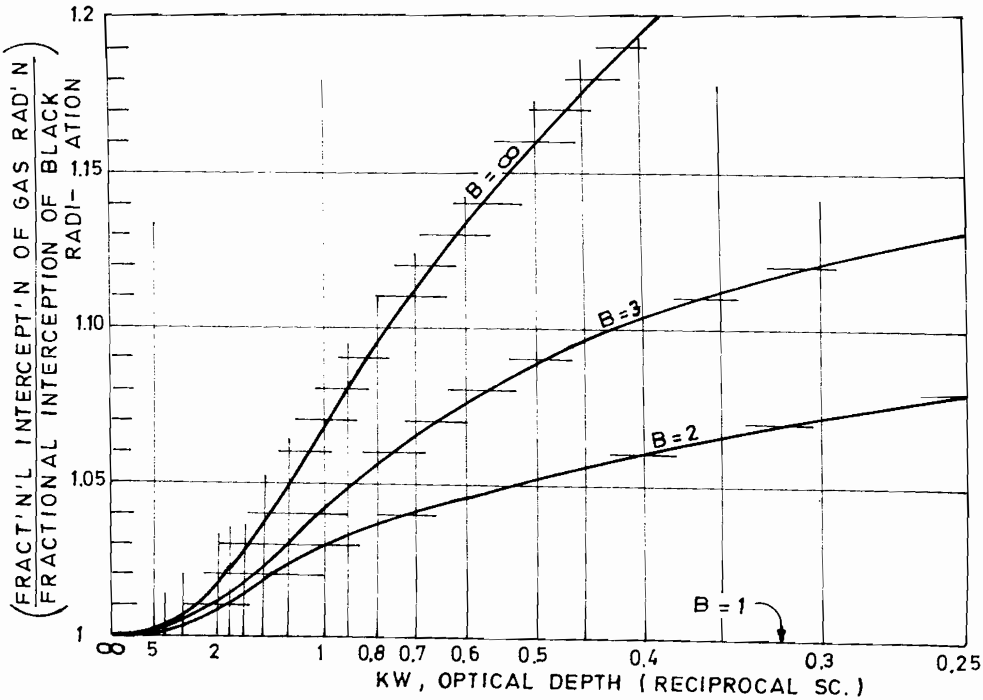

Figure 8. Ratio F_{gas}/F_{iso}, (fractional interception of a gas radiation)/ (fractional interception of isotropic radiation) by a tube raw. Tube center-to-center distance / diameter $= B$; optical thickness of gas a slab $= KW$.

From the above discussion it is clear that in using Equation (15)-(22) and (24) to predict the performance of a furnace with tubes mounted on walls, A_r refers only to bare refractory, A_1 is the total area of the tube planes which, with their refractory backing act like a surface at T_1 and of emissivity ε_1 given by Eq. (25). There remains the evaluation of gas emissivity, which depends on the mean beam length. for rectangular parallepipeds varying from cubes to the space between infinite parallel planes an average mean beam length of 0.83x4 times the system mean hydraulic radius is an excellent approximation for absorption strengths in the range of industrial furnaces ($KL_m \sim$ 1-3).

The use of the one-gas-zone model for furnace chambers in which the tubes are enveloped in combustion gases rather than mounted in planes near refractory back-walls raises some difficult questions, such as how to define the sink area A_1 and what mean beam length L_m to use in evaluating ε_g. No longer is the system representable as an equivalent box, with the sink A_1 represented by continuous plane surfaces. A rigorous treatment of this problem has not to my knowledge been made. The multizone method could be used to determined the best recommentations for a one-zone model. In the interim an approximation will be suggested.

