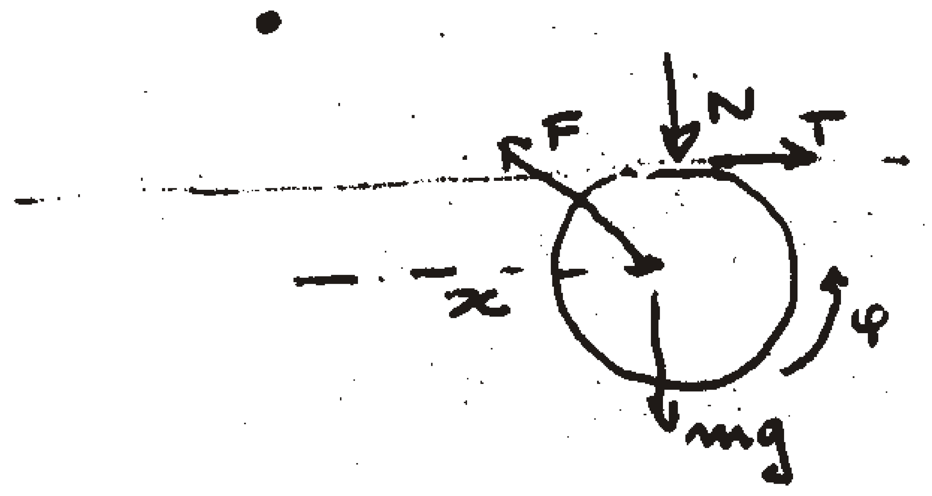
2º parcial MECÁNICA NEWTONIANA - 14/07/05

1)



$$\dot{x} = R\dot{\varphi}$$

$$0 = k(a+R) - N - mg$$

$$m\ddot{x} = -kx + T$$

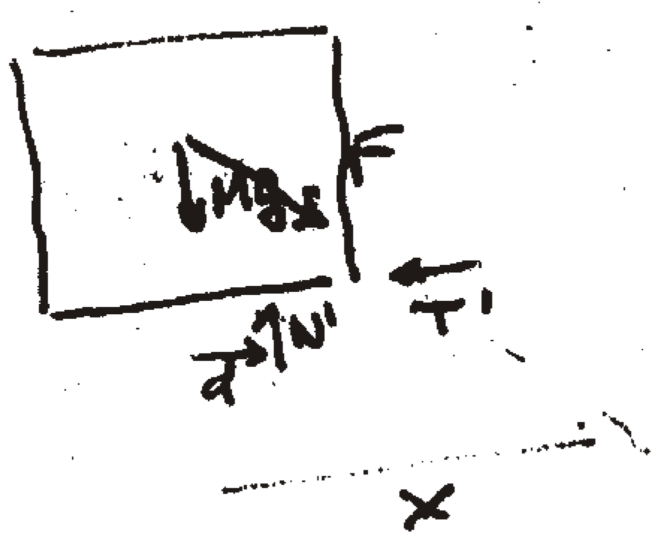
$$\frac{1}{2} m R^2 \ddot{\varphi} = -RT$$

$$\frac{3}{2} m \ddot{x} = -kx$$

a) i) $N = k(a+R) - mg$, $N > 0$, $\Rightarrow$ $k(a+R) - mg > 0$

ii) $\frac{3}{2} m\ddot{x} + kx = 0$, $\dot{x} = V_0 \cos\left(\sqrt{\frac{2k}{3m}}\, t\right)$, $x = V_0 \sqrt{\frac{3}{2}\frac{m}{k}}\ \mathrm{sen}\left(\sqrt{\frac{2}{3}\frac{k}{m}}\, t\right)$

b)



$$N' = Mg + k(a+R)$$

$$T' = kx$$

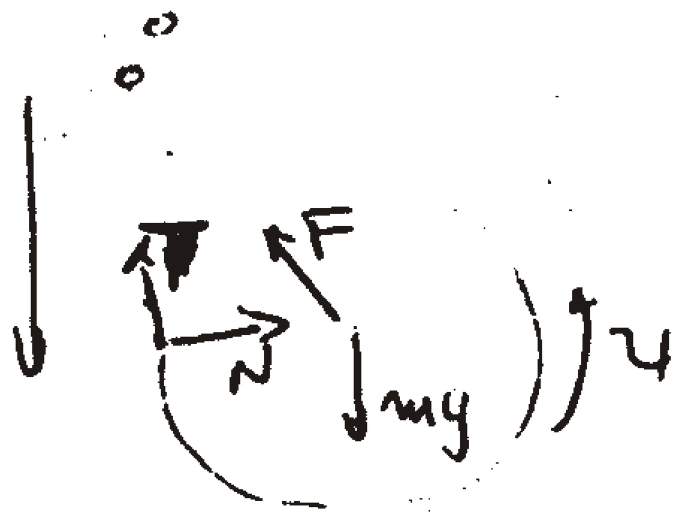
$$|T'| < f N' \ , \quad |x| \le V_0 \sqrt{\frac{3}{2}\frac{m}{k}}$$