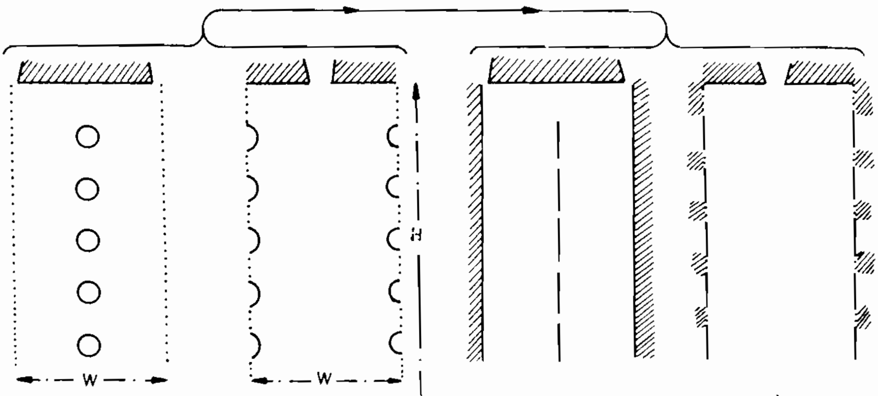

Fig. 9. Section through one cell of a furnace chamber filled wiht spaced parallel vertical tube screens, and its two alternative visualizaitons

Consider a furnace in which equally spaced parallel tube screens are mounted, with firing between each pair, sufficient in number to justify the assumption that a repeating pattern is typical. Figures. 9(a) and (b) depict the two choices avaiable for picturing the repeating pattern. If there are several zones on either side of the one pictured, the dotted lines are planes of no net radiative flux. Specular mirrors replacing the dotted lines would make both sketches truly representative, and therefore completely equivalent. It is tempting to visualize perfectly diffuse mirrors instead, because for a gray-gas system with refractory surfaces in radiative equilibrium such surfaces are the equivalent of perfectly diffuse mirrors. A little consideration shows, however, that diffuse and specular mirrors produce different distributions of flux,particlularly when the top and bottom refractory surfaces of the enclosure are significant. Because the scale of variation in image detail is small compared to W and particluarly because H/W is usually large, the assumption that the dotted lines are replaceable by refractory surfaces, thereby defining a chamber independent of the rest of the system, is probably a good one. But it provides two alternative proceedures between which to choose.

Step one is to replace the tubes by plane interrupted surfaces - vertical strips - which intercept the same radiation that the tubes would. Since most of the radiation is from the gas, the height of a replacing strip should be the tube pitch P (the center-to-center spacing) multiplied by the F_{gas} of Eq. (30) - the F_{iso} of Eq. (26), multiplied if warranted, by a correction factor from Fig. 8. (This will hereafter be called F, without subscript). Figures. 9(a) and (b) then become (a') and (b'). With the number of tubes in a screen equal to n, the total scren height is nP, generally a little less than H; and the area A_1 is $2nPF$ (all areas are per unit dimension normal to the plane of the illustration). The tabulation below gives, for each of the two cases, values of A_1, A_r, A_T, $\overline{lr}/A_T (\equiv F_{r1}A_r/A_T)$ for use in Eq. (15) or (18), and the transverse dimension involved in determining mean beam length L_m.

	Case (a')	Case (b')
A_1	$2nPF(\cong 2HF)$	$2nPF(\cong 2HF)$
A_r	$2(H+W)$	$2[H-nPF+W]$
$A_T=A_1+A_r$	$2(H+W)+nPF$	$2(H+W)$
$\left.\dfrac{\overline{lr}}{A_T}, \text{ clear-gas value} \atop \text{for use in Eq. (15)}\right\}$	$\dfrac{A_1F_{1r}}{A_T}=\dfrac{nPF}{H+W+nPF}\equiv C\ (=0=S=0)$	$\dfrac{A_1(1-F_{11})}{2(H+W)}=\dfrac{F\{nP-[\sqrt{(nP)^2+W^2}-W]F\}}{H+W}$
Basis for L_m, related to transmittances associated with;		
$\overline{s_rs_1}$	Based on $W/2$	Based on W
$\overline{s_1s_1}$	Inapplicable. $\overline{s_1s_1}=0$	Based on W
$\overline{s_rs_r}$	Based on W	Based on W

Some of the terms need explanation. For case (b'), the term $[\sqrt{(nP)^2+W^2}-W]F$ in the expression for $\overline{lr}/A_T$ comes from setting $\overline{lr}\,(\equiv A_1F_{1r})$ equal to $A_1(1-F_{11})$ and from visualizing A_1F_{11} to consist of the exchange-area between two continuous parallel planes of height nP and separeted by W$(=\sqrt{(nP)^2+W^2}-W)$, then multiplying once by F to allow for emitter area and once more to allow for receiver area.