i) $$f > \frac{V_0 \sqrt{\frac{3}{2} mk}}{Mg + k(a+R)}$$

ii) $d N' = kxa$, $d = \dfrac{kxa}{Mg + k(a+R)}$

$|d| < a$, $|x| \le V_0 \sqrt{\frac{3}{2}\frac{m}{k}}$, $\dfrac{V_0 \sqrt{\frac{3}{2} mk}}{Mg + k(a+R)} < 1$.

2) Inicialmente hay deslizamiento.



$\dot z(0)=v_0$, $\dot\varphi(0)=0$, $T=fN$

$$m\ddot z = -kz + mg - fN$$

$$-mr\omega^2 = -kr + N$$

$$\frac{1}{2} m r^2 \ddot\varphi = -r f N$$

$$N = \left(\frac{k}{m} - \omega^2\right) m r$$

a) i) $$\ddot z + \frac{k}{m} z = g - f\left(\frac{k}{m} - \omega^2\right) r$$

$$\ddot\varphi = -2f\left(\frac{k}{m} - \omega^2\right)$$

ii) $\frac{k}{m} - \omega^2 > 0$.

b) $f=1$, $\frac{k}{m} - \omega^2 = \frac{g}{r}$, $\ddot z + \frac{k}{m} z = 0$, $\ddot\varphi = -2 g/r$

$$\dot z = v_0 \cos\left(\sqrt{\tfrac{k}{m}}\, t\right) \qquad \dot\varphi = -2\frac{g}{r} t$$

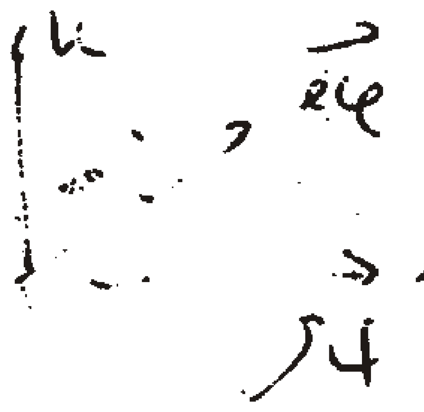
$$z = v_0 \sqrt{\tfrac{m}{k}}\, \mathrm{sen}\left(\sqrt{\tfrac{k}{m}}\, t\right) , \qquad \varphi = \varphi_0 - \frac{g}{r} t^2$$

c) Desliza mientras $\dot z + r\dot\varphi > 0$

Empieza a rodar sin deslizar en t_D que verifica la ecuación:

$$v_0 \cos\left(\sqrt{\tfrac{k}{m}}\, t_D\right) - \frac{2g}{r} t_D = 0$$

d)