The gas emissivity affects $\overline{GS}_1$ through its appearance in the the direct-exchange areas in Eq.(14). These may be written in terms of gas transmittance τ as follows, with primes indicating that gas absorption in present.

$$\begin{aligned} \overline{gs}_1 &= A_1-\overline{ll}'-\overline{lr}'=A_1-\overline{s_1s_1}\,\tau_{11}-\overline{s_1s_r}\,\tau_{1r} \\ \overline{gs}_r &= A_r-\overline{rl}'-\overline{rr}'=A_r-\overline{s_rs_1}\,\tau_{1r}-\overline{s_rs_r}\,\tau_{rr} \\ \overline{s_rs_1}' &= \overline{rl}'=\overline{s_rs_1}\,\tau_{1r} \end{aligned} \tag{32}$$

All exchange areas on the r.h.s. are clear-gas values; those in Eq. (14) include gas emission or absorption. It is seen that $\overline{GS}_1$ involves three different values of gas transmittance $\tau(\equiv 1-\alpha_g, = 1-\varepsilon_g$ if the gas is gray), differentiated by the subscripts identifying the surfaces at the two ends of a beam.

Case (b'). In this case the three τ's are identical, and Eq.(15) may be used directly, with ε_g based on a rectangular parallelepiped H high and W wide. Substitution of values from the table into (15) yields

$$\frac{1}{\overline{GS}_1}=\frac{1}{2nPF}\left[\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{\varepsilon_g\left(1+\cfrac{1}{\cfrac{nPF}{H+W-nPF}+\cfrac{\varepsilon_g}{1-\varepsilon_g}\,\cfrac{nP}{nP-F(\sqrt{(nP)^2+W^2}-W)}}\right)}\right] \tag{33}$$

As a numerical example, if $nP=H$, $W/H=2/3$, $\varepsilon_g=0.3$, $B=2$, and $F=0.6576\times$ $\times\ 1.02=0.67$ (use of Fig. 8), the above yields

$$\frac{1}{\overline{GS}_1}=\frac{1}{2nPF}\left[\left(\frac{1}{\varepsilon_1}-1\right)+1.91\right]$$

If $\varepsilon_1=0.9, \overline{GS}_1=2nPF/2.021$

Case (a'). Although two different τ's (and ε_g's) are involved, assume temporarily that a single value of ε_g may be used. Substitution from the table into Eq. (15) yields

$$\frac{1}{\overline{GS}_1}=\frac{1}{2nPF}\left[\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{\varepsilon_g\left(1+\cfrac{1}{\cfrac{nPF}{H+W}+\cfrac{\varepsilon_g}{1-\varepsilon_g}}\right)}\right] \tag{34}$$

The same numerical example gives

$$\frac{1}{\overline{GS}_1}=\frac{1}{2nPF}\left[\left(\frac{1}{\varepsilon_1}-1\right)+1.511\right]$$

If $\varepsilon_1=0.9$, $\overline{GS}_1=2nPF/1.622$, which is 24.5% highter than case (b').
Consider the other ε_g involved. τ_{r1} is based on the slab width $W/2$, τ_{rr} on W (the screen effect is allowed for in the F's). If the gas is gray and A_r is dominantly on the walls,

$$\tau_{rr}=\tau_{rl}\tau_{lr},\text{ or } [1-\varepsilon_g(W)]=[1-\varepsilon_g(W/2)]^2.$$

Then

$$\varepsilon_g(W/2)=1-\sqrt{1-\varepsilon_g(W)} \tag{35}$$

Introducing $\varepsilon_g(W/2)$ into (33) instead of $\varepsilon_g(W)$

$$\frac{1}{\overline{GS}_1}=\frac{1}{2nPF}\left[\left(\frac{1}{\varepsilon_1}-1\right)+2.286\right]$$

If $\varepsilon_1=0.9$, $\overline{GS}_1=2nPF/2.397$, which is 15.7 percent lower than case (b').
Clearly, the two bounds set by use of the limiting values of ε_g are too far apart to be useful, and it is necessary to go back to more basic Eq. (14). Either the substitution of exchange areas from (32) into (14) or the use of a different approach [7] yields

$$\frac{1}{\overline{GS}_1}=\frac{1}{A_1}\left[\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{1-\frac{\overline{ll}'}{A_1}-\frac{(\overline{lr}')^2/A_1}{A_r-\overline{rr}'}}\right] \tag{36}$$