$$\vec\omega_{disco} = \omega \vec k - \dot\varphi \vec e_\varphi$$

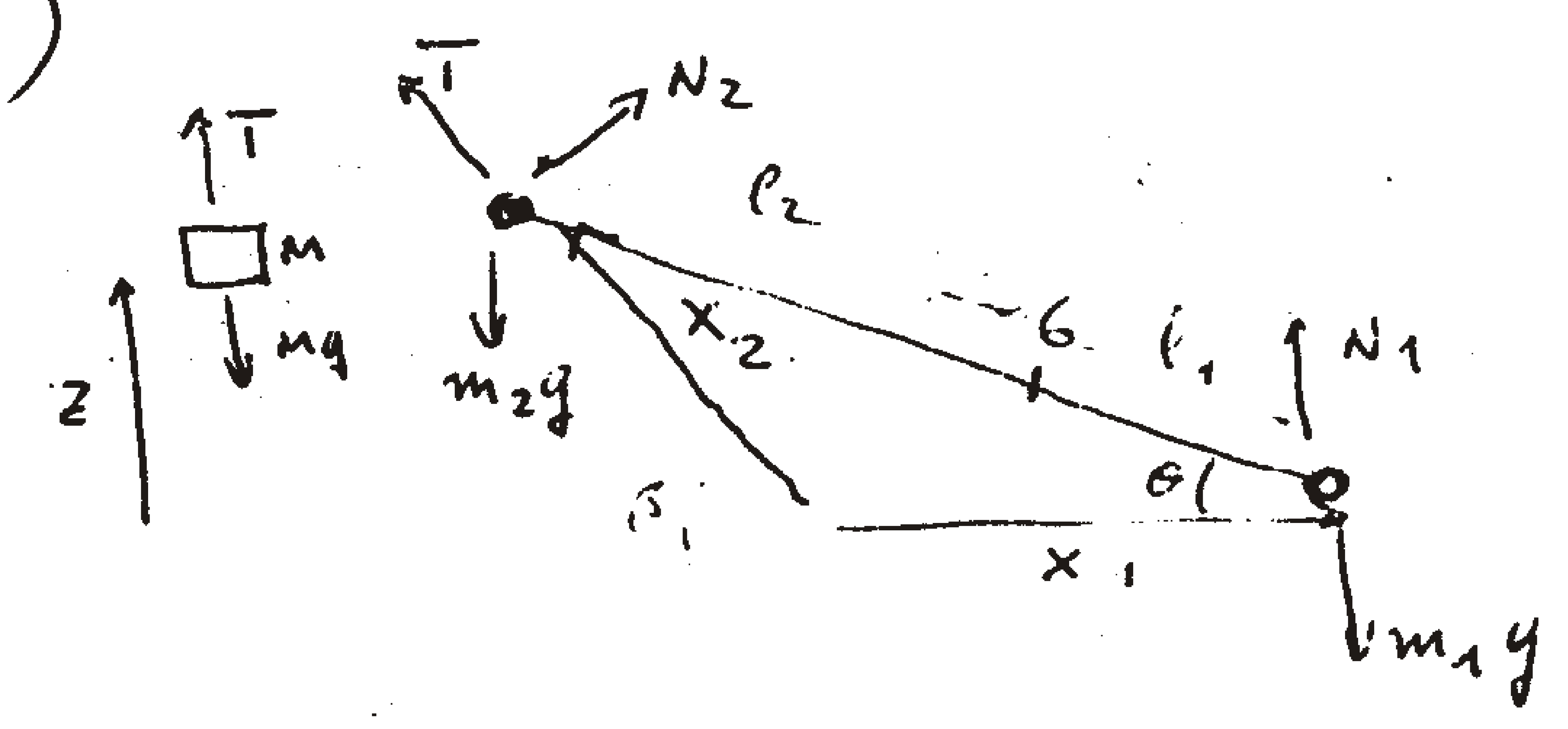
$$\vec L_G = \frac{1}{4} m r^2 \omega \vec k - \frac{1}{2} m r^2 \dot\varphi \vec e_\varphi$$

$$\dot{\vec L}_G = -\frac{1}{2} m r^2 \ddot\varphi \vec e_\varphi + \frac{1}{2} m r^2 \dot\varphi \omega \vec e_r$$

$$\dot{\vec L}_G = r T \vec e_\varphi + \vec M$$

$$\vec M = \frac{1}{2} m r^2 \dot\varphi \omega \vec e_r = - m g r \omega t \vec e_r$$

3)



a) $\ell_1 = 2\ell \dfrac{m_2}{m_1+m_2}$, $\ell_2 = 2\ell \dfrac{m_1}{m_1+m_2}$

$I = m_1 \ell_1^2 + m_2 \ell_2^2 = 4\ell^2 \dfrac{m_1 m_2}{m_1+m_2}$

$$\mathbb{I}_G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

b)

$$\dot z + \dot x_2 = 0$$

$$x_2 \operatorname{sen}\beta = 2\ell \operatorname{sen}\theta$$

$$x_1 = 2\ell\cos\theta - x_2\cos\beta$$

$$x_G = x_1 - \ell_1 \cos\theta$$

$$Y_G = \ell_1 \operatorname{sen}\theta$$

$$M\ddot z = T - Mg$$

$$(m_1+m_2)\ddot x_G = N_2 \operatorname{sen}\beta - T\cos\beta$$

$$(m_1+m_2)\ddot Y_G = N_2\cos\beta + T\operatorname{sen}\beta + N_1 - (m_1+m_2)g$$

$$I\ddot\theta = N_2 \ell_2 \cos(\beta-\theta) + T\ell_2 \operatorname{sen}(\beta-\theta) - \ell_1 N_1 \cos\theta$$

c) El sistema es conservativo.

$$U = Mgz + m_2 g\, 2\ell\operatorname{sen}\theta = 2g\ell\operatorname{sen}\theta\left[m_2 - \frac{M}{\operatorname{sen}\beta}\right] + cte.$$

$\dfrac{\partial U}{\partial\theta} = 0$, $0<\theta<\beta$ $\Rightarrow$ $M = m_2 \operatorname{sen}\beta$,

y en ese caso todas las configuraciones son de equilibrio

d) $T + U = E$

$$\frac{1}{2}M\dot z^2 + \frac{1}{2}m_1\dot x_1^2 + \frac{1}{2}m_2\dot x_2^2 + 2\left(m_2 - \frac{M}{\operatorname{sen}\beta}\right)g\ell\operatorname{sen}\theta = cte.$$