From first principles and definitions

$$\overline{ll}'=A_1(1-F_{lr})\tau_{ll}$$

$$\overline{lr}')^2/A_1=A_1(F_{lr}\tau_{lr})^2$$

$$A_r-\overline{rr}'=A_r[1-(1-F_{rl})\tau_{rr}]=A_r(1-\tau_{rr})+A_lF_{lr}\tau_{rr}$$

Insertion of these into (36) yields

$$\frac{1}{\overline{GS}_1}=\frac{1}{A_1}\left[\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{1-(1-F_{lr})\tau_{ll}-\frac{(F_{lr}\tau_{lr})^2}{(A_r/A_l)+F_{lr}\tau_{rr}}}\right] \tag{37}$$

This is still general. For case (a'), from the relation before (35), $\tau_{lr}{}^2=\tau_{rr}=1-$ $-\varepsilon_g(W)$. In addition, $F_{1r}=1$ and $A_r/A_1=(H+W)/nPF$; and substitution into (37)

$$\frac{1}{\overline{GS}_1}=\frac{1}{A_1}\left[\left(\frac{1}{\varepsilon_1}-1\right)+\frac{1}{1-\frac{1}{1+\frac{H+W}{nPF}\,\frac{\varepsilon_g}{1-\varepsilon_g}}}\right]=\frac{1}{A}\left[\frac{1}{\varepsilon_1}+\frac{1-\varepsilon_g}{\varepsilon_g}\,\frac{npF}{H+W}\right] \tag{38}$$

Numerical substitution gives

$$\frac{1}{\overline{GS}_1}=\frac{1}{2nPF}\left(\frac{1}{\varepsilon_1}-1\right)+1.938$$

If $\varepsilon_1=0.9$, $\overline{GS}_1=2nPF/2.049$, which is but 1.3% lower than case (b'). The excellent agrement between Eqs. (33) and (38) is no measure of the error introduced by the diffuse-mirror assumption (As $W/H\to 0$, both (a') and (b') reduce to: $A_1/\overline{GS}_1=(1/\varepsilon_1)+(FnP/H)\,(1-\varepsilon_g)/\varepsilon_g)$. Because case (a'), Eq. (38), is simpler, its use is recommended. It is to be remembered that both derivations apply rigorously to two-dimensional systems; many reformer furnaces approximate that condition. Approximate allowance for the third both derivations apply rigorously to two-dimensional systems; many reformer furnaces approximate that condition. Approximate allowance for the third dimension comes from adding the end enclosure areas to A_r and adjusting $\overline{ll}$ in the conventioanal manner of allowing for opposed rectangles rather than opposed strips.

There remains the term ε_1, the effective emissivity of the strips which have replaced the tubes. The recesses formed by the tubes will give them an effective emissivity greater than that of plane metal. In the absence of gas, the interchange area between a tube row of true area A_2 and true emissivity of ε' and two parallel black plates on either side of it, of total area A_B and coextensive with the plane of the tubes, is given by

$$\frac{1}{\frac{1}{A_2}\left(\frac{1}{\varepsilon'}-1\right)+\frac{1}{A_BF}}$$

For black plates surrounding gray strips of emissivity ε_1 and area A_BF replacing the tubes, the interchange area is $A_BF\varepsilon_1$. The two interchange areas are then equated to find ε_1. Replacement of A_B/A_2 by $2B/\pi$, where B is the ratio of center-to-center distance to diameter, gives

$$\varepsilon_1 = \frac{1}{1 + \frac{2BF}{\pi}\left(\frac{1}{\varepsilon_l{}'} - 1\right)} \tag{39}$$

This is the effective emissivity of A_1. When $B = 2$ and ,$\varepsilon'_1 = 0.8$, $F = 0.66$ and $\varepsilon_1 = 0.084$.

Other tube arrangements. Since furnaces with one central vertical screen of horizontal tubes between refractory walls are the complete equivalent of case (a'), they do not present a new problem. If the central screen consists of two rows of tubes the only change is the increase in the value of the interception factor F defining the fraction of the central plane occupied by equivalent strips. If one cell consists of a fired section bounded by a refractory wall on one side and a tube screen on the other, with another cell on the other side of the tube screen, there is a lack of symmetry which makes the replacement of the tube row by mixed strips of refractory and sink not quite right; but such a model, if modified, is recommended. If, when calculations of the impinging performance of that cell and the one next to it have been completed, the flux densities onto the two sides of the tube row are different, there is then justification to for repeatining the calculations allowing for net flux between the two cells.

The statement has been made that the gray-plus-clear gas model is more realistic and by implication better than the gray-gas model; but the only solution given for the former was restricted to the speckled box-type furnace, Eq (24). It may be shown [2,h] that, if the constraint is put on the gray-plus-clear gas model that its refractory surfaces diffusely reflect all radiation incident on them (thereby failing to take advantage of shifting the incident gas radiation to the spectral windows of the gas on re-emission through gas to sink), the flux is obtained quite simply from the relation.

$$\overline{GS}\ \text{(gray-plus-clear-gas model, with refractory surface perfect diffuse reflectors)} = a_g \times (\overline{GS}\ \text{based on a gray gas of emissivity}\ \varepsilon_g/a_g) \tag{39}$$

Furthermore, the result for the real-gas real-refractory model-when it is available -, with $\varepsilon_r = 0.5$, always lies roughly half-way between (39) and that obtained for the simple gray-gas mode. The arithmetic mean of (39) and the gray-gas model is therefore suggested for those cases, such as the ones last discussed above, which are too complex to formulate rigorously. How important it is to make such a correction can only be established by correlation of a considerable body of furnace data.

Models (a') and (b') presented here have not been tested with furnace data. It will be especially necessary to allow for the temperature difference Δ in tube-screen furnaces with fuel fired between screens because of the absence of strong back-mixing with a reach comparable to the total gas-flow path. The fitting by Lobo and Evans [6] of a model similar to that based on Eq. (15) was done without the use of a Δ. Whether the striking success of that model-when fitted to and applied to box furnaces-indicated that Δ was in fact zero in such furnaces or whether there were compensating errors, such as the gray-gas assumption, was never established.It is reported that that model has in a number of cases been less than satisfactory in application to modern furnaces with interior tubes. The reason may lie in the incorrect evaluation of $\overline{GS}_1$.

A correlation of performance data on modern furnaces, using a model such as that herein described, is highly desirable.

NOMENCLATURE

A_0	area of furnace openings losing radiation
$A_r(A_p)$	area of refractory (of tube plane)
A_S	effective sink area (old $(\overline{GS})_{R,c}$)
A_1	area of stock or sink in furnace chamber. If tubes on wall, plane of tubes
$A_{1,e}$	that part of A_1 exclusive of any tube curtain across the gas-exit passage.
a_g	energy fraction of black-body spectrum occupied by gray gas in a gray plus clear mixture
B	ratio of center-to-center tube spacing to diameter
C	"cold" fraction of furnace enclosure are, $A_1/(A_1+A_r)$
$\overline{c}_{p,g}$	specific heat of combustion gases, mean value from gas-chamber exit to base temperature
D'	reduced firing density, $\dot{H}_F/\sigma A_S T_F^4(1-T_0')$
d	dimensionless constant in relation $\Delta' = (1-1/d)Q'$
$\varepsilon 3$	third exponential integral, $\int_1^\infty (e^{-xt}/t^3)dt$
B	black emissive power, σT^4
$\overline{F}$	factor for radiation loss through walls, based on inside and outside temperatures and area of opening
$\overline{F}_{\text{iso(gas)}}$	fraction of isotropic (gas) radiation intercepted by tube row, directly plus by interception of returning beam from background
F_{xy}	fraction of radiation from surface x which is intercepted by surface y
$\overline{g_x s_y}$	direct-interchange area between gas x and surface y
$\overline{G_x S_y}$	total-interchange area, ratio of net radiative flux between gas zone x and surface zone y, allowing for reflections products at all surfaces,to the difference in black emissive powers (σT^4) of x and y. Sometimes, allowance made for refractory aid without appending sub-R.
$(\overline{GS})_R$	total-interchange area between gas and surface zones, with aid given by equilibrium refractory surfaces included
$H(\overline{T_1})$	enthalpy of stream 1 (sink stream) dependent on temperature
$H_{1,i(o)}$	enthalpy of stream 1 at inlet (outlet)

H_F	enthalpy of any entering streams affecting firing rate, including fuel, air, and recirculated flue gas if any, above dead state of completely burned gaseous products at T_0.
h_i	convection heat-transfer coefficient on inside surface of refractory walls
$h_{c+r,0}$	heat transfer coefficient by convection plus radiation, on outside walls of refractory surfaces
K	absorption coefficient, l^{-1}
k	thermal conductivity of refractory walls
L	dimension of parallelepipes
L_m	mean beam length for gas radiation
L'_O	dimensionless loss coefficient for radiative flux through furnace openings, $\overline{F} A_0/A_s$
L'_r	dimensionless loss coefficient for heat loss through refractory walls, $U_r A_r/\sigma A_S T_F{}^3$
m_g	mass flow rate of combustion gases
n	number of tubes in a tube row
P	pitch of tubes in row, center-to-center distance
p	partial pressure of gas-radiating components, atm.
Q_G	rate of heat transfer from combustion gases
Q'	reduced rate of heat transfer from combustion gases, $(Q_G / \dot{H}_F)(1-T_0')$
r	ratio of temperature rise of stock surface to the sum of inlet and outlet temperatures, $(T_{1,o}-T_{1,i})/(T_{1,o}+T_{1,i})$.
S	speckledness; 1 for surface with A_1 and A_r intimately mixed
$\overline{s_x s_y}$	direct-interchange area for radiative exchange between surfaces x and y no gas absorption included
$\overline{s_x s_y}'$	same as above, except that gas absorption is included
$\overline{xy}\,(\overline{xy})'$	shorthand for $\overline{s_x s_y}$ $(\overline{s_x s_y}')$
T_F	adiabatic pseudo–flame temperature, based on Eq. following [2]
T_g	mean radiating temperature of combustion gases
T_1	mean temperature of stock or sink surface
T_0	base temperature (also ambient)
U_r	overall coefficient of heat transfer from combustion gases through refractory wall to ambient
W	refractory wall thickness. Also thickness of gas slab, Figs. 7-9
$\overline{I_r}$	shorthand for $\overline{s_1 s_r}$ $,\equiv A_1 F_{1r} \equiv A_r F_{r1}$
α	gas absorptivity
Δ	gas-radiating temperature minus gas temperature leaving combustion chamber
$\varepsilon_g(\varepsilon_1)$	gas emisivity (effective emissivity (emittance, absorptance) of surface A_1)
ε_1'	true emissivity of tube surface
n	furnace efficiency, (flux to stock or sink)/H_F
σ	Stefan-Boltzman constant
τ	gas transmissivity ($\equiv$ 1-absorptivity, $= 1-\varepsilon_g$ if gas gray)
ψ	angle. See Fig. 7

Subscripts

i,o	inlet, outlet
r	refractory
l	sink surface, or stock surface
T	total, applied to area

Primes

On T_g, T_1, T_o designate the ratio those temperatures to T_F.

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Author's comments, after 13 years, on the paper "First Estimates of Industrial Furnace Performance – the One – Gas – Zone Model Reexamined"

The dominant theme, combining an energy balance based on the gas – exit temperature T_E with a rigorously formulated heat transfer relation based on a mean gas – radiating temperature T_G through an empirical relation between T_E and T_G is, by hindsight, clouded by much chaff. A few desired changes or omissions:

1. Since the energy balance and heat transfer relations force the solution of a limited quartic in T_G, there is no particular merit in forcing the convection term $hA_1(T_G - T_1)$ into a fourth – power temperature expression for combining with $(GS_1)_R$.

2. The recommended relation between T_E and T_G could have been stated much more clearly: $T_G = AT_E + (1 - a)T_F$, with "a" in the neighborhood of 3/4 but increasing toward 1 as the firing rate goes down.

3. The attempted quantitative definition of "spleckledness" – material following Eq. 18 and running to two paragraphs before Eq. (23) – was an abberation, better forgotten. Arrangement of the sink and refractory surfaces of a furnace chamber in "speckled" vs. sink – in – a – plane form are not bounding arrangements determining the view factor F_{1R}. Speckledness is near the center of possible arrangements, and Eq. (24) is recommended for determining $(GS_1)_R$, except for metallurgical or glass furnaces where the sink is generally planar